Madrid June 12-16

ILAS 2023

Abstracts' Book

25th Conference of the International Linear Algebra Society

June 12th - 16th, 2023

Escuela Técnica Superior de Ingenieros de Montes, Forestales y del Medio Natural
Universidad Politécnica de Madrid. Madrid, Spain

ILAS International Linear Algebra Society

Montes

POLITÉCNICA

UNIVERSIDAD POLITÉCNICA DE MADRID

25th Conference of the International Linear Algebra Society

ILAS 2023

Escuela Técnica Superior de Ingenieros de Montes, Forestales y del Medio Natural

Universidad Politécnica de Madrid

Madrid, Spain

June 12 - 16, 2023

Diseño, maquetación y corrección: Solana e Hijos, A.G., S.A.U.

ISBN.: 978-84-XXXXX-XX-X
Depósito Legal: M-XXXXX-2023
Imprime: Solana e Hijos, A.G., S.A.U.
Impreso en España / Printed in Spain

INTRODUCTION

Linear Algebra and Matrix Analysis are long established mathematical disciplines that play a fundamental role in modern applications. Matrices and Hypermatrices appear everywhere in data analysis, quantum computing, networks, and scientific computing. In addition, many fundamental mathematical problems are still open in these disciplines and many more arise every year.

In this scenario, the International Linear Algebra Society (ILAS) is a global professional organization solely dedicated to scientists, professionals and educators interested in Linear Algebra and its Applications.

Among many other activities, ILAS organizes conferences that, since 1989, have taken place over North America, Europe, Asia and South America. These conferences feature high profile speakers (including prize winners) that present the latest developments in Linear Algebra research along with its wide range of applications. These conferences provide an opportunity for linear algebraists all over the world to present their work and to interact with members of the community.

Local Organizing Committee

Luis Miguel Anguas (Saint Louis University)
Roberto Canogar (Universidad Nacional de Educación a Distancia)
Fernando De Terán (Universidad Carlos III de Madrid, Chair)
Froilán M. Dopico (Universidad Carlos III de Madrid)
Ana M. Luzón (Universidad Politécnica de Madrid)
Ana Marco (Universidad de Alcalá)
José Javier Martínez (Universidad de Alcalá)
Manuel A. Morón (Universidad Complutense de Madrid)
Raquel Viaña (Universidad de Alcalá)

Scientific Committee

Raymond H. Chan (City University of Hong Kong)
Fernando De Terán (Universidad Carlos III de Madrid, Chair)
Gianna Del Corso (Università di Pisa)
Shaun Fallat (University of Regina)
Heike Fassbender (Technische Universität Braunschweig)
Elias Jarlebring (KTH Stockholm)
Linda Patton (Cal Poly University)
Jennifer Pestana (University of Strathclyde)
João Queiró (Universidade de Coimbra)
Naomi Shaked-Monderer (Max Stern Yezreel Valley College)
Zdenek Strakos (Charles University)
Daniel Szyld (Temple University, ILAS President)
Raf Vandebril (KU Leuven, ILAS Vice-President for conferences)

Proceedings

LAA is proud to announce a special issue on the occasion of the 25th Conference of the International Linear Algebra Society (ILAS 2023). Papers corresponding to talks given at the conference should be submitted by December 1st 2023 via the Elsevier Editorial System. Special editors for this ILAS 2023 issue are:
Erin Carson (Charles University)
Fernando De Terán (Universidad Carlos III de Madrid)
Vanni Noferini (Aalto University)
João Queiró (Universidade de Coimbra)
Volker Mehrmann (TU Berlin) is the responsible Editor-in-Chief of LAA for this special issue.

Sponsors

We would like to thank the following organizations for their direct or indirect support of the Conference:

- ILAS sponsors the conference and provides support for reception cocktail and the overall budget. It also sponsors the Hans Schneider Prize (Nicholas J. Higham), the ILAS Taussky-Todd Prize (Stefan Güttel), and the ILAS Richard A. Brualdi Early Career Prize (Michael Tait).
- Elsevier has provided 5 grants for students' expenses and another 5 grants for Early Career speakers.
- SIAM sponsors the SIAG/LA speaker (Elias Jarlebring).
- Taylor & Francis supports the LAMA lecturer (Vanni Noferini).
- Universidad Complutense de Madrid provides housing at the students' dormitories.
- Universidad Nacional de Educación a Distancia (UNED).
- Universidad de Alcalá.
- Departamento de Física y Matemáticas de la Universidad de Alcalá.
- Sociedad Española de Matemática Aplicada (SeMA).
- Departamento de Matemáticas de la Escuela de Montes, Forestales y del Medio Natural de la Universidad Politécnica de Madrid.
- Saint Louis University.
- Universidad Carlos III de Madrid.
- Ayuntamiento de Madrid, through the Madrid Convention Bureau, provides support for the congress visit.
- Comunidad Autónoma de Madrid provides the congress kit (bag, pen, notebook, fan, and brochures) and support for the congress visit.

GENERAL INFORMATION

Instructions and suggestions for speakers

- All talks are 20 minutes long plus 5 minutes for questions and another 5 minutes for allowing attendees to move from one room to another.
- There is a computer with a projector in each room for your presentation.
- All rooms are equipped with white or blackboards.
- It is always a good idea to load your presentation in PDF format on a USB stick.

Instructions for chairs

- The chair of the session is responsible for the session to run smoothly.
- The organizers of the minisymposia (or those they delegate on) are expected to chair their sessions.
- The chairs are encouraged to be in the room early enough to check that all the speakers are present, that all video equipment is working as expected, and that all the speakers' presentations are ready.
- Please signal the speaker once the 20 minutes are almost over to allow some minutes for questions.
- If a speaker does not show up, please do not reschedule the other lectures in the session.

Welcome cocktail

There will be a reception cocktail on Monday, at 7pm, right after the end of the lectures. It will be held at the venue. All attendees (and accompanying persons) are welcome to the cocktail, which is included in the registration fee.

Lunches

As it is customary in ILAS Conferences, lunches are not included in the registration fee, and attendees are expected to get it on their own. Besides the canteen of the *Escuela de Forestales*, there are several lunch options near the conference venue.

Canteens:

- Canteen of the Faculty of Biological and Geological Sciences (*Facultad de Ciencias Biológicas y Geológicas*) of UCM. 3' by foot from the venue.
- Canteen of the *Centro de investigaciones biológicas Margarita Salas* (CSIC). 5' by foot from the venue.
- Canteen of the faculty of pharmacy (*Facultad de Farmacia*) of UCM. 7' by foot from the venue.
- Canteen of the faculty of medicine (*Facultad de Medicina*) of UCM. 11' by foot from the venue.
- Canteen of the faculty of journalism (*Facultad de Ciencias de la Información*) of the UCM. 10' by foot from the venue.

– Canteen of the school of agronomic engineering (ETSIAAB) of UPM. 14′ by foot from the venue.
– Canteen of the school of aeronautical engineering (ETSIAE) of UPM. 16′ by foot from the venue.

All canteens have vegan and vegetarian menus.

Restaurants:

There are several restaurants in C/ Almansa, about 14′ away (by foot) from the venue. Another options are:

– El Momento Gastro Bar, P.° de Juan XXIII, 22, 28040. 10′ by foot from the venue.
– Cafetería Mara, P.° de Juan XXIII, 15, 28040 Madrid (for a fast food). 9′ by foot from the venue.

Conference dinner

The conference dinner will start at 20:30 on Wednesday in La Masía de José Luis (P.° de la Prta del Ángel, 3, 28011). The dinner includes a welcome drink with appetizers and a three-course menu with beverages (wine and water included) and coffee. Those attendees having allergies or food restrictions that were not indicated when registering to the conference are encouraged to inform the organizers as soon as possible.

There will be an after-dinner speaker.

Excursion

There will be a guided tour on Wednesday, from 6:30pm to 7:30pm (approximately), through a neighborhood of Madrid city called the "Madrid de los Austrias" (the "Austrias" refers to the dynasty, a branch of the House of Habsburg, that ruled Spain within the XVI and XVII centuries). This tour is included in the registration fee, but only those who have replied affirmatively to the questionnaire will be allowed to participate.

ILAS does not discriminate on the basis of race, color, age, ethnicity, religion, national origin, pregnancy, sexual orientation, gender identity, genetic information, sex, marital status, or disability. If you have a concern regarding this, please contact the conference email: ilas2023@uc3m.es or the ILAS officers whose names are indicated at: https://ilasic.org/inclusiveness-statement/

ILAS2023: Program at a glance (contributed talks)

	Monday	Tuesday	Wednesday	Thursday
15:00-15:30	AULA 3. Spectral geometric mean, geometric mean, and Kantorovich constant. **Shigeru Furuichi.**	AULA 3. Linear dynamical systems for constructing observable convolutional codes with good decodable properties. **Noemí De Castro-García.**	AULA 3. Parallel-in-time solver for the all-at-once Runge-Kutta discretization. **Ángeles Martínez Calomardo.**	
15:00-15:30	AULA 5. Factorization of completely positive matrices by alternating minimization. **Harry Oviedo.**	AULA 5. Matrix Nearness Problems with Rank-Structured Positive Definite Matrices. **Luijing Chen.**	AULA 5. Dispersion Entropy for Graph Signals. **John Stewart Fabila Carrasco.**	AULA 5. Fast multiplication, determinants, inverses and eigendecomositions of arrowhead and diagonal-plus-rank-one matrices over associative fields. **Nevena Jakovčević Stor.**
15:00-15:30	AULA 6. Optimized Higher Order Dynamic Mode Decomposition Analysis of Electrocardiography Datasets. **Andrés Bell.**	AULA 6. Problems related to data analysis in non-Euclidean spaces: iterative filtering for signals defined on the sphere. **Roberto Cavassi.**		
15:00-15:30	AULA 7. The inverse Horn problem. **João Queiró.**	AULA 7. Combined matrices of diagonally equipotent matrices. **Rafael Bru.**	AULA 7. Sequences of lower and upper bounds for the spectral radius of a nonnegative matrix and applications. **Aikaterini Aretaki.**	AULA 7. On almost semimonotone matrices and the linear complementarity problem. **Bharat Pratap Chauhan.**
15:00-15:30	AULA 10. Bi-Additive models and Symmetry. **Sandra Ferreira.**	AULA 10. CP decomposition and low-rank approximation of antisymmetric tensors. **Erna Begovic.**	AULA 10. The Varchenko Determinant for Complexes of Oriented Matroids. **Sophia Keip**.	AULA 10. Stopping criteria for the coarsest-grid solver in multigrid V-cycle methods. **Petr Vacek.**
15:00-15:30	AULA 11. Monodromy group of decomposable Blaschke products of degree 2^n. **M. Eugenia Celorrio.**	AULA 11. Linear algebra in the category of linear systems. **Miguel Carriegos.**	AULA 11. Generating efficient vectors for pairwise comparison matrices. **Susana Furtado.**	AULA 11. The combinatory under hyperinvariant subspaces. **Eulalia Montoro.**
15:00-15:30	AULA 12. Signed graphs with maximum nullity two. **Marina Arav.**	AULA 12. On the max k-cut problem and the smallest signless Laplacian eigenvalue of a graph. **Leonardo de Lima.**	AULA 12. On the smallest positive eigenvalue of bipartite graphs. **Subhasish Behera.**	AULA 12. Wiener Index and Eccentricity after Edge contraction. **Joyentanuj Das.**
15:00-15:30	AULA 15. Design of an estimator with orthogonal projections for a linear regression model and its strong consistency. **Kensuke Aishima.**	AULA 15. Symmetrization Techniques in Image Deblurring. **Paola Ferrari.**	AULA 15. Partial Smoothness of the Numerical Radius at Matrices whose Fields of Values are Disks. **Michael Overton.**	AULA 15. The geometry of numerical ranges over finite fields. **Kristin Camenga.**
15:00-15:30	AULA 16. Doubly Structured Mappings and Backward errors for Matrix pencils arising in Optimal Control. **Mohit Kumar Baghel.**	AULA 16. Sturm-Liouville problem and linear transformation on eigenpolynomials. **Luis Miguel Angüas.**	AULA 16. New perturbation bounds for eigenvalues of quadratic eigenvalue problem for efficient damping optimization. **Ranjar Kumar Das.**	AULA 16. Properties of the shell of a square matrix and Shell-Extremal Eigenvalues. **Christos Chorianopoulos.**
15:00-15:30	AULA 6F. Total graphs of gain graphs. **Matteo Cavaleri.**	AULA 6F. Is there a Kemeny's constant for second-order random walks? **Dario Fasino.**	AULA 6F. Generating acyclic symmetric matrices with the minimum number of distinct eigenvalues. **Luiz Emilio Alem.**	AULA 6F. Rank distribution of graphs over the field of two elements. **Badriah Safarji.**

15:00-15:30	AULA SEMINARIOS. Verified error bounds for all eigenvalues and basis of invariant subspaces of a real symmetric matrix. **Shinya Miyajima.**	AULA SEMINARIOS. A defect-correction algorithm for quadratic matrix equations, with applications to quasi-Toeplitz matrices. **Beatrice Meini.**	AULA SEMINARIOS. Structured solutions of the reduced biquaternion matrix equations with applications. **Neha Bhadala.**	AULA SEMINARIOS. Quantum walk-based ranking algorithms for directed networks. **Paola Boito.**
15:30-16:00	AULA 3. New quantum divergence and barycenter with the spectral geometric mean. **Miran Jeong.**	AULA 3. Algebraic decoding for convolutional codes over modular integer rings. **Ángel Luis Muñoz Castañeda.**	AULA 3. How perturbations propagate along the solutions of linear ordinary differential equations: a relative error analysis. **Asma Farooq.**	
15:30-16:00	AULA 5. On some extensions of the class of Q-matrices. **Sushmitha P.**	AULA 5. Quasiseparable representations of Green matrices. **Yuli Eidelman.**	AULA 5. The (multivariate) Pascal matrix. **Helena Cobo.**	AULA 5. Linear preservers of semipositive matrices. **Sachindran Jayamaran.**
15:30-16:00	AULA 6. Challenges and opportunities in solving Navier-Stokes equations in patient-specific left heart model. **Mahesh Nagargoje**.	AULA 6. Rational approximation for the recovery of short exponential sums. **Nadiia Derevianko.**		
15:30-16:00	AULA 7. On Completely Mixed Matrix Games. **Gomatam Ravindran.**	AULA 7. The range of combined matrices and doubly stochastic matrices. **Begoña Cantó.**	AULA 7. Obtaining the Jordan structure of a totally nonnegative matrix from the Jordan structures of an upper block echelon matrix. **Rafael Cantó.**	AULA 7. Trifactorization of pattern symmetric nonnegative matrices. **Damjana Kokol Bukovšek.**
15:30-16:00	AULA 10. Orbit-injective Covariant Quantum Channels. Orbit-injective Covariant Quantum Channels. **Degunag Han.**	AULA 10. On Q-tensor. **Sunil Kumar.**	AULA 10. Matrices Similar to Centrosymmetric Matrices. **Rubén Martínez Avendaño.**	AULA 10. High-dimensional multi-view clustering. **Alaeddine Zahir.**
15:30-16:00	AULA 11. On interpolation with finite Blaschke products. **Sergei Kalmykov.**	AULA 11. Tropical Matrix Exponential. **Ali M. Askar**	AULA 11. Decompositions of matrices into torsion matrices and zero-square matrices. **Esther García.**	AULA 11. The Waring problem for matrix algebras. **Peter Semrl.**
15:30-16:00	AULA 12. No cycle-spliced bipartite signed graph with nullity $\eta(\Sigma)=c(\Sigma)$. **Suliman Khan.**	AULA 12. Existence of Characteristic-like Vertices on Trees with Matrix Weights. **Sumit Mohanty.**	AULA 12. Locating Eigenvalues of Unicyclic Graphs. **Rodrigo Braga.**	AULA 12. Topologically-induced suppression of explosive synchronization on graphs. **Manuel Miranda.**
15:30-16:00	AULA 15. Restarted pseudo-Lanczos bidiagonalization for the hyperbolic SVD. **Jose E. Román.**	AULA 15. Parallel High-Resolution Compact PFFT-Type Algorithms vs. Low-Dimensional Eigenvectors Solvers for 3D Subsurface Scattering Problems.. **Yuri Gryazin.**	AULA 15. Computational aspects related to Serre's reduction of underdetermined linear functional systems. **Mohamed Salah Boudellioua.**	AULA 15. On the numerical range of some structured matrices. **Rute Lemos.**
15:30-16:00	AULA 16. Canonical Forms for Strictly Regular Matrix Polynomials. **Richard Hollister.**	AULA 16. On a matrix perspective of Sobolev-type inner products and higher-order recurrence relations. **Edmundo J. Huertas.**	AULA 16. Optimizing the Rayleigh quotient with symmetric constraints and its application to perturbations of structured polynomial eigenvalue problems. **Anshul Parjapati.**	AULA 16. Geometric Estimates of Kernel Matrix Eigenvalues. **Mikhail Lepilov.**
15:30-16:00	AULA 6F. Complete resolution of the circulant nut graph order-degree existence problem. **Ivan Damnjanović.**	AULA 6F. On the directional derivative of Kemeny's constant. **Kate Lorenzen.**	AULA 6F. Number of non-isomorphic graphs obtained from a tree by switches. **Rosário Fernandes.**	AULA 6F. A low rank ODE for spectral clustering stability. **Stefano Sicilia.**

15:30-16:00	AULA SEMINARIOS. Old Song, New Verse -- Easier Spectral Questions via Algebraic Restrictions. **Jeffrey Stuart.**	AULA SEMINARIOS. Recursion formulas for determinants of k-Tridiagonal Toeplitz Matrices. **Eugene Agyei-Kodie**.	AULA SEMINARIOS. Recent Progress in GMRES-Based Iterative Refinement for Weighted and Generalized Least-Squares Problems. **Eda Oktay.**	AULA SEMINARIOS. Quantum Hitting Time According to a Given Distribution. **Gianna M. Del Corso.**
16:00-16:30	AULA 3. Multi-variable Wasserstein means of positive definite operators. **Vatsalkumar Mer.**	AULA 3. An Algorithm to Compute a Minimal Input-State-Output Representation of a Convolutional Code. **Verónica Requena.**	AULA 3. Characterization of a sparse problem with stochastic coefficients to solve elliptic BVPs. **Jorge Morón Vidal.**	
16:00-16:30	AULA 5. On a question of Bhatia, Friedland and Jain. **Mandeep Singh.**	AULA 5. Frames and Finite-rank Integral Representations of Positive Operator-Valued Measures. **Jean-Pierre Gabardo.**	AULA 5. Spread Code Constructions from Abelian Non-Cyclic Groups. **Xaro Soler-Escrivá.**	AULA 5. Determinants of some special matrices. **Yogesh Kapil.**
16:00-16:30	AULA 6. Developing an efficient aeronautical design tool using modal decomposition and deep learning for fluid dynamics analysis. **Ashton Ian Hetherington.**	AULA 6. Finite time horizon mixed control of vibrational systems. **Zoran Tomljanovic.**		
16:00-16:30	AULA 7. Approximation of the smallest eigenvalue of large hermitian matrices dependent on parameters. **Mattia Manucci.**	AULA 7. On combinatorial matrix majorizations. **Alexander Guterman.**	AULA 7. Simplifying the compensation criteria for the real nonnegative inverse eigenvalue problem. **Roberto Canogar.**	AULA 7. Learning Co-embedding for Multi-type Data based on Integrated Symmetric Nonnegative Matrix Factorization. **Haesun Park.**
16:00-16:30	AULA 10. Geometry of sub-algebras of Hol(Γ ∪ Int(Γ)) and zeros of holomorphic functions. **Babhrubahan Bose.**	AULA 10. Approximating manifold-valued functions. **Simon Jacobson.**	AULA 10. Jordan Structure and Stability of Schur Canonical Form. **Anastasiia Minenkova.**	AULA 10. NFFT in Parameter Learning for Nonlocal Image Denoising Models. **Andrés Miniguano Trujillo.**
16:00-16:30	AULA 11. Frame structure of Szego kernels in Hardy space of unit circle and Rational Approximation of ECG signals. **Anusree Sreedhara.**	AULA 11. Cyclic matrices, polynomial interpolation, and Sylvester equation over division rings. **Vladimir Bolotnikov.**	AULA 11. Self-dual polyhedral cones and their slack matrices. **João Gouveia.**	AULA 11. Linear maps preserving (p, k) norms of tensor products of matrices. **Run Zheng**.
16:00-16:30	AULA 12. A topological characterization of signed graphs with stable positive semidefinite maximum nullity at most two. **Hein van der Holst.**	AULA 12. Laplacian spectra of cographs: A twin reduction perspective. **Sane Umesh Reddy.**	AULA 12. Recovering the Spectrum of a Graph Having Most of its Eigenvalues Shared by a Vertex Deleted Subgraph. **Alexander Farrugia.**	AULA 12. Quantifying the Topological Stability of a Simplicial Complex. **Anton Savostyanov.**
16:00-16:30	AULA 15. Solving linear systems of the form $(A+\gamma UU^ T)\,{\bf x } = { \bf b}$. **Chiara Faccio.**	AULA 15. Computation of the von Neumann entropy of large matrices via trace estimators and rational Krylov methods. **Michele Rinelli.**	AULA 15. V-AISM, an Approximate Inverse LU Preconditioner. **José Mas.**	AULA 15. An envelope for the spectrum of a square matrix. **Panayiotis Psarrakos.**
16:00-16:30	AULA 16. Polynomial approximations for the matrix logarithm with computation graphs. **Jorge Sastre.**	AULA 16. Matrix version of a three-term recurrence formula with rational coefficients for q-Hermite Sobolev-type orthogonal polynomials. **Víctor Soto Iarrosa.**	AULA 16. An inexact matrix-Newton method for solving eigenvector-dependent nonlinear eigenvalue problems. **Tom Werner.**	AULA 16. Spaces of matrices with a bounded number of eigenvalues. **Klemen Sivic.**

16:00-16:30	AULA 6F. On the minimal least eigenvalues of circulant graphs. **Bašić Milan**.	AULA 6F. An interactive user-friendly software supporting research in graph theory. **Kristina Kostić**.	AULA 6F. A geometric construction of isospectral graphs for the discrete magnetic Laplacian. **Fernando Lledó.**	AULA 6F. Graph Degeneracy and Orthogonal Vector Representations. **Lon Mitchell.**
16:00-16:30	AULA SEMINARIOS. Homotopy method for singular multiparameter eigenvalue problems. **Zhijun Wang.**	AULA SEMINARIOS. Symbol-Based Convergence Analysis in (Block) Multigrid Methods for saddle-point problems. **Isabella Furci.**	AULA SEMINARIOS. On Strongly Infinitely Divisible Matrices. **Samir Mondal.**	AULA SEMINARIOS. Numerical Computation of Quantum Graph Spectra. **Anna Weller.**

EDIFICIO NUEVO (ANEXO)	EDIFICIO MONTES (PRINCIPAL)	AULARIO	EDIFICIO FORESTALES

ILAS 2023: Program at a glance (MS lectures)

	Monday	Tuesday	Wednesday	Thursday	Friday
11:10-11:40	AULA 15. A new low-rank solver for algebraic Riccati equations based on the matrix sign function and principal pivot transforms. **Peter Benner** (MSI04).	AULA 15. Numerical solution of a class of quasi-linear matrix equations. **Valeria Simoncini** (MSI04).	AULA 6. The power and the limits of visualizations. **Damjan Kobal** (MSC01).	AULA SEMINARIOS. Rigid spectra - a surprising consequence of invertible subtrees. **Seth Meyer** (MSI01).	AULA 10. Low-rank tensor frames for the high-accuracy solution of elliptic and parabolic PDEs. **Vladimir Kazeev** (MSI02).
11:10-11:40	AULA 5. On the minimum number of Toeplitz factors of a matrix. peaker: **Daniel Seco** (MSI05).	AULA 5. Contractive realizations of rational functions on polynomially defined domains and contractive determinantal representations of stable polynomials. **Victor Vinnikov** (MSI05).	AULA 3. Green's functions every where. **Enric Monsó** (MSC04).	AULA 10. Submatrices with the best-bounded inverses: revisiting the hypothesis. **Yuri Nesterenko** (MSI02).	AULA 5. Nonnegative polynomials, sums of squares and sums of nonnegative circuit polynomials - a story of three convex cones. **Moritz Schick** (MSI05).
11:10-11:40	AULA 6. Comparative Judgement and student engagement with proof writing in linear algebra. **Anthony Cronin** (MSC01).	AULA 6. Give an example of... **Rachel Quinlan** (MSC01).	SALÓN DE ACTOS. Quantum computing and graph theory. **Chi-Kwong Li** (MSC08).	AULA 5. Ranks of linear matrix pencils separate simultaneous similarity orbits. **Igor Klep** (MSI05).	AULA 16. On the Rellich eigendecomposition of para-Hermitian matrices on the unit circle. **Giovanni Barbarino** (MSC09).
11:10-11:40	AULA SEMINARIOS. On the spectra and algebraic connectivity of token graphs of a cycle. **Cristina Dalfó** (MSC02).	AULA SEMINARIOS. On classes of diminimal trees. **Carlos Hoppen** (MSC02).	AULA 16. Error representation of block rational Krylov methods by means of rational matrices. **Angelo A. Casulli** (MSC09).	AULA 3. Recovering piecewise constant conductances on networks with boundary. **Álvaro Samperio** (MSC04).	AULA 6F. Quantum concentration inequalities. **Daniel Stilck França** (MSC13).
11:10-11:40	AULA 3. Powers of Sparsest Matrices Realising the Karpelevic Arcs. **Priyanka Joshi** (MSC03).	AULA 11. Algebraic properties of operations on positive definite cones in operator algebras corresponding to various versions of Heron means. **Lajos Molnár** (MSC06).	AULA SEMINARIOS. Advanced Krylov Subspace methods with applications to Bayesian inverse problems. **Malena Sabaté Landman** (MSC10).	SALÓN DE ACTOS. Extremal Singular Graphs and Nut Graphs. **Irene Sciriha** (MSC08).	AULA 7. The Degree-Distance and Transmission-Adjacency Matrices. **Carlos Alfaro** (MSC14).
11:10-11:40	AULA 16. Stabilization of port-Hamiltonian systems by low rank output feedback. **Volker Mehrmann** (MSC05).	SALÓN DE ACTOS. Reminiscences of Steve Kirkland. **Richard A. Brualdi** (MSC08).	AULA 10. Numerical linear algebra aspects of the Dynamic Mode Decomposition. **Zlatko Drmac** (MSC12).	AULA 16. Linearization of meromorphic matrix-valued functions. **Rafikul Alam** (MSC09).	AULA 15. Linear systems of moment differential equations. **Alberto Lastra** (MSC16).
11:10-11:40	SALÓN DE ACTOS. Stochastic Matrices Realising the Boundary of the Karpelevic Region. **Helena Smigoc** (MSC08).	AULA 16. Spectral Localization and the Infinite Elementary Divisor Structure of Matrix Polynomials. **D. Steven Mackey** (MSC09).	AULA 6F. Recoverability of quantum channels via hypothesis testing. **Anna Jenčová** (MSC13).	AULA 7. Root-Max Problems, Hybrid Expansion-Contraction, and Optimization of Passive Systems. **Tim Mitchell** (MSC11).	AULA 12. Similarity via transversal intersection of manifolds. **Zhongshan Li** (MSC17).
11:10-11:40	AULA 10. On the Loewner framework for model reduction. **Athanasios C. Anthoulas** (MSC12).	AULA 3. Rational Krylov for Stieltjes matrix functions with Kronecker structure. **Leonardo Robol** (MSC15).	AULA 15. The Christoffel function: Some applications and connections. **Jean Bernard Lasserre** (MSC16).	AULA 6F. Entangled subspaces and their characterization. **Remigiusz Augusiak** (MSC13).	AULA 3. Updating a Sequence of Orthogonal Rational Functions. **Raf Vandebril** (MSC22).
11:10-11:40	AULA 6F. Diagonal Unitary and Orthogonal Symmetries in Quantum Theory. **Ion Nechita** (MSC13).	AULA 7. Riordan Group Involutions. **Luis Shapiro** (MSC18).	AULA 7. Natural partial orderings and associated Riordan poset matrices. **Gi-Sang Cheon** (MSC18).	AULA 15. Bernstein-Szegö measures in the plane. **Jeffrey Geronimo** (MSC16).	AULA 6. Completion of operator matrices with application to solving operator equations. **Dragana Cvetkovic-Ilic** (MSC25).
11:10-11:40	AULA 7. Vertical Recurrence Relation of Riordan Arrays, Quasi-Riordan Group and its Subgroups and Extended Subgroups. **Tian-Xiao He** (MSC18):	AULA 6F. Inevitability of Negative Quantum Conditional Entropy. **Gilad Gour** (MSC23).	AULA 5. Mixed-precision eigenvalue solver on GPUs. Toshiyuki Imamura (MSC21).	AULA 6. Blocks of endomorphism algebras via quasi-hereditary algebras. **Stephen Donkin** (MSC24).	AULA 11. Searching for Rigidity in Algebraic Starscapes. **Gabriel Dorsfman-Hopkins** (MSC26).
11:40-12:10	AULA 15. Chebyshev HOPGD for parameterized linear systems. **Siobhan Correnty** (MSI04).	AULA 15. Inexact low-rank ADI for large-scale Sylvester equations. **Patrik Kürschner** (MSI04).	AULA 6. The Challenges of Teaching Elementary Linear Algebra in a Modern Matrix Based Way. **Frank Uhlig** (MSC01).	AULA SEMINARIOS. New results on graph partition and Fiedler theory. **Enide Andrade** (MSI01).	AULA 10. Low-rank nonnegative matrix and tensor approximations: alternating projections and how to make them faster. **Stanislav Budzinskiy** (MSI02).
11:40-12:10	AULA 5. The wonders of the Cesàro matrix. **William T. Ross** (MSI05).	AULA 5. Hankel forms over a free monoid. **Michael T. Jury** (MSI05).	AULA 3. Capacity on graphs: submodularity and simplex geometry. **Karel Devriendt** (MSC04).	AULA 10. Superfast iterative refinement of Low Rank Approximation of a Matrix. **Victor Pan** (MSI02).	AULA 5. Projection Theorems in Free Semialgebraic Geometry. **Tim Netzer** (MSI05).
11:40-12:10	AULA 6. The structure and nature of linear algebra. **Sepideh Stewart** (MSC01).	AULA 6. A study of quadratic forms in Linear Algebra with GeoGebra. **André Lucio Grande** (MSC01).	SALÓN DE ACTOS. Pretty good state transfer among large set of vertices. **Ada Chan** (MSC08).	AULA 5. Clark measures associated with RIFs. **Linus Bergqvist** (MSI05).	AULA 16. Computing the nearest (structured) singular matrix polynomial. **Miryam Gnazzo** (MSC09).
11:40-12:10	AULA SEMINARIOS. Cospectrality results on generalized Johnson and Grassmann graphs. **Robin Simoens** (MSC02).	AULA SEMINARIOS. On Sidorenko's conjecture for determinants and Gaussian Markov random fields. **Peter Csikvari** (MSC02).	AULA 16. Error analysis of compact Arnoldi methods for linearized polynomial eigenvalue problems. **Javier Pérez** (MSC09).	AULA 3. A Riesz Decomposition Theorem for Schrödinger Operators on Graphs. **Florian Fischer** (MSC04).	AULA 6F. Thermalization in quantum spin systems. **Antonio Pérez-Hernández** (MSC13).
11:40-12:10	AULA 3. Connecting the Hermite-Biehler Theorem to the Nonnegative Inverse Eigenvalue Problem. **Richard Ellard** (MSC03).	AULA 11. Non-homogeneous gradient equations for sum of squares of Wasserstein metric. **Jinmi Hwang** (MSC06).	AULA SEMINARIOS. Matrix-Free Hyperparameter Optimization for Gaussian Processes. **Theresa Wagner** (MSC10).	SALÓN DE ACTOS. Limit points of Laplacian spectral radii of graphs. **Vilmar Trevisan** (MSC08).	AULA 7. Cospectral graphs by edge deletion. **Chris Godsil** (MSC14).
11:40-12:10	AULA 16. Eigenvalues of rank one perturbations of singular M-matrices. **André Ran** (MSC05).	SALÓN DE ACTOS. Rank one perturbations for cone reachability and holdability. **Michael Tsatsomeros** (MSC08).	AULA 10. One can hear the impedance and loss profiles of a string: from the discrete to continuum dissipative inverse problem. **Vladimir Druskin** (MSC12).	AULA 16. Rational approximation and linearisation for nonlinear eigenvalue problems and nonlinear systems. **Karl Meerbergen** (MSC09).	AULA 15. A matrix approach to the linearization and connection coefficients of orthogonal polynomial sequences. **Luis Verde-Star** (MSC16).

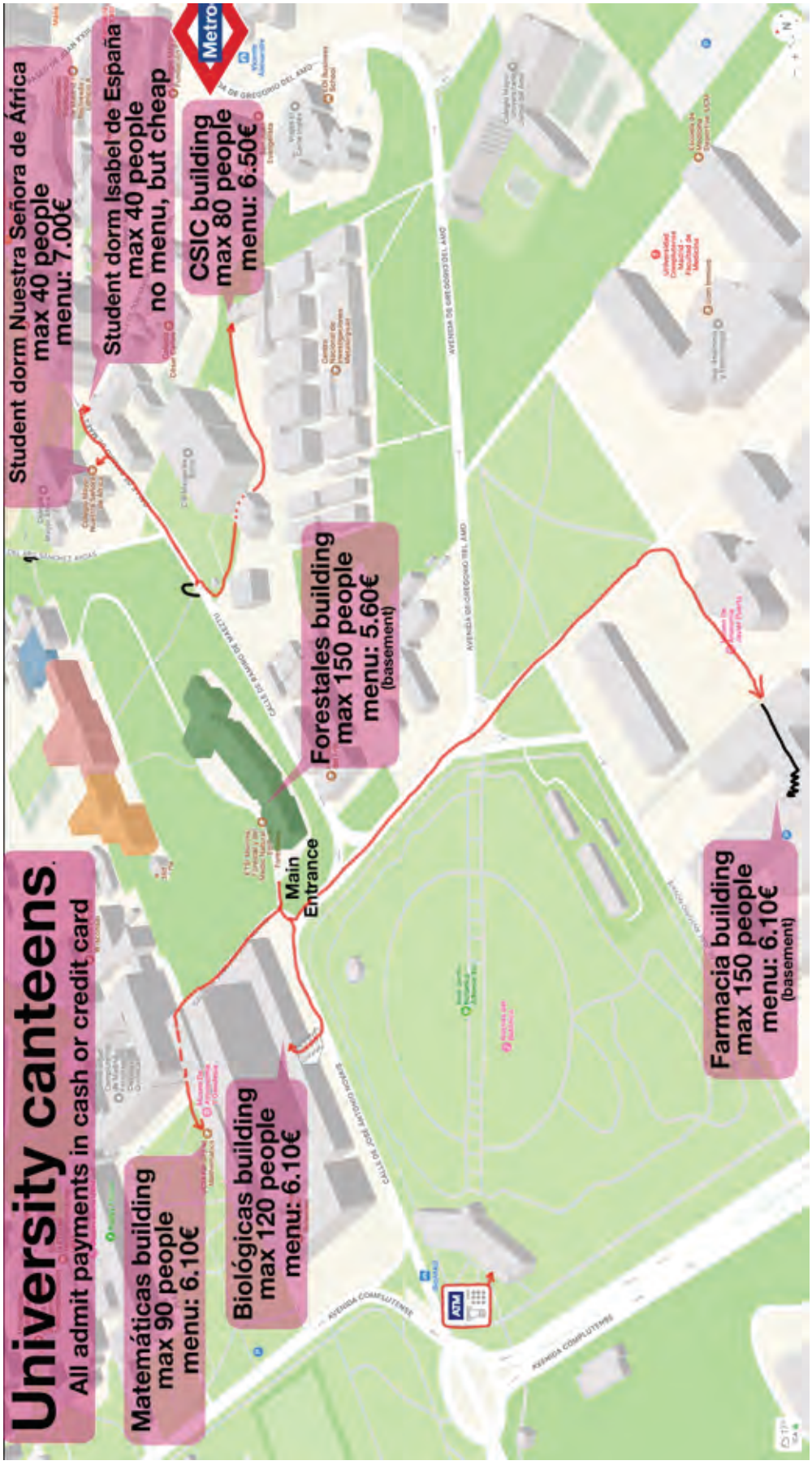
11:40-12:10	SALÓN DE ACTOS. On Kemeny's constant and its applications. **Emanuele Crisostomi** (MSC08).	AULA 16. Filters connecting spectrally equivalent nonsingular polynomial matrices. **Silvia Marcaida** (MSC09).	AULA 6F. Recoverability of quantum Fisher information. **Haojian Li** (MSC13).	AULA 7. Locating Eigenvalues of Quadratic Matrix Polynomials. **Shreemayee Bora** (MSC11	AULA 12. The bifurcation lemma for strong properties in the inverse eigenvalue problem of a graph. **Jephian C.-H. Lin** (MSC17).
11:40-12:10	AULA 10. From matrix equations to surrogate models. **Jens Saak** (MSC12).	AULA 3. Sketched and truncated polynomial Krylov methods for matrix equations. **Marcel Schweitzer** (MSC15).	AULA 15. Lax-type pairs in the theory of bivariate orthogonal polynomials. **Teresa E. Pérez** (MSC16).	AULA 6F. A complete hierarchy of linear systems for certifying quantum entanglement of subspaces. **Benjamin Lovitz** (MSC13).	AULA 3. Algorithmic aspects of the Bessmertnyı realization theorem for multivariate rational matrix functions. **Aaron Welters** (MSC22).
11:40-12:10	AULA 6F. Positive maps and entanglement in real Hilbert spaces. **Mizanur Rahaman** (MSC13).	AULA 7. Exponential Riordan matrices and decomposition of Hankel matrices. **Emanuele Munarini** (MSC18).	AULA 7. Abstract cell complexes and Riordan matrices. **Luis-Felipe Prieto-Martínez** (MSC18).	AULA 15. Time-and-band limiting for exceptional orthogonal polynomials. **Mirta M. Castro Smirnova** (MSC16).	AULA 6. The η-(anti-)Hermitain solution to a constrained Sylvester-type matrix equation over the generalized commutative quaternions. **Qing-Wen Wang** (MSC25).
11:40-12:10	AULA 7. The binary Pascal matrix and associated algebras. **Nikolaos Pantelidis** (MSC18).	AULA 6F. Measurement sharpness and incompatibility as quantum resources. **Francesco Buscemi** (MSC23).	AULA 5. Acceleration of iterative refinement for symmetric eigenvalue decomposition with clustered eigenvalues. **Yuki Uchino** (MSC21).	AULA 6. Exact Borel subalgebras of stratified algebras. **Teresa Conde** (MSC24).	AULA 11. Eigenvectors of the block Kronecker formulation of Mandelbrot matrices. **Piers W. Lawrence** (MSC26).
12:10-12:40	AULA 15. A mixed-precision algorithm for the Sylvester equation. **Massimiliano Fasi** (MSI04).	AULA 15. Deflating subspaces of palindromic pencils and the T-Riccati matrix equation. **Bruno Iannazzo** (MSI04).	AULA 6. Linear Algebra teaching in engineering degrees. **Marta Peña** (MSC01).	AULA SEMINARIOS. Integer eigenvalues of n-Queens graph. **Inês Serôdio Costa** (MSI01).	AULA 10. Tensor product algorithms for Bayesian inference of networks from epidemiological data. **Dmitry Savostyanov** (MSI02).
12:10-12:40	AULA 5. Packages of curves associated with the numerical range. **Pamela Gorkin** (MSI05).	AULA 5. Realizations of rational inner functions in the full Fock space. **Robert T. W. Martin** (MSI05).	AULA 3. Generalized diffusion equation on graphs/networks. **Fernando Díaz-Díaz** (MSC04).	AULA 10. Adaptive Undersampling in Spectromicroscopy. **Oliver Townsend** (MSI02).	AULA 5. Positivity of state polynomials with applications. **Jurij Volcic** (MSI05).
12:10-12:40	AULA 6. Proble solving before instruction. **Avi Berman** (MSC01).	AULA 6. Linear Algebra Education Reform, A Retrospective. **Steven J. Leon** (MSC01).	SALÓN DE ACTOS. Fractional revival on graphs. **Xiaohong Zhang** (MSC08).	AULA 5. Facial structure of matrix convex sets. **Tea Strekelj** (MSI05).	AULA 16. Nearest singular pencil via Riemannian optimization. **Lauri Nyman** (MSC09).
12:10-12:40	AULA SEMINARIOS. Constructing cospectral hypergraphs. **Antonina Khramova** (MSC02).	AULA SEMINARIOS. Spectra of trees. **Thomas Jung** (MSC02).	AULA 16. Rectangular multiparameter eigenvalue problems. **Bor Plestenjak** (MSC09).	AULA 3. Group Inverse and equilibrium measure on Random Walks. **Álvar Martín-Llopis** (MSC04).	AULA 6F. Spectral gap for AKLT models on arbitrary decorated graphs. **Angelo Lucia** (MSC13).
12:10-12:40	AULA 3. Nonnegative Jacobi matrix realizations in low dimension. **Andrés M. Encinas** (MSC03).	AULA 11. Geometric means on some matrix manifolds. **Luis Machado** (MSC06).	AULA SEMINARIOS. Probabilistic Rounding Error Analysis in Numerical Linear Algebra. **Nicholas J. Higham** (MSC10).	SALÓN DE ACTOS. Smallest positive eigenvalue of graphs. **Sasmita Barik** (MSC08).	AULA 7. Phantom mates of strongly cospectral vertices. **Krystal Guo** (MSC14).
12:10-12:40	AULA 16. Rank one perturbations of matrices with applications in graph theory. **Michal Mojtylak** (MSC05).	SALÓN DE ACTOS. Refined inertias of full and hollow positive sign patterns. **Minerva Catral** (MSC08).	AULA 10. Randomized POD-Beyn algorithm for nonlinear eigenvalue problems – analysis and perspectives. **Luka Grubisic** (MSC12).	AULA 16. Computing zeros of rational functions and matrices. **María C. Quintana** (MSC09).	AULA 15. Eigenvalues of infinite Hermitian matrices and Sobolev orthogonal polynomials. **Carmen Escribano** (MSC16).
12:10-12:40	SALÓN DE ACTOS. Markov chains: theory and applications. **Jane Breen** (MSC08).	AULA 16. Isomorphisms between Ansatz Spaces over Classical Polynomial Bases. **Vasilije Perovic** (MSC09).	AULA 6F. Monogamy of entanglement between cones and DPS-like hierarchies. **Martin Plávala** (MSC13).	AULA 7. Large-Scale Minimization of the Pseudospectral Abscissa. **Emre Mengi** (MSC11).	AULA 12. The liberation set of a graph. **Polona Oblak** (MSC17).
12:10-12:40	AULA 10. Data-driven balancing: what to sample for different types of balanced reduced models. **Serkan Gugercin** (MSC12).	AULA 3. Quantum Krylov Methods: What's the Deal?. **Roel Van Beeumen** (MSC15).	AULA 15. Discrete Darboux Transformations Leading to Nonstandard Orthogonality. **Maxim Derevyagin** (MSC16).	AULA 6F. Refuting spectral compatibility of quantum marginals. **Felix Huber** (MSC13).	AULA 3. Structured Matrices Approach for Legendre Pairs. **Ilias Kotsireas** (MSC22).
12:10-12:40	AULA 6F. When are quantum states antidistinguishable? **Jamie Sikora** (MSC13).	AULA 7. Combinatorics on the negative part of Riordan matrices. **Minho Song** (MSC18).	AULA 7. From Alexandroff spaces to Riordan matrices. **Pedro J. Chocano** (MSC18).	AULA 15. On generating Sobolev orthogonal polynomials. **Niel van Buggenhout** (MSC16).	AULA 6. Singular value decomposition of commutative quaternion tensors. **Yang Zhang** (MSC25).
12:10-12:40	AULA 7. Total positivity of Riordan arrays. **Roksana Slowik** (MSC18).	AULA 6F. A new distance between pure states of qudits. **Tomasz Miller** (MSC23).	AULA 5. GEMM-based numerical algorithm for accurate matrix multiplication. **Ozaki Katsuhisa** (MSC21).	AULA 6. Compression of bounded complexes and Auslander-Reiten sequences. **María José Souto Salorio** (MSC24).	AULA 11. Numerical Examples on Backward Stability of Algebraic Linearizations. **Eunice Y. S. Chan** (MSC26).
12:40-13:10	AULA 15. Efficient iterative methods for the solution of Generalized Lyapunov Equations: Block vs. point Krylov projections, and other controversial decisions. **Daniel Szyld** (MSI04).	AULA 15. Compress-and-restart block Krylov subspace methods for Sylvester matrix equations. **Kathryn Lund** (MSI04).	AULA 6. Magic tricks as a source of examples in Linear Algebra. **Fernando Blasco** (MSC01).	AULA SEMINARIOS. Approximate Graph Colouring and Crystals. **Lorenzo Ciardo** (MSI01).	AULA 10. Deep Importance Sampling Using Tensor Approximations. **Sergey Dolgov** (MSI02).
12:40-13:10	AULA 5. A moment theoretic approach to estimate the cardinality of certain algebraic varieties. **Raúl E. Curto** (MSI05).	AULA 5. Spectrahedral Shadows and Completely Positive Maps on Real Closed Fields. **Mario Kummer** (MSI05).	AULA 3. Spectral Gap Problems of Periodic Jacobi Operators. **V B Kiran Kumar** (MSC04).	AULA 10. A Nyström-like randomized algorithm for low-rank approximation of tensors. **Alberto Bucci** (MSI02).	AULA 16. Computing a compact local Smith McMillan form. **Paul Van Dooren** (MSC09).
12:40-13:10	AULA 6. Exciting Eigenvectors: Seeing is Believing. **D. Steven Mackey** (MSC01).	AULA 6. Virtual reality for the teaching of linear geometry. **José L. Rodríguez** (MSC01):	SALÓN DE ACTOS. Perfect state transfer on trees with small diameter. **Steve Kirkland** (MSC08).	AULA 5. Free Extreme points of free spectrahedrops and generalized free spectrahedra. **Eric Evert** (MSI05).	AULA 7. Coalescing sets for a cospectral construction. **Joel Jeffries** (MSC14).

12:40-13:10	AULA 3. A combinatorial characterization of C-realizable lists in the nonnegative inverse eigenvalue problem. **Julio Moro** (MSC03).	AULA SEMINARIOS. Algebraic connectivity of maximal outerplanar graphs. **Claudia M. Justel** (MSC02).	AULA 16. Eigenvector error bounds and perturbation for nonlinear eigenvalue problems. **Françoise Tisseur** (MSC09).	SALÓN DE ACTOS. Some bounds for the energy of complex unit gain graphs. **Rajesh Kannan** (MSC08).	AULA 15. A matrix approach to bounded point evaluation and zeros of Sobolev orthogonal polynomials. **Raquel Gonzalo** (MSC16).
12:40-13:10	AULA 16. Solving singular generalized eigenvalue problems: perturbation, projection and structure preservation. **Christian Mehl** (MSC05).	AULA 11. Operator means of positive definite compact operators and their properties. **Sushil Singla** (MSC06).	AULA SEMINARIOS. Mixed precision randomized Nyström approximation. **Erin C. Carson** (MSC10).	AULA 16. Randomized sketching of nonlinear eigenvalue problems. **Daniel Kressner** (MSC09).	AULA 11. Bohemian Doubly Companion Matrices. **Robert M. Corless** (MSC26).
12:40-13:10	SALÓN DE ACTOS. Kemeny's constant and Braess edges. **Sooyeong Kim** (MSC08).	SALÓN DE ACTOS. A Short Survey on the Scrambling Index of Primitive Digraphs. **Mahmud Akelbek** (MSC08).	AULA 6F. Abstract cone systems. **Mirte van der Eyden** (MSC13).	AULA 7. Optimal Rational Matrix Function Approximation Using the Arnoldi Algorithm. **Anne Greenbaum** (MSC11).	
12:40-13:10	AULA 10. An Eigensystem Realization Algorithm for Continuous-Time Systems and Its Connection with the Hankel Operator. **Igor Pontes Duff** (MSC12).	AULA 16. Diagonalizable Matrix Polynomials. **Ion Zaballa** (MSC09).	AULA 15. Inverse Darboux transformations and Sobolev inner products. **Francisco Marcellán** (MSC16).	AULA 6F. Mutually unbiased measurements andtheir applications in quantum information. **Máté Farkas** (MSC13).	
12:40-13:10	AULA 6F. Free spectrahedra in quantum information theory. **Andreas Bluhm** (MSC13).	AULA 3. A new Legendre polynomial approach for computing the matrix exponential. **Shazma Zahid** (MSC15).	AULA 7. Combinatorial statistics on Catalan words avoiding consecutive patterns. **José L. Ramírez** (MSC18).	AULA 6. Shintani descent for supercharacters of finite algebra groups. **Carlos André** (MSC24).	
12:40-13:10	AULA 7. TBA. **Ana Luzón/Manuel A. Morón** (MSC18).	AULA 7. Properties of Riordan quotients. **Paul Barry** (MSC18).	AULA 5. Robust iterative solvers. **Roman Iakymchuk** (MSC21).		
12:40-13:10		AULA 6F. Quantum Wasserstein semi-distances and applications. **Michal Eckstein** (MSC23).			
17:00-17:30	AULA 5. A semidefinite program for least distortion embeddings of flat tori into Hilbert spaces. **Marc Christian Zimmerman** (MSI03).	AULA SEMINARIOS. Semidefinite and eigenvalue bounds for bicliques and biidependent sets. **Luis Felipe Vargas** (MSC02).		AULA SEMINARIOS. On weight partitions of graphs and their applications. **Aida Abiad** (MSI01).	
17:00-17:30	AULA 15. On a new family of low-rank algorithms for large-scale algebraic Riccati equations. **Heike Fassbender** (MSI04).	AULA 16. Generic skew-symmetric matrix polynomials with bounded rank and fixed even grade. **Andril Dmytryshyn** (MSC05).		AULA 10. A simple yet effective tensor-based ODE model for Deep Learning. **Davide Palitta** (MSI02).	
17:00-17:30	AULA SEMINARIOS. On the spectra of weighted digraphs. **Miriam Pisonero** (MSC02).	AULA 11. Approximation results for generalized operator means. **Miklós Pálfia** (MSC06).		AULA 16. Jordan-like decompositions of linear relations. **Henrik Winkler** (MSC05).	
17:00-17:30	AULA 3. Universal Realizability on the border. **Carlos Marijuán** (MSC03).	AULA 6. Rational extrapolation methods, Anderson acceleration, and solution of systems of equations. **Claude Brezinski** (MSC07).		AULA 6. The Short-term Rational Lanczos Method and Applications. **Stefano Pozza** (MSC07).	
17:00-17:30	AULA 16. Minimal rank factorizations of low rank polynomial matrices. **Froilán M. Dopico** (MSC05).	AULA 12. Minimum rank bounds for cobipartite graphs and zero-nonzero patterns. **Louis Deaett** (MSC17).		AULA 7. Model Order Reduction in Gas Network Simulation and the Role of Eigenvalues. **Sara Grundel** (MSC11).	
17:00-17:30	AULA 6F. Matrix/Operator Mean Lagniappe. **Jimmie Lawson** (MSC06).	AULA 7. Accurate eigenvalues of some generalized sign regular matrices via relatively robust representations. **Rong Huang** (MSC19).		AULA 3. A $\star$-product solver for linear nonautonomous fractional differential equations. **Fabio Durastante** (MSC15).	
17:00-17:30	AULA 6. Efficient computation of the Wright function. **Lidia Aceto** (MSC07).	AULA 5. Fan-Theobald-von Neumann systems. **Muddappa Gowda** (MSC20).		AULA 15. Spectral theory for bounded banded matrices with positive bidiagonal factorization and mixed multiple orthogonal polynomials. **Ana Foulquié-Moreno** (MSC16).	
17:00-17:30	AULA 10. Optimal reduced-order modeling for structured linear systems. **Petar Mlinaric** (MSC12).	AULA 10. A Low-complexity Algorithm in Navigating Unmanned Aerial Systems. **Sirani M. Perera** (MSC22).		AULA 6F. Hyperdeterminant, Fermionic Fock space and entanglement. **Frédéric Holweck** (MSC23).	
17:00-17:30	AULA 12. On the number of distinct eigenvalues allowed by a sign pattern. **Kevin Vander Meulen** (MSC17).	AULA 6F. Apolarity for border rank and applications. **Jaroslaw Buczynski** (MSC23).		AULA 5. Representation theory of quantum algebras at roots of unity through linear algebra techniques. **Stéphane Launois** (MSC24)	
17:00-17:30	AULA 7. Some optimal properties related to Total Positivity. **Juan Manuel Peña** (MSC19).	AULA 15. On Eigenvalue Gaps of Integer Matrices. **Jamie Pommersheim** (MSC26).		AULA 11. Galois group actions and rational solutions of p(X) = A. **Gerrit Goosen** (MSC25).	
17:30-18:00	AULA 5. The Difference-of-Convex Algorithm and Quantum Conditional Entropy. **Oisín Faust** (MSI03).	AULA SEMINARIOS. Probing the Structure of Graph Nullspaces with Zero Loci. **Joshua Cooper** (MSC02).		AULA SEMINARIOS. Combinatorics behind signed graphs. **Milica Andelic** (MSI01).	
17:30-18:00	AULA 15. On computing modified moments for half–range Hermite and Pollaczek–Hermite weights in floating point arithmetic. **Nicola Mastronardi** (MSI04).	AULA 16. Generic Hermitian matrix pencils with bounded rank. **Fernando De Terán** (MSC05).		AULA 10. A statistical POD approach for feedback boundary optimal control in fluid dynamics. **Luca Saluzi** (MSI02).	
17:30-18:00	AULA SEMINARIOS. Perfect state transfer in quantum walks on orientable maps. **Vincent Schmeits** (MSC02).	AULA 11. Regression on the manifold of fixed rank positive semidefinite matrices. **Hosoo Lee** (MSC06).		AULA 16. Kernel and range representation of matrix pencils. **Carsten Trunk** (MSC05).	

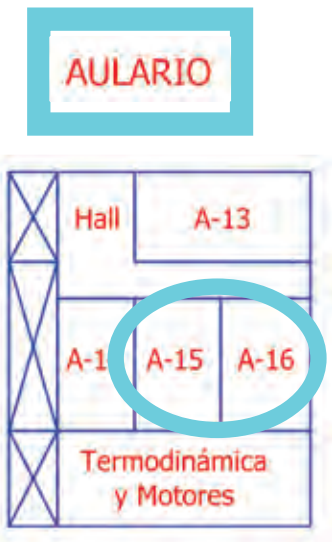
17:30-18:00	AULA 3. Smigoc's glue for universal realizability on the left half-plane. **Ricardo L. Soto** (MSC03).	AULA 6. On generalized inverse of a vector, with applications to vector epsilon algorithm. **Ahmed Salam** (MSC07).
17:30-18:00	AULA 16. Combinatorics in matrix pencils completion and rank perturbation problems. **Marko Stosic** (MSC05).	AULA 12. The difficulty of minimum rank 3. **Alathea Jensen** (MSC17).
17:30-18:00	AULA 6F. The Endpoint Geodesic Problem on Symmetric Spaces with Applications. **Knut Hüper** (MSC06).	AULA 7. Bidiagonal decompositions of singular sign regular matrices of signature (1, . . . , 1, −1). **Plamen Koev** (MSC19).
17:30-18:00	AULA 6. Numerical approximation of the symbol of an operator with local spectral mean values evaluations. **Jean-Paul Chehab** (MSC07).	AULA 5. A Fiedler-type determinantal inequality in Euclidean Jordan algebras. **David Sossa** (MSC20).
17:30-18:00	AULA 10. H2 optimal model reduction for simply connected domains. **Alessandro Borghi** (MSC12).	AULA 10. A Vandermonde Neural Operator: Extending the Fourier Neural Operator to Nonequispaced Distributions. **Levi Lingsch** (MSC22).
17:30-18:00	AULA 12. Orthogonal Realizations of Random Sign Patterns. **Bryan Curtis** (MSC17).	AULA 6F. On the complexity of finding tensor ranks. **Mohsen Aliabadi** (MSC23).
17:30-18:00	AULA 7. Bidiagonal decomposition of rectangular totally positive Lagrange-Vandermonde matrices and applications. **Ana Marco** (MSC19).	AULA 15. On the orthogonal decomposition of real square matrices over the co-Latin and semi-magic symmetry classes. **Matthew Lettington** (MSC26).
18:00-18:30	AULA 5. Deterministic Approximation Algorithms for Volumes of Spectrahedra. **Mahmut Levent Doğan** (MSI03).	AULA SEMINARIOS. Clique complexes of strongly regular graphs and their eigenvalues. **Sebastian M. Cioaba** (MSC02).
18:00-18:30	AULA 15. Balanced Truncation Model Reduction of Parametric Differential-Algebraic Systems. **Matthias Voigt** (MSI04).	AULA 16. An interlacing result for Hermitian matrices in Minkowski space. **Madeleine van Straaten** (MSC05).
18:00-18:30	AULA SEMINARIOS. Spectra of normal Cayley graphs. **Soffia Arnadottir** (MSC02).	AULA 11. Matrix Means on Grassmann Manifolds. **Tin-Yau Tam** (MSC06).
18:00-18:30	AULA 3. More on polynomials preserving nonnegative matrices. **Raphael Loewy** (MSC03).	AULA 6. Computing the generalized rational minimax approximation. **Nir Sharon** (MSC07).
18:00-18:30	AULA 16. Minimal rank perturbations of matrix pencils. **Marija Dodig** (MSC05).	AULA 12. Generic realisability and applications. **Rupert Levene** (MSC17).
18:00-18:30	AULA 6F. Majorization and properties on Spectral geometric mean. **Luyining Gan** (MSC06).	AULA 7. Tropical totally positive matrices. **Adi Niv** (MSC19).
18:00-18:30	AULA 6. Efficient Inversion of Matrix ϕ-Functions of Low Order. **Luca Gemignani** (MSC07).	AULA 5. Jordan automorphisms and derivatives of symmetric cones. **Michael Orlitzky** (MSC20):
18:00-18:30	AULA 10. On multi-objective optimization of model reduction for port-Hamiltonian systems. **Jonas Nicodemus** (MSC12).	AULA 10. Computing Approximate Solutions of Ill-Conditioned Linear Systems in Low and Mixed Precision. **James Nagy** (MSC22).
18:00-18:30	AULA 12. Zq-Forcing Game for Some Families of Graphs. **Shahla Nasserasr** (MSC17).	AULA 6F. Tensor optimal transport. **Shmuel Friedland** (MSC23).
18:00-18:30	AULA 7. Accurate computations with rectangular totally positive collocation matrices of the Lupas-type (p,q)-analogue of the Bernstein basis. **Raquel Viaña** (MSC19).	AULA 15. Inner Bohemian matrices. **Juana Sendra** (MSC26).
18:30-19:00	AULA 5. Classifying Linear Matrix Inequalities via Abstract Operator Systems. **Tim Netzer** (MSI03).	AULA 16. Spectral enclosures and resolvent estimates for matrix and operator polynomials. **Christiane Tretter** (MSC05).
18:30-19:00	AULA 15. A delayed shift technique for M-matrix algebraic Riccati equations. **Federico Poloni** (MSI04).	AULA 6. Perfect shifts for Hessenberg-Hessenberg pencils. **Marc Van Barel** (MSC07).

AULA 6. A tensor bidiagonalization method for higher-order singular value decomposition with applications. **Anas El Hachimi** (MSC07).	
AULA 7. Structured eigenvalue optimization via rank-1 ODEs. **Nicola Guglielmi** (MSC11).	
AULA 3. Rational approximation with minimal sampling for Helmholtz-like problems. **Davide Pradovera** (MSC15).	
AULA 15. A generalisation of the Hermite–Biehler theorem. **Mikhail Tyaglov** (MSC16).	
AULA 6F. Quantum Wasserstein energy distance. **Rafal Bistron** (MSC23).	
AULA 5. U(h)-free modules and weight representations. **Eduardo Monteiro Mendonça** (MSC24).	
AULA 11. Automated proofs of operator statements. **Clemens Hofstadler** (MSC25).	
AULA SEMINARIOS. Multiplicative structures generated by alternating sign matrices. **Rachel Quinlan** (MSI01).	
AULA 10. Learning Feynman diagrams with tensor trains. **Yuriel Núñez Fernández** (MSI02).	
AULA 16. Weyr characteristics perturbation results for matrix pencils. **Francisco Martínez-Pería** (MSC05).	
AULA 6. Error bounds for the approximation of matrix functions with rational Krylov methods. **Igor Simunec** (MSC07)	
AULA 7. Tributes to Michael Overton on the Occasion of His 70th Birthday (MSC11).	
AULA 3. Rational approximations of BEM systems for the 3D scalar Helmholtz equation. **Simon Dirckx** (MSC15).	
AULA 15. Jacobi matrices on binary trees: multilevel interpolations and boundedness. **Vladimir Lysov** (MSC16).	
AULA 5. The graphs of reduced words of apermutation. **Ricardo Mamede** (MSC24).	
AULA 11. Trace Minimization Principles. **Ren-Cang Li** (MSC25).	
AULA 6F. On perfect tensors and multipartite entanglement. **Karol Zyczkowski** (MSC23).	
AULA SEMINARIOS. Alternating Sign Matrices and Generalizations. **Geir Dahl** (MSI01).	
AULA 10. A weighted subspace exponential kernel for support tensor machines. **Kirandeep Kour** (MSI02).	

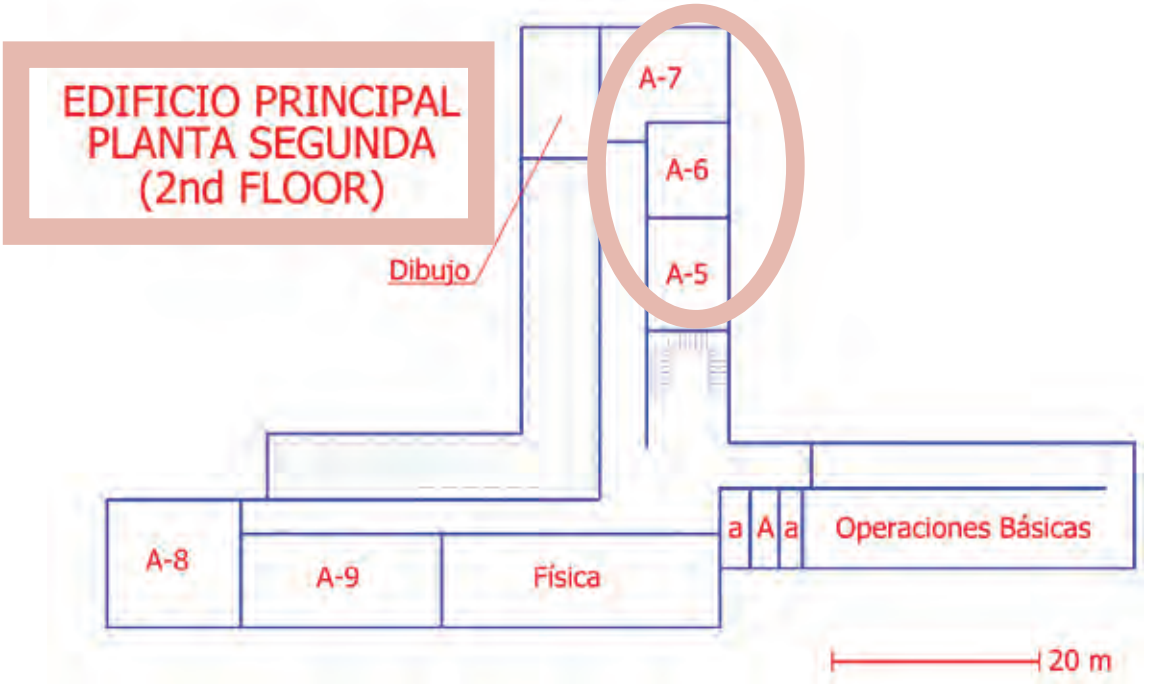
	EDIFICIO NUEVO (ANEXO)	EDIFICIO MONTES (PRINCIPAL)	AULARIO	EDIFICIO FORESTALES
18:30-19:00	AULA SEMINARIOS. NEPS of Complex Unit Gain Graphs. **Francesco Belardo** (MSC02).	AULA 12. The number of distinct eigenvalues of joins of graphs. **Mark Kempton** (MSC17).		AULA 16. Small rank perturbations of H-expansive matrices. **Dawie Janse van Rensburg** (MSC05).
18:30-19:00	AULA 16. Rank-one perturbation of linear relations via matrix pencils. **Alicia Roca** (MSC05).	AULA 7. Linear Algebra in Approximation Theory: a new hope. **José-Javier Martínez** (MSC19):		AULA 6. Applications of trace estimation techniques. **Yousef Saad** (MSC07).
18:30-19:00	AULA 6F. Linearity of Cartan and Wasserstein geodesics. **Sejong Kim** (MSC06).	AULA 5. On certain properties of the second order cone and some of its generalizations. **Roman Sznajder** (MSC20).		AULA 7. Tributes to Michael Overton on the Occasion of His 70th Birthday (MSC11).
18:30-19:00	AULA 6. Structured-barycentric forms and the AAA framework for modeling second-order dynamics from data. **Ion Victor Gosea** (MSC07).	AULA 10. Solving an inverse eigenvalue problem using a divide-and-conquer method. **Natalia Bebiano** (MSC22).		AULA 3. Polynomial preconditioning with Faber polynomials for the Helmhotz equation. **Olivier Sète** (MSC15).
18:30-19:00	AULA 10. Parametric Linearization of Nonlinear Dynamical Systems Subject to Periodic Inputs. **Giovanni Coni** (MSC12).	AULA 6F. Entropic characterization of the spectral radius of nonnegative tensors and beyond. **Stéphane Gaubert** (MSC23).		AULA 5. Jordan type Artinian Gorenstein algebras and related invariants. **Pedro Macias Marques** (MSC24).
18:30-19:00	AULA 7. On the total positivity of Gram matrices of polynomial bases. **Esmeralda Mainar** (MSC19).	AULA 15. Computing the maximum spread of a Bohemian symmetric matrix with entries in [a, b]. **Rafael Sendra** (MSC26).		
19:00-19:30		AULA 6. A Rational Preconditioner for Multi-dimensional Riesz Fractional Diffusion Equations. **Mariarosa Mazza** (MSC07)		AULA 10. Empirical Tensor Train Approximation in Optimal Control. **Mathias Oste**r (MSI02).
19:00-19:30		AULA 5. Hadamard product and related inequalities in the Jordan spin algebra. **Juyoung Jeong** (MSC20)		AULA 6. Extrapolation methods for choosing a regularization parameter. **Giuseppe Rodriguez** (MSC07).
19:00-19:30		AULA 15. On the the eigenvalues of (Bohemian) Q-matrices and P-matrices. **Laureano González-Vega** (MSC26)		AULA 5. Carnot graded Lie algebras and chain ideal lattices. **Pilar Benito** (MSC24).
19:00-19:30				

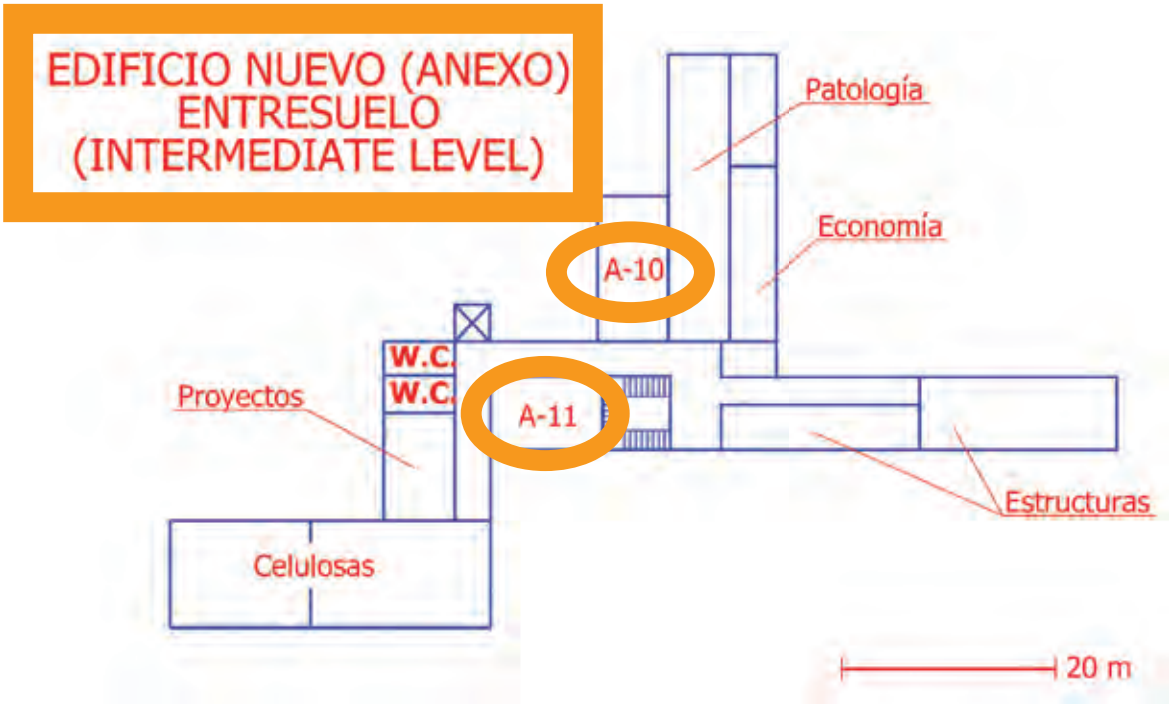

University canteens
All admit payments in cash or credit card
Student dorm Nuestra Señora de África
max 40 people
menu: 7.00€
Student dorm Isabel de España
max 40 people
no menu, but cheap
CSIC building
max 80 people
menu: 6.50€
Forestales building
max 150 people
menu: 5.60€
(basement)
Main Entrance
Matemáticas building
max 90 people
menu: 6.10€
Biológicas building
max 120 people
menu: 6.10€
Farmacia building
max 150 people
menu: 6.10€
(basement)
Metro
ATM
AVENIDA COMPLUTENSE


Aulario
A16
A15
A7
(2n floor)
A6 (2n floor)
A5 (2n floor)
A3
Montes
A12
(1st floor)
A10
(intermediate level)
A11
(intermediate level)
Nuevo
Salón de Actos
(main Hall)
A6F
(ground floor)
Aula Seminario
(1st floor)
Forestales
CALLE DE RAMIRO DE MAEZTU
ETSI Montes, Forestal y del Medio Natural Edificio Forestales
Main Entrance


AULARIO
Hall
A-13
A-1
A-15
A-16
Termodinámica y Motores


EDIFICIO PRINCIPAL
PLANTA BAJA
(GROUND FLOOR)
A-3
Sala Informática de Alumnos
Servicio Informático
Gestión Económica
A-2
W.C.
Aula de Seminarios: Fernando Peña
a edificio nuevo
Secretaría
HALL
S. de Actos
A-1
Edafología
20 m

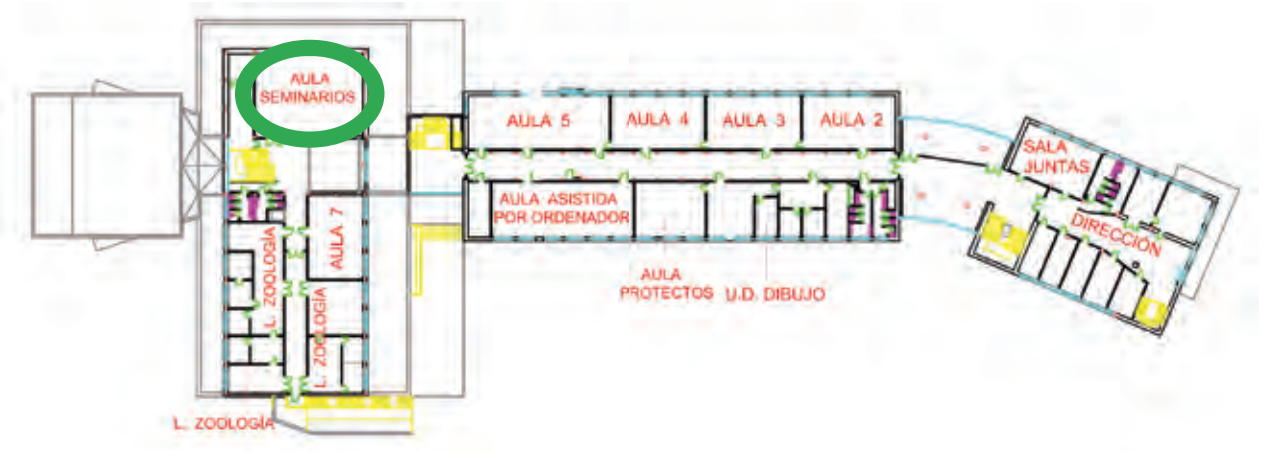

EDIFICIO PRINCIPAL
PLANTA SEGUNDA
(2nd FLOOR)
A-7
A-6
A-5
Dibujo
A-8
A-9
Física
a
A
a
Operaciones Básicas
20 m


EDIFICIO NUEVO (ANEXO)
ENTRESUELO
(INTERMEDIATE LEVEL)
Patología
Economía
A-10
W.C.
W.C.
A-11
Proyectos
Estructuras
Celulosas
20 m


EDIFICIO NUEVO (ANEXO)
PLANTA PRIMERA
(1st FLOOR)
Anatomía
W.C.
W.C.
A-12
Derecho
Mecánica
I-3
Topografía
20 m

EDIFICIO FORESTALES
PLANTA BAJA
(GROUND FLOOR)

EDIFICIO FORESTALES
PLANTA PRIMERA
(1st FLOOR)



CONTENTS

List of Communications

COMMUNICATIONS

Plenary talks

Towards a more sensible theory of stability in Numerical Linear Algebra

Carlos Beltrán[1], *Vanni Noferini*[2], *Nick Vannieuwenhoven*[3]

[1] *Dpt. MATESCO, Universidad de Cantabria, Spain*
E-mail: beltranc@unican.es
[2] *Dpt. Mathematics and Systems Analysis, Aalto University, Finland*
E-mail: vanni.noferini@aalto.fi
[3] *Dpt. Computer Science, KU Leuven, Belgium*
E-mail: nick.vannieuwenhoven@kuleuven.be

Abstract

We are all used to the nowadays quite standard definitions of backward, mixed and forward stability. But, to which point are these definitions really sound? Spoiler: this is not a simple question. Here is an even harder to answer one: it is a basic procedure to use the output of a (presumably) stable algorithm as an input for another (presumably) stable algorithm. Under which hypotheses can we grant that the concatenation of both algorithms is itself a stable algorithm? We will discuss these questions and give reasonable answers to them.

Acknowledgements: Work (partially) supported by Grant PID2020-113887GB-I00 funded by MCIN/ AEI /10.13039/501100011033, Banco Santander and Universidad de Cantabria grant 21.SI01.64658, Academy of Finland grant (Suomen Akatemian päätös 331240), a Postdoctoral Fellowship of the Research Foundation—Flanders (FWO) with project 12E8119N and a FWO Grant for a long stay abroad V401518N.

References

[1] C. Beltrán, V. Noferini and N. Vannieuwenhoven *When can forward stable algorithms be composed stably?*. To appear.

Balancing Inexactness in Matrix Computations

Erin C. Carson[1]

[1] *Department of Numerical Mathematics, Faculty of Mathematics and Physics, Charles University, Czech Republic*
E-mail: carson@karlin.mff.cuni.cz

Abstract

On supercomputers that exist today, achieving even close to the peak performance is incredibly difficult if not impossible for many applications. Techniques designed to improve the performance of matrix computations - making computations less expensive by reorganizing an algorithm, making intentional approximations, and using lower precision - all introduce what we can generally call "inexactness". The questions to ask are then:

1. With all these various sources of inexactness involved, does a given algorithm still get close enough to the right answer?

2. Given a user constraint on required accuracy, how can we best exploit and balance different types of inexactness to improve performance?

Studying the combination of different sources of inexactness can thus reveal not only limitations, but also new opportunities for developing algorithms for matrix computations that are both fast and provably accurate. We present few recent results toward this goal, involving mixed precision randomized decompositions and mixed precision sparse approximate inverse preconditioners.

Acknowledgements: Work partially supported by ERC Starting Grant No. 101075632 and the Exascale Computing Project (17-SC-20-SC), a collaborative effort of the U.S. Department of Energy Office of Science and the National Nuclear Security Administration.

Randomized sketching of Krylov methods in numerical linear algebra

Stefan Güttel

Department of Mathematics, The University of Manchester, United Kingdom
E-mail: stefan.guettel@manchester.ac.uk

Abstract

Many large-scale computations in numerical linear algebra are powered by Krylov methods, including the solution of linear systems of equations, least squares problems, linear and nonlinear eigenvalue problems, matrix functions and matrix equations, etc. We will discuss some recent ideas to speed up Krylov methods for these tasks using randomized sketching, and highlight some of the key challenges for future research.

Matrix Stories

Nicholas J. Higham

[1] *Department of Mathematics, University of Manchester, Manchester, M13 9PL, UK*
E-mail: nick.higham@manchester.ac.u

Abstract

The study of particular matrices has often opened up fruitful research directions in numerical linear algebra and matrix analysis. I will give a variety of examples of such matrices, describing their properties, their applications, and stories behind them. I will also describe the Anymatrix project, which makes these and many other matrices available and easily searchable by properties in MATLAB.

Acknowledgements: This work was supported by the Royal Society.

References

[1] N. J. Higham and M. Mikaitis. Anymatrix: An extensible MATLAB matrix collection. *Numer. Algorithms*, 90(3):1175–1196, 2021.

Minimum Number of Distinct Eigenvalues of Graphs

Shahla Nasserasr

School of Mathematical Sciences, Rochester Institute of Technology, USA
E-mail: shahla@mail.rit.edu

Abstract

For a graph G on n vertices, let $\mathcal{S}(G)$ be the set of all $n \times n$ real symmetric matrices such that their nonzero off-diagonal entries represent the weights of the edges of G. The inverse eigenvalue problem for a graph G (IEP-G) asks to determine all possible spectra of matrices in $\mathcal{S}(G)$.

A list of positive integers $\mathbf{m} = (m_1, m_2, \ldots, m_k)$ is realized as an ordered multiplicity list for the graph G if there is a matrix in $\mathcal{S}(G)$ with k distinct eigenvalues such that the ith largest eigenvalue has the multiplicity m_i, for $i = 1, 2, \ldots, k$.

One of the relaxations of the IEP-G is to determine the minimum length among all realizable multiplicity lists of a graph. This parameter is denoted by $q(G)$ and it is called the minimum number of distinct eigenvalues of G.

In this presentation, we will review interesting advances and techniques from a number of recent developments regarding $q(G)$.

References

[1] Bahman Ahmadi, Fatemeh Alinaghipour, Michael S. Cavers, Shaun Fallat, Karen Meagher, and Shahla Nasserasr. Minimum number of distinct eigenvalues of graphs. *Electron. J. Linear Algebra*, 26: 673–691 (2013).

[2] Leslie Hogben, Jephian C.-H. Lin, and Bryan L. Shader. *Inverse problems and zero forcing for graphs*. volume 270 of Mathematical Surveys and Monographs. American Mathematical Society, Providence, RI, 2022.

[3] Wayne Barrett, Shaun Fallat, H. Tracy Hall, Leslie Hogben, Jephian C.-H. Lin, and Bryan L. Shader. Generalizations of the strong Arnold property and the minimum number of distinct eigenvalues of a graph. *Electron. J. Combin.*, 24(2): Paper No. 2.40, 28, (2017).

Spectral Turán problems

Michael Tait[1]

[1] *Department of Mathematics & Statistics, Villanova University, USA*
E-mail: michael.tait@villanova.edu

Abstract

In this talk we will consider a spectral version of the classical Turán problem: given a fixed graph F, how large can the largest eigenvalue of the adjacency matrix be over all n-vertex graphs which do not contain F as a subgraph? As the largest eigenvalue of the adjacency matrix is an upper bound for the average degree of a graph, any upper bound on this quantity also gives an upper bound on the Turán number $\mathrm{ex}(n, F)$, and in fact several theorems in this area imply and strengthen classical results in extremal graph theory. We will discuss recent progress on this problem including what the similarities and differences between it and the classical Turán problem are and what future work may be done.

This talk will include joint work with Sebastian Cioabă, Dheer Noal Desai, Lihua Feng, Liying Kang, Yongtao Li, Zhenyu Ni, Jing Wang, and Xiao-Dong Zhang

Acknowledgements: Work (partially) supported by National Science Foundation grant DMS-2011553

References

The role of the field in some questions of matrix algebra

Rachel Quinlan

[1] *University of Galway, Ireland*
E-mail: rachel.quinlan@universityofgalway.ie

Abstract

Given a field $\mathbb{F}$ and a matrix property $\mathcal{P}$, one can investigate the maximum possible dimension of a subspace of $M_{m\times n}(\mathbb{F})$ in which every (non-zero) element has property $\mathcal{P}$, and try to identify those subspaces that attain this maximum. This formulation provides a rich source of interesting problems, many of which have a long and influential history. Sometimes the answers to these questions are independent of the field $\mathbb{F}$, for example if $\mathcal{P}$ is an upper bound on rank, or if $m = n$ and $\mathcal{P}$ is nilpotence. Sometimes the answers are highly dependent on $\mathbb{F}$, for example if $\mathcal{P}$ is a lower bound on rank, or if $m = n$ and $\mathcal{P}$ is non-nilpotence. We will discuss the role of field properties in some of these cases.

Fields that are algebraically closed are great for linear algebra, for example because every square matrix is similar to a unique Jordan canonial form. Fields that are real are also very nice, for example because they admit a distinction between positive and negative elements. Finite fields allow opportunities for counting. Fields that possess extensions of finite degree are excellent too, because such an extension is a finite dimensional vector space, with the extra algebraic machinery of a field multiplcation that plays well with the vector space structure. If $\mathbb{K}$ is a field extension of $\mathbb{F}$ of degree n, then $\mathbb{K}$ is isomorphic as a vector space to any other n-dimensional space over $\mathbb{F}$. It follows that any $\mathbb{F}$-vector space V of dimension n can be endowed with an $\mathbb{F}$-bilinear field multiplication arising from $\mathbb{K}$.

In the talk, we will consider how this idea can be used to uncover large subspaces of various matrix spaces in which rank behaves in a controlled way, specifically over fields which admit cyclic Galois extensions of all degrees. These fields comprise a broad class, including for example all finite fields and all finite extensions of $\mathbb{Q}$.

Root vectors

Vanni Noferini[1]

[1] *Aalto University, Finland*
E-mail: vanni.noferini@aalto.fi

Abstract

Root vectors are a classical, albeit somewhat underappreciated, topic in linear algebra in the regular case [4, 11]. More recently, they have proven to be a powerful tool in the singular case as well [1, 2, 8, 9, 10]. In this presentation, we will explore their applicability in increasingly general situations, including classical eigenvalue problems, generalized eigenvalue problems [9], polynomial and rational matrices [1, 2, 4, 10], as well as analytic and meromorphic matrices [8, 11].

Let us first consider the most basic case of classical eigenvalue problems, focusing on root vectors that are polynomials. If $\mathbb{F}$ is an algebraically closed field and $A \in \mathbb{F}^{n\times n}$ has an eigenvalue $\lambda \in \mathbb{F}$, a *root polynomial* for A at λ of order ℓ is defined as a vector $v(x) \in \mathbb{F}[x]^n$ such that

$$(1)\ (A - xI)v(x) = (x-\lambda)^\ell w(x) \text{ with } w(\lambda) \neq 0; \qquad (2)\ v(\lambda) \neq 0.$$

Such a root polynomial can be seen as a generating function for a Jordan chain of A at λ. Indeed, if we expand it as $v(x) = v_0 + v_1(x-\lambda) + v_2(x-\lambda)^2 + v_3(x-\lambda)^3 + \ldots$, then it is easy to see that $v_0, v_1, \ldots, v_{\ell-1}$ is a Jordan chain of length ℓ for A associated with the eigenvalue λ. A kind of converse statement also holds: for example, if v_0, v_1, v_2 is a Jordan chain of lenght 3 at λ then one has $(A-\lambda I)v_0 = 0$, $(A-\lambda I)v_1 = v_0$, and $(A-\lambda I)v_2 = v_1$. Hence, $(A-xI)(v_0 + (x-\lambda)v_1 + (x-\lambda)^2 v_2) = (x-\lambda)^3(-v_2)$ so that $v(x) = v_0 + (x-\lambda)v_1 + (x-\lambda)^2 v_2$ is a root polynomial of order 3 for A at λ.

One can further extend this idea by constructing *maximal sets* of root polynomials, which correspond to generating functions for canonical sets of Jordan chains. This process involves several steps:

1. A set of root polynomials $\{v_i(x)\}_{i=1}^s$ at λ for A, of orders $\ell_1 \geq \cdots \geq \ell_s$, is called λ-independent if the constant matrix $\begin{bmatrix} v_1(\lambda) & \ldots & v_s(\lambda) \end{bmatrix}$ has full column rank;

2. A λ-independent set of root polynomials at λ for A is called complete if there are not λ-independent sets of larger cardinality;

3. A complete set of root polynomials at λ for A is called maximal if it cannot be modified by replacing one root polynomial with another of larger order while still maintaining the completeness property.

It can be proven that the orders of a maximal set of root polynomials are precisely the partial multiplicities of the eigenvalue λ for the matrix A. Thus, a maximal set serves as a condensed source of relevant information about the eigenvalue λ, including partial multiplicities and (generalized) right eigenvectors.

The concept of a maximal set of root vectors, which include root polynomials as a special case, can be extended beyond the classical eigenvalue problem represented by the pencil $A - xI$. Specifically, maximal sets of root vectors exist, and exhibit similar properties to those discussed earlier in the context of the generalized eigenvalue problem $(A + xB)$, the polynomial eigenvalue problem $(P(x) \in \mathbb{F}[x]^{m\times n})$, the rational eigenvalue problem $(R(x) \in \mathbb{F}(x)^{m\times n})$, and other nonlinear eigenvalue problems involving matrices over the ring of analytic functions or the field of meromorphic functions.

All these generalizations, unlike the classical eigenvalue problem, encompass the *singular* case. For instance, a pencil $A + xB$ is regular if it is square and $\det(A + xB) \not\equiv 0$, while it is singular otherwise. Analogous definitions apply to polynomial, rational, analytic, and meromorphic matrices. The application of the concept of a canonical set of Jordan chains becomes problematic in the singular case, as it is not immediately clear how to extend the definition of eigenvectors (let alone chains). However, the notion of a maximal set of root vectors is flexible enough to adapt to singular (linear or nonlinear) eigenvalue problems. The starting point is the generalization of the notion of a root polynomial. Suppose $M(x)$ is a minimal basis [3] for the singular pencil $A + xB \in \mathbb{F}[x]1^{m\times n}$, and define $\ker_\lambda(A + xB)$ as the linear span of the columns of $M(\lambda)$. Then, a vector $v(x) \in \mathbb{F}[x]$ is termed a *root polynomial* for $A + xB$ at λ of order ℓ if:

$$(1)\ (A + xB)v(x) = (x - \lambda)^\ell w(x) \text{ with } w(\lambda) \neq 0; \qquad (2)\ v(\lambda) \notin \ker_\lambda(A + xB).$$

Maximal sets are then defined similarly to the regular case, with the exception that for λ-independence one requires $\begin{bmatrix} M(\lambda) & v_1(\lambda) & \dots & v_s(\lambda) \end{bmatrix}$ to have full rank. It is important to note that, in the regular case (or more generally when the pencil $A + xB$ has full column rank), the block $M(\lambda)$ is empty. One significant application is the rigorous definition of eigenvectors also for singular pencils: an eigenvector is a nonzero element of the quotient space $\ker(A + \lambda B)/\ker_\lambda(A + xB)$. It should be noted that, if $A + xB$ has full column rank, then $\ker_\lambda(A + xB) = \{0\}$ is trivial and the conventional definition of an eigenvector is thus regained. Eigenvectors of singular pencils have numerous computational applications [5, 6, 7].

Finally, root vectors can also be defined for rational matrices, using a connection with valuation theory [10], and for analytic (and meromorphic) matrices, using a connection with module theory [8]. In all these settings, by utilizing root vectors, eigenvectors can still be defined also for matrices that do not have full column rank.

References

[1] F. Dopico, V. Noferini. The $\mathbb{DL}(P)$ vector space of pencils for singular matrix polynomials. Preprint.

[2] F. Dopico, V. Noferini. Root polynomials and their role in the theory of matrix polynomials *Linear Algebra Appl.* 584:37–78, (2020).

[3] G. D. Forney Jr.. Minimal bases of rational vector spaces, with applications to multivariable linear system. *SIAM J. Control* 13, 493–520, (1975).

[4] I. Gohberg, P. Lancaster, L. Rodman. *Matrix Polynomials*, SIAM, 2009. Unabridged republication of book first published by Academic Press in 1982.

[5] M. E. Hochstenbach, C. Mehl and B. Plestenjak. Solving singular generalized eigenvalue problems. Part II: Projection and agumentation. Preprint.

[6] D. Kressner and I. Šain Giblić. Singular quadratic eigenvalue problems: linearization and weak condition numbers. *BIT Numer. Math.* 63, 18 (2023).

[7] M. Lotz and V. Noferini. Wilkinson's bus: Weak condition numbers, with an application to singular polynomial eigenproblems. *Found. Comput. Math* 20, 1439–1473 (2020).

[8] V. Noferini. Invertible bases and root vectors for analytic matrix-valued functions Preprint.

[9] V: Noferini, P. Van Dooren. On computing root polynomials and minimal bases of matrix pencils *Linear Algebra Appl.* 658:86–115, (2023).

[10] V. Noferini, P. Van Dooren. Root vectors of polynomial and rational matrices: theory and computation *Linear Algebra Appl.* 656:510–540, (2023).

[11] V. Trofimov. The root subspace of operators that depend analytically on a parameter. Mat. Issled 3, 117–125, (1968).

Eigenvalue nonlinearities and eigenvector nonlinearities

Elias Jarlebring[1]

[1] *KTH Royal Institute of Technology, Sweden*
E-mail: eliasj@kth.se

Abstract

The following two generalizations of the standard eigenvalue problem have received considerable attention in the numerical linear algebra community: The eigenvalue nonlinear eigenvalue problem $A(\lambda)x = 0$, where $A : \mathbb{C} \to \mathbb{C}^{n\times n}$ is typically a holomorphic or meromorphic function of the scalar λ, and the eigenvector nonlinear eigenvalue problem $A(x)x = \lambda x$, where either $A : \mathbb{C}^n \to \mathbb{C}^{n\times n}$ is assumed to be homogeneous, $A(\alpha x) = A(x)$, or we explicitly require a normalization condition, e.g., $x^H x = 1$. We summarize how these problems arise in applications, for example delay differential equations, acoustics, quantum physics and data science. Application-driven numerical developments are presented, as well as a review of general numerical linear algebra, and theoretical approaches for both types of problems in the context of specific structures.

References

[1] P. Henning and E. Jarlebring, *The Gross-Pitaevskii equation and eigenvector nonlinearities: Numerical methods and algorithms*, arXiv preprint, 2023.

[2] P. Upadhyaya, E. Jarlebring, F. Tudisco, *The self-consistent field iteration for p-spectral clustering*, arxiv preprint, 2021

[3] R. Claes, E. Jarlebring, K. Meerbergen, P. Upadhyaya, *Linearizability of eigenvector nonlinearities*, SIAM J. Matrix Anal. Appl., 43:764-786, 2022

[4] E. Jarlebring, K. Meerbergen and W. Michiels, *A Krylov method for the delay eigenvalue problem*, SIAM J. Sci. Comput., 32(6):3278-3300, 2010

Determinantal Representations in Theory and Applications

Cynthia Vinzant [1]

[1] *Department of Mathematics, University of Washington, Seattle, WA, USA*

Abstract

A linear determinantal representation expresses a multivariate polynomial as the determinant of a square matrix whose entries are linear forms. The study of determinantal representations dates back to 19th century classical algebraic geometry and has since found applications in partial differential equations, operator theory, convex optimization, and complexity theory. I will survey some of the classical and recent theory of determinantal representations with a focus on applications in linear algebra and matrix theory, including numerical ranges and the principal minor map.

References

[1] Abeer Al Ahmadieh and Cynthia Vinzant. Characterizing principal minors of symmetric matrices via determinantal multiaffine polynomials. (`arXiv:2105.13444`)

[2] Abeer Al Ahmadieh and Cynthia Vinzant. Determinantal representations and the image of the principal minor map. (`arXiv:2205.05267`)

[3] Alfred Cardew Dixon. Note on the reduction of a ternary quantic to a symmetrical determinant. *Cambr. Proc.*, 11:350–351, 1902.

[4] J. William Helton and Victor Vinnikov. Linear matrix inequality representation of sets. *Comm. Pure Appl. Math.*, 60(5):654–674, 2007. *Cambr. Proc.*, 11:350–351, 1902.

[5] Faye Pasley Simon and Cynthia Vinzant. Invariant hyperbolic curves: determinantal representations and applications to the numerical range. (`arXiv:2102.01726`)

[6] Daniel Plaumann and Cynthia Vinzant. Determinantal representations of hyperbolic plane curves: an elementary approach. *J. Symbolic Comput.* 57 (2013), 48–60.

[7] Victor Vinnikov. LMI representations of convex semialgebraic sets and determinantal representations of algebraic hypersurfaces: past, present, and future. In *Mathematical methods in systems, optimization, and control*, volume 222 of *Oper. Theory Adv. Appl.*, pages 325–349. Birkhäuser/Springer Basel AG, Basel, 2012.

Contributed talks

Recursion formulas for determinants of k-Tridiagonal Toeplitz Matrices

Eugene Agyei-Kodie[1], *Hamide Dogan*[2]

[1] *Department of Mathematical Sciences , University of Texas at El Paso , USA*
E-mail: eagyeikodi@miners.utep.edu
[2] *Departement of Mathematical Sciences , University of Texas at El Paso , USA*
E-mail: hdogan@utep.edu

Abstract

As defined in [2], we consider an n × n Toeplitz matrix , T, to be of the form:

$$T_{ij} = \begin{cases} a & ; \quad i = j \\ b & ; \quad |i-j| > k \\ c & ; \quad |i-j| < k \\ 0 & \text{otherwise} \end{cases}$$

Toeplitz matrices with its spectral properties are of great essence to physics, statistics and signal processing. Moreover, Toeplitz matrices help model problems including computation of spline functions, signal and image processing, polynomial and power series computations etc. Over the years, there have been studies on Toeplitz matrices such as recursion of determinants of 2-tridiagonal Toeplitz matrix [1] and tridiagonal 2-Toeplitz matrices [3]. In our study, we investigate the determinant of a k-tridiagonal Toeplitz matrices for $k > 2$. By extending the work of Borowska et al.[1] , we identified recursion formulas for determinants of all $k-$ tridiagonals Toeplitz matrices. Thus, we propose to share our findings at the 2023 ILAS Conference.

References

[1] J. Borowska, L. Łacińska. Eigenvalues of 2-tridiagonal Toeplitz matrix, *Journal of Applied Mathematics and Computational Mechanics,* vol.14, no.14,2015

[2] M. Elouafi, On a relationship between Chebyshev polynomials and Toeplitz determinants,*Applied Mathematics and Computation,* vol.229, pp. 27-33, 2014.

[3] M.J Gover, The eigenproblem of a tridiagonal 2-Toeplitz matrix, *Linear Algebra and its Applications,* vol. 197, pp. 63-78, 1994.

Design of an estimator with orthogonal projections for a linear regression model and its strong consistency

Kensuke Aishima[1]

[1] *Faculty of Computer and Information Sciences, Hosei University, Japan*
E-mail: aishima@hosei.ac.jp

Abstract

This study deals with an application of numerical linear algebra to statistical analysis. We aim to construct an estimator for a certain type of linear regression model related to errors-in-variables models that arise in many scientific and engineering computations, such as data fitting problems. Such a regression model gives rise to the total least squares (TLS) problem or its variants, which can be solved numerically using the singular value decomposition (SVD). Some total least squares estimators for the corresponding regression models have strong consistency in the statistical sense as in [1, 3], where the theory is based on the consistency analysis of Gleser [2] in 1981. In this talk, we discuss a unified regression model that covers the above, and then construct a new estimator with orthogonal projections, leading to a straightforward proof of its strong consistency.

Acknowledgements: This study was supported by JSPS KAKENHI Grant Nos. JP17K14143, JP21K11909, and JP22K03422.

References

[1] K. Aishima, Consistent estimation for an errors-in-variables model based on constrained total least squares problems, JSIAM Letters **14** (2022), 111–114.

[2] L. J. Gleser, Estimation in a multivariate "errors in variables" regression model: large sample results, Ann. Statist., **9** (1981), 24–44.

[3] S. Van Huffel and H. Zha, The restricted total least squares problem: formulation, algorithm, and properties, SIAM J. Matrix Anal. Appl., **12** (1991), 292–309.

Tropical Matrix Exponential

Himadri Mukherjee[1], *Askar Ali M*[2]

[1] *Dpt. of Mathematics, BITS PILANI Goa Campus, India*
E-mail: himadrim@goa.bits-pilani.ac.in
[2] *Dpt. of Mathematics, BITS PILANI Goa Campus, India*
E-mail: p20190037@goa.bits-pilani.ac.in

Abstract

Let $\mathbb{T}$ be the tropical field $\mathbb{R} \cup \{-\infty\}$, equipped with binary operations $a \oplus b = max\{a, b\}$, $a \otimes b = a + b$, for $a, b \in \mathbb{R} \cup \{-\infty\}$. We have studied the convergence of general matrix power series of the form $\sum_i^{\oplus} a_i \otimes A^i$, in Tropical algebra (Max-Plus algebra). In that light, we have given a series expansion for matrix exponent e^A, for $A \in M_n(\mathbb{T})$. We also have established the relation of matrix exponent to the eigenvalue-eigenvector problem. A finite vector v (at least one coordinate of v is finite) is called a generalised eigenvector of A, of order m, if $A^{(m)} \otimes v = \lambda^m \otimes v$, but, $A^{(m-1)} \otimes v \neq \lambda^{m-1} \otimes v$ (different from an already existing notion). Further, a matrix A is called quasi-robust if, for any finite vector x, for some positive integer k, $A^{(k)} \otimes x$ is a generalised eigenvector of A, of some order m. We prove that an irreducible matrix is quasi-robust if and only if it is periodic. Also, for an irreducible square matrix, we have analysed when its exponent is quasi-robust.

Acknowledgements: The research of the second author was supported by the University Grant Commission- Ministry of Human Resource Development, New Delhi, India, through CSIR-UGC fellowship.

References

[1] P. Butkovic, *Max-Linear Systems: Theory and Algorithms*, Springer, Berlin, 2010.

[2] P. Butkovic, R.A. Cuninghame-Green, *On matrix powers in max-algebra*, Linear Algebra Appl., 421, 370–381, 2007.

[3] Akian M., Bapat R., Gaubert S., *Max-plus algebra*, In: Hogben L., Brualdi R., Greenbaum A., Mathias R. (eds.), *Handbook of Linear Algebra*, Chapman and Hall, London, 2006.

[4] Develin M., Santos F., Sturmfels B., *On the rank of a tropical matrix*, Combinatorial and computational geometry, 52, Math. Sci. Res. Inst. Publ., 213–242, Cambridge University Press, Cambridge, 2005.

[5] Izhakian Z., Johnson M., Kambites M., *Tropical matrix groups*, Semigroup Forum, 96, 2018.

Generating acyclic symmetric matrices with the minimum number of distinct eigenvalues

Luiz Emilio Allem[1], *Lucas Siviero Sibemberg*[2]

[1] *UFRGS - Universidade Federal do Rio Grande do Sul, Instituto de Matemática, Porto Alegre, Brazil*
E-mail: emilio.allem@ufrgs.br
[2] *UFRGS - Universidade Federal do Rio Grande do Sul, Instituto de Matemática, Porto Alegre, Brazil*
E-mail: lucas.siviero@ufrgs.br

Abstract

Any symmetric matrix $M = (m_{ij}) \in \mathbb{F}^{n\times n}$ over a field $\mathbb{F}$ may be associated with a simple graph G with vertex set $[n] = \{1, \ldots, n\}$ such that distinct vertices i and j are adjacent if and only if $m_{ij} \neq 0$. We say that G is the *underlying graph* of M. Let $\mathcal{S}(G)$ be the set of real symmetric matrices whose underlying graph is G. We deal with the possible number of distinct eigenvalues of acyclic symmetric matrices. More precisely, given a tree T, we wish to study the quantity

$$q(T) = \min\{|\mathrm{DSpec(A)}| : A \in \mathcal{S}(T)\},$$

where DSpec(A) denotes the set of *distinct* eigenvalues of A. In [4] it is proved that if T is a tree with diameter d and $A \in \mathcal{S}(T)$, then $q(T) \geq d + 1$. The same authors suspected that, for every tree T of diameter d, there exists a matrix $A \in \mathcal{S}(T)$ with exactly $d + 1$ distinct eigenvalues. However, this turns out to be false. Barioli and Fallat [1] constructed a tree T with 16 vertices such that $\mathrm{diam}(T) = 6$, but $q(T) = 8$. It is now known that $q(T) = d + 1$ for every tree T of diameter d if and only if $d \leq 5$ [3]. For diameter $d \geq 6$, it is thus natural to characterize the trees T for which $q(T) = \mathrm{diam}(T) + 1$, which are known as *diameter minimal* (or *diminimal*, for short).

One of the main tools used to address this problem in [3] is the construction of trees using an operation called *branch duplication* [2]. The intuition is that, for any fixed positive integer d, there is a finite set $\mathcal{S}_d$ of (unlabelled) trees of diameter d, called the *seeds of diameter* d, with the property that any (unlabelled) tree of diameter d may be obtained from one of the seeds of diameter d by a sequence of branch duplications. As it turns out, for any tree T of diameter d there is a single seed of diameter d from which it can be obtained, so that the seeds are precisely the trees that cannot be obtained from smaller trees through branch duplication.

In this talk we present a constructive procedure that, given any $d \geq 6$ and any tree T that is obtained from a given set of seeds through branch duplication, produces a symmetric matrix $A \in \mathbb{R}^{n\times n}$ with underlying tree T with the

property that $q(T) = |\mathrm{DSpec}(A)| = d + 1$. This means that, in addition to exploring the existence of such a matrix, we also address its computability. In particular, the procedure allows us to produce such a matrix A with integral spectrum, i.e., with the property that its spectrum consists entirely of integers.

In a git repository https://github.com/Lucassib/Diminimal-Graph-Algorithm and https://lucassib-diminimal-graph-algorithm-st-app-0t3qu7.streamlit.app/, readers can access our method.

Acknowledgements: Work partially supported by MATH-AMSUD under project GSA, brazilian team financed by CAPES under project 88881.694479/2022-01. L. E. Allem acknowledges the support of FAPERGS 21/2551- 0002053-9.

References

[1] Francesco Barioli and Shaun Fallat. On two conjectures regarding an inverse eigenvalue problem for acyclic symmetric matrices. *The Electronic Journal of Linear Algebra*, 11: 41–50, (2004).

[2] Charles R. Johnson, Jacob Lettie, Sander Mack-Crane and Alicja Szabelska-Bersewicz. Branch duplication in trees: uniqueness of sedes and enumeration of sedes *Proyecciones (Antofagasta)* 39: 451 – 465 (2020).

[3] Charles R. Johnson and Carlos M. Saiago. *Eigenvalues, Multiplicities and Graphs*. Cambridge University Press, Cambridge, UK, 2018.

[4] Anónio Leal-Duarte and Charles R Johnson. On the minimum number of distinct eigenvalues for a symmetric matrix whose graph is a given tree. *Mathematical Inequalities and Applications*, 5: 175–180 (2002).

Sturm-Liouville problem and linear transformation on eigenpolynomials

Luis Miguel Anguas[1], *D. Barrios Rolanía*[2], *J. C. García-Ardila*[3]

[1] *Dpt. Math and Computer Sciences, Saint Louis University, Madrid Campus, Spain*
E-mail: luismiguel.anguas@slu.edu
[2] *Dpt. Matemática Aplicada a la Ingeniería Industrial, Universidad Politécnica de Madrid, Spain*
E-mail: dolores.barrios.rolania@upm.es
[3] *Dpt. Matemática Aplicada a la Ingeniería Industrial, Universidad Politécnica de Madrid, Spain*
E-mail: juancarlosgarciaa@upm.es

Abstract

In this talk we consider families of polynomials that are eigenfunctions of a finite order differential operator

$$L \equiv \sum_{i=0}^{N} a_i(x)\partial_x^i$$

More precisely, we consider the Sturm-Liouville problem associated to L

$$\sum_{i=1}^{N} a_i(x)\partial_x^i P_n(x) = \lambda_n P_n(x)\,, \quad \forall n \in \mathbb{N}.$$

where $\{\lambda_n\}$ are the eigenvalues and $\{P_n\}$ the eigenfunctions.
The aim of this talk is to provide necessary conditions under which transformations given by $P_n^{(1)}(x) = P_n(x) + \gamma_n P_{n-1}(x)$ give rise to new families of eigenfunctions of another finite order differential operator $\tilde{L}$ with the same sequence of eigenvalues $\{\lambda_n\}$. In particular, we prove that Darboux transformations do not always lead to new families of eigenfunctions. The case of Hermite polynomials is approached.

Acknowledgements: Work partially supported by Agencia Estatal de Investigación, Ministerio de Ciencia e Innovación, Spain, under grants PID2019-106362GB-I00 and PID2021-122154NB-I0 and the Comunidad de Madrid multi-annual agreement with the Universidad Rey Juan Carlos under the grant Proyectos I+D para Jóvenes Doctores, Ref. M2731, project NETA-MM.

Signed graphs with maximum nullity two

Marina Arav

[1] *Department of Mathematics and Statistics, Georgia State University, US*
E-mail: marav@gsu.edu

Abstract

A signed graph is a pair (G, Σ) where G is an undirected graph (we allow parallel edges but no loops) and $\Sigma \subseteq E(G)$. The edges in Σ are called odd and the other of E even. If (G, Σ) is a signed graph with vertex-set $V = \{1, \ldots, n\}$, $S(G, \Sigma)$ is the set of all $n \times n$ real symmetric matrices $A = [a_{i,j}]$ with $a_{i,j} > 0$ if i and j are adjacent and connected by only odd edges, $a_{i,j} < 0$ if i and j are adjacent and connected by only even edges, $a_{i,j} \in \mathbb{R}$ if i and j are adjacent and connected by both even and odd edges, $a_{i,j} = 0$ if i and j are not adjacent, and $a_{i,i} \in \mathbb{R}$ for all vertices i. The parameter $M(G, \Sigma)$ is defined as the largest nullity of any matrix $A \in S(G, \Sigma)$. In 2021, Arav, Hall, van der Holst, and Li gave a characterization of 2-connected signed graphs (G, Σ) with $M(G, \Sigma) \leq 2$. In this talk, we discuss a full characterization of signed graphs (G, Σ) with $M(G, \Sigma) \leq 2$.

Sequences of lower and upper bounds for the spectral radius of a nonnegative matrix

Aikaterini Aretaki[1], *Iro Oikonomou*[2], *Maria Adam*[2]

[1] *Department of Mathematics, University of Thessaly, 35100 Lamia, Greece*
E-mail: kathy@uth.gr
[2] *Department of Computer Science and Biomedical Informatics, University of Thessaly, 35131 Lamia, Greece*
E-mail: madam@dib.uth.gr, iro.oikonomou99@gmail.com

Abstract

In this work, we expand classical Frobenius results [5] upon consecutive k-th powers of nonnegative matrices, for a positive integer $k \geq 1$, so as to establish sequences with respect to k of new lower and upper bounds for their spectral radius. With the aid of the average $(k+1)$-row sums and taking the extreme entries of the matrix, we present new bounds that generalize existing formulae in [1, 2, 3, 4, 6, 7, 8] and produce new tighter approximations for the spectral radius.

The monotonicity and convergence properties of the constructed sequences are explored and certain conditions are stated under which the new bounds are sharper than the Frobenius' bounds and other existing formulae. We further characterize the cases of equality in the aforesaid bounds, when the matrix is irreducible. Throughout, we perform illustrative numerical examples to showcase the efficiency of our proposed bounds and make comparisons among them.

Acknowledgements: Work supported by "ParICT_CENG: Enhancing ICT research infrastructure in Central Greece to enable processing of Big data from sensor stream, multimedia content, and complex mathematical modeling and simulations" (MIS 5047244).

References

[1] M. Adam, D. Aggeli, Aik. Aretaki. Some new bounds on the spectral radius of nonnegative matrices. *AIMS Math.*, 5: 701-716 (2019).

[2] M. Adam, Aik. Aretaki. Bounds for the spectral radius of nonnegative matrices and generalized Fibonacci matrices. *Special Matrices*, 10: 308-326 (2022).

[3] M. Adam, N. Assimakis, F. Babouklis. Sharp bounds on the spectral radius of nonnegative matrices and comparison to the Frobenius' bounds. *In. J. of Circuits, Systems and Signal Processing*, 14: 423-434 (2020).

[4] X. Duan, B. Zhou. Sharp bounds on the spectral radius of a nonnegative matrix. *Linear Algebra Appl.*, 439: 2961-2970 (2013).

[5] G. Frobenius, Über matrizen aus nicht negativen elementen, *Sitzungsber. Kön. Preuss. Akad. Wiss. Berlin*, 465-477 (1912).

[6] H. Lin, B. Zhou. On sharp bounds for spectral radius of nonnegative matrices. *Linear Mult. Alg.*, 65: 1554-1565 (2017).

[7] A. Melman. Upper and lower bounds for the Perron root of a nonnegative matrix, *Linear Mult. Algebra*, 61: 171-181 (2013).

[8] R. Xing, B. Zhou. Sharp bounds for the spectral radius of nonnegative matrices. *Linear Algebra Appl.*, 449: 194-209 (2014).

Nearest matrix with a prescribed eigenvalue whose Weyr partition is bounded below

Gorka Armentia[1], Juan-Miguel Gracia[1], Francisco-Enrique Velasco[1]

[1] Dpt. of Mathematics, University of the Basque Country UPV/EHU, Faculty of Pharmacy, Spain
E-mail: gorka.armentia@ehu.eus
E-mail: juanmiguel.gracia@ehu.eus
E-mail: franciscoenrique.velasco@ehu.eus

Abstract

According to [4], the distance from a matrix $A \in \mathbb{C}^{n\times n}$ to the set of matrices Z which have a prescribed complex number z_0 as a multiple eigenvalue is given by

$$\min_{\substack{Z\in\mathbb{C}^{n\times n}\\ (1,1)\prec\prec \mathsf{w}(z_0,Z)}} \|Z-A\| = \max_{t\geq 0} \sigma_{2n-1}\left(\begin{bmatrix} z_0 I_n - A & tI_n \\ O & z_0 I_n - A \end{bmatrix}\right), \qquad (1)$$

where $\prec\prec$ stands for weak majorization and $\mathsf{w}(z_0, Z)$ denotes the partition in the Weyr characteristic of z_0 as an eigenvalue of the matrix Z.

Our ongoing research (see [1]) tries to generalize (1) under the more restrictive condition of $(2,1) \prec\prec \mathsf{w}(z_0, Z)$. Indeed, we have already proved that

$$\min_{\substack{Z\in\mathbb{C}^{n\times n}\\ (2,1)\prec\prec \mathsf{w}(z_0,Z)}} \|Z-A\| = \max_{t\geq 0} \sigma_{2n-2}\left(\begin{bmatrix} z_0 I_n - A & tI_n \\ O & z_0 I_n - A \end{bmatrix}\right), \qquad (2)$$

except when the function on the right side attains its maximum value only at $t = 0$.

Our ultimate goal would be to extend (1) and (2) to the cases where $\mathsf{w}(z_0, Z)$ weakly majorizes (k, k) or $(k + 1, k)$, with $k \geq 1$.

Acknowledgements: This work is part of project PID2021-124827NB-I00, funded by MCIN/AEI/0.13039/501100011033/ and by "ERDF A way of making Europe", and it is also supported by the University of the Basque Country UPV/EHU through grant GIU21/020.

References

[1] G. Armentia, J. M. Gracia, F. E. Velasco. Nearest matrix with a prescribed eigenvalue of bounded multiplicities. *Linear Algebra Appl.*, 592: 188–209 (2020).

[2] Kh. D. Ikramov, A. M. Nazari. On the distance to the closest matrix with triple zero eigenvalue. *Mathematical Notes*, 73, No. 4: 511–520 (2003).

[3] R. A. Lippert. Fixing multiple eigenvalues by a minimal perturbation. *Linear Algebra Appl.*, 432: 1785–1817 (2010).

[4] A. N. Malyshev. A formula for the 2-norm distance from a matrix to the set of matrices with multiple eigenvalues. *Numer. Math.*, 83, No. 3: 443–454 (1999).

[5] E. Mengi. Locating a nearest matrix with an eigenvalue of prespecified algebraic multiplicity. *Numer. Math.*, 118: 109–135 (2011).

Doubly Structured Mappings and Backward errors for Matrix pencils arising in Optimal Control

Mohit Kumar Baghel[1], *Punit Sharma*[1]

[1] *Mathematics Department, Indian Institute of Technology Delhi, India*
E-mail: mohitbaghel18@gmail.com, punit.sharma@maths.iitd.ac.in

Abstract

For a given class of structured matrices $\mathbb{S}$, we find necessary and sufficient conditions on vectors $x, w \in \mathbb{C}^{n+m}$ and $y, z \in \mathbb{C}^n$ for which there exists $\Delta = [\Delta_1 \ \Delta_2]$ with $\Delta_1 \in \mathbb{S}$ and $\Delta_2 \in \mathbb{C}^{n,m}$ such that $\Delta x = y$ and $\Delta^* z = w$. We also characterize the set of all such mappings Δ and provide sufficient conditions on vectors x, y, z, and w to investigate a Δ with minimal Frobenius norm. The structured classes $\mathbb{S}$ we consider include (skew)-Hermitian, (skew)-symmetric, pseudo(skew)-symmetric, J-(skew)-symmetric, pseudo(skew)-Hermitian, positive (semi)definite, and dissipative matrices, see [2]. These mappings are then used in computing the structured eigenvalue/eigenpair backward errors of matrix pencils arising in optimal control.

The minimal norm solutions to such doubly structured mappings can be very handy in the perturbation analysis of matrix pencils arising in control systems [1, 3]. In particular, for the computation of structured eigenvalue/eigenpair backward errors of matrix pencils $L(z)$ of the form

$$L(z) = M + zN := \begin{bmatrix} 0 & J-R & B \\ (J-R)^* & 0 & 0 \\ B^* & 0 & S \end{bmatrix} + z \begin{bmatrix} 0 & E & 0 \\ -E^* & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad (1)$$

where $J, R, E, Q \in \mathbb{C}^{n,n}$, $B \in \mathbb{C}^{n,m}$ and $S \in \mathbb{C}^{n,m}$ satisfy $J^* = -J$, $R^* = R$ is positive semidefinite, $E^* = E$, and $S^* = S$ is positive definite.

Our work is motivated by [3], where the eigenpair backward errors have been computed while preserving the block and symmetry structures of pencils of the form $L(z)$, where only the Hermitian structure was considered on R. The definiteness structure on R describes the energy dissipation in the system and guarantees the stability of the underlying port-Hamiltonian system. This makes it essential to preserve the definiteness of R to preserve the system's port-Hamiltonian structure. For more details one can have a look on the preprint the references in [4].

Acknowledgements: Supported by Department of Science and Technology, Ministry of Science and Technology, India

References

[1] C. Mehl, V. Mehrmann, and P. Sharma. Stability radii for linear Hamiltonian systems with dissipation under structure-preserving perturbations. *SIAM Journal on Matrix Analysis and Applications*, 37 (4): 1625–1654, 2016

[2] D.S. Mackey, N. Mackey, and F. Tisseur. Structured mapping problems for matrices associated with scalar products. Part I: Lie and Jordan algebras. *SIAM journal on matrix analysis and applications*, 29 (4): 1389–1410, 2008

[3] C. Mehl, V. Mehrmann, and P. Sharma, Structured eigen- value/eigenvector backward errors of matrix pencils arising in optimal control, Electronic Journal of Linear Algebra, 34 (2018)

[4] M.K. Baghel, P. Sharma, Doubly structured mapping problems of the form $\Delta x = y$ and $\Delta^* z = w$ (2022), arxiv, https://arxiv.org/abs/2208.12429

CP decomposition and low-rank approximation of antisymmetric tensors

Erna Begović[1], *Lana Periša*[2],

[1] *University of Zagreb, Croatia*
E-mail: ebegovic@fkit.hr
[2] *Rimac Technology, Croatia*
E-mail: lana.perisa@rimac-technology.com

Abstract

In this talk we explore the antisymmetric tensors, their CP decomposition and the low-rank approximation algorithms. For a given antisymmetric tensor $\mathcal{A} \in \mathbb{R}^{n\times n\times n}$ we are looking for its low-rank antisymmetric approximation which is represented via only three vectors. First, we discuss a suitable low-rank format of the approximation $\tilde{\mathcal{A}}$ of $\mathcal{A}$,

$$\tilde{\mathcal{A}} = \frac{1}{6}(x \circ y \circ z + y \circ z \circ x + z \circ x \circ y - x \circ z \circ y - y \circ x \circ z - z \circ y \circ x),$$

where $x, y, z \in \mathbb{R}^n$. Then we propose an alternating least squares structure-preserving algorithm for finding such approximation. The algorithm is based on solving a minimization problem in each tensor mode. We compare our algorithm with a "naive" idea which uses a posteriori antisymmetrization. Additionally, we study the tensors with partial antisymmetry, that is, antisymmetry in only two modes. The algorithms are implemented in Julia programming language and their numerical performance is examined.

Acknowledgements: Work supported by Croatian Science Foundation under the project UIP-2019-04-5200.

On the smallest positive eigenvalue of bipartite graphs

Sasmita Barik[1], *Subhasish Behera*[2], *Sukanta Pati*[3]

[1] *School of Basic Sciences, IIT Bhubaneswar, India*
E-mail: sasmita@iitbbs.ac.in
[2] *School of Basic Sciences, IIT Bhubaneswar, India*
E-mail: sb52@iitbbs.ac.in
[3] *Department of Mathematics, IIT Guwahati, India*
E-mail: pati@iitg.ac.in

Abstract

The smallest positive eigenvalue $\tau(G)$ of a simple graph G is same as the smallest positive eigenvalue of its adjacency matrix $A(G)$. Let $\mathscr{G}_n$ be the class of all connected bipartite graphs with $n = 2m$ vertices, having a unique perfect matching. In 1984, Godsil obtained the extremal graph for the minimum smallest positive eigenvalue in class $\mathscr{G}_n$. Here, we discuss the upper bounds and extremal graphs for the $\tau(G)$, where $G \in \mathscr{G}_n$. Further, we talk about the limit points of set Ψ, where $\Psi = \{\tau(G)|G \in \mathscr{G}_{2m}, m = 1, 2, \ldots\}$.

Acknowledgements: The second author acknowledges the financial support from CSIR, Government of India.

References

[1] S. Barik, S. Behera. On the smallest positive eigenvalue of bipartite unicyclic graphs with a unique perfect matching. *Discret. Math.*, 346(2): 113252 (2023).

[2] S. Barik, S. Behera. On the smallest positive eigenvalue of bipartite unicyclic graphs with a unique perfect matching II. *Linear Multilinear Alg.*, Accepted.

[3] D. Cvetkovíc, M. Doob, I. Gutman. On graphs whose spectral radius does not exceed $\sqrt{2+\sqrt{5}}$. *ARS Combinatoria*, 14: 225–239 (1982).

[4] C. D. Godsil. Inverses of trees. *Combinatorica*, 5: 33–39 (1985).

[5] A. J. Hoffman. On limit points of spectral radii of nonnegative symmetric integral matrices, in: Y. Alavi et al. (Eds). *Graph Theory and its Applications, Lecture Notes in Mathematics, Springer, New York*, 303: 165–172 (1972).

[6] Y. Hong. Sharp lower bounds on the eigenvalues of trees. *Linear Algebra Appl.*, 113: 101–105 (1989).

[7] S. Pavlíková, J. Krč-jediný. On the inverse and dual index of a tree. *Linear Multilinear Alg.*, 28: 93–109 (1990).

[8] S. Rani, S. Barik. On alternating paths and the smallest positive eigenvalue of trees. *J. Comb. Optim.*, 39: 589–601 (2019).

[9] S. Rani, S. Barik. Upper bounds on the smallest positive eigenvalue of trees. *Annals of combinatorics*, DOI: 10.1007/s00026-022-00619-x.

[10] J. H. Smith. Some properties of the spectrum of a graph, in: Richard Guy (Editions). *Combinatorial Structures and Their Applications*, 403–406 (1970).

[11] F. J. Zhang, A. Chang. Acyclic molecules with greatest HOMO-LUMO separation. *Discret. Appl. Math.*, 98: 165–171 (1999).

Optimized Higher Order Dynamic Mode Decomposition Analysis of Electrocardiography Datasets

Andrés Bell-Navas[1], *Jesús Garicano-Mena*[1,2], *Soledad Le Clainche*[1,2]

[1] **ETSI Aeronáutica y del Espacio, Universidad Politécnica de Madrid, 28040, Madrid, Spain**
[2] **Center for Computational Simulation (CCS), 28660, Boadilla del Monte, Spain**
E-mail: a.bell@upm.es jesus.garicano.mena@upm.es soledad.leclainche@upm.es

Abstract

Higher Order Dynamic Mode Decomposition (HODMD) is a data-driven method which can be reinterpreted as an approach to factorise data matrixes describing the evolution in time of a determined process [1] [2]. It has been traditionally used in the field of fluid dynamics and to analyse complex non-linear dynamical systems for the modeling of diverse industrial applications [2] [3]. Recently, HODMD has been applied for the first time to medical image datasets in [4], suggesting a great potential as a feature extraction technique to identify characteristic patterns of different heart diseases in echocardiography images and predict their evolution. Due to the economical acquisition of medical imaging (in special EKG images), it is foreseeable to have to deal with large databases. Therefore, the necessity to implement HODMD in the medical field in the most efficient way becomes more important. In this work, several strategies to optimize the performance of the HODMD algorithm [4] have been explored. In particular, diverse implementations of the different computational kernels controlling the overall performance (eigenvalue decomposition, and higher order singular value decomposition - HOSVD) have been analysed. Considering also the actual context in which a tendency to leverage computational resources from the cloud can be perceived, the use of different computational architectures has been explored as well. Two databases of echocardiography images have been used for the analysis of the different implementations. The first database has been obtained with respect to a long axis view (LAX), and the second one with respect to a short axis view (SAX).

Table 1 presents the performance of the HODMD algorithm under different implementations according to the libraries used when the LAX and SAX images are used. In all the implementations considered, the relative root mean square error (RRMSE) of the HODMD reconstruction is negligible (in the order of 1e−14), representing only the machine error. Concerning the computational cost (represented with the average time of HOSVD $\bar{t}_{HOSVD}$ and the average time of HODMD $\bar{t}_{HODMD}$), the implementation using the Torch library and a standard CPU from a local computer (row named as 'Torch CPU') is the most efficient one. This is due to that the associated time for HOSVD $\bar{t}_{HOSVD}$, supposing the hardest stage in the HODMD technique, is considerably lower

than those of the other implementations. Meanwhile, when a high-performance GPU from the cloud is used (row named as 'Torch GPU'), the computational cost is a little higher than the case with Torch CPU. This is because no local resources are employed, which may slow down the algorithm. In addition, a preload of the data has to be performed prior to HOSVD and HODMD, which may contribute to the increase of the global time as well.

Table 1: Comparison of the performance of different implementations of the HODMD algorithm using LAX and SAX medical images.

LAX			
Implementation	RRMSE	$\bar{t}_{HOSVD}$(s)	$\bar{t}_{HODMD}$(s)
MATLAB	9.22e−14	7.78e+01	2.50e−01
NumPy	8.40e−14	1.84e+02	5.61e−01
Torch CPU	4.37e−14	5.38e+01	1.27e+00
Torch GPU	9.84e−13	6.23e+01	2.87e+00
SAX			
Implementation	RRMSE	$\bar{t}_{HOSVD}$(s)	$\bar{t}_{HODMD}$(s)
MATLAB	1.27e−14	8.04e+01	2.47e−01
NumPy	5.07e−14	1.83e+02	6.86e−01
Torch CPU	4.87e−14	5.63e+01	2.15e+00
Torch GPU	8.13e−13	6.40e+01	2.51e+00

As a conclusion, among the explored implementations of the HODMD algorithm, the one based on Torch executed with a standard CPU in a local computer is the fastest. This encourages and motivates the use of free software, and, thus, open science. As future work, a combination of libraries could be used with the overarching goal to optimize the computational performance of the HODMD technique. On the other hand, if a GPU from a local computer was used, it would be likely to further reduce the computational cost.

Acknowledgements: The authors acknowledge the support provided by Grant TED2021-129774B-C21 and by Grant PLEC2022-009235, funded by MCIN/AEI /10.13039/501100011033 and by the European Union "NextGenerationEU"/PRTR.

References

[1] Begiashvili, B., Groun, N., Garicano-Mena, J., Le Clainche, S., & Valero, E. (2023). Data-driven modal decomposition methods as feature detection techniques for flow problems: A critical assessment. Physics of Fluids. https://doi.org/10.1063/5.0142102

[2] Vega, J. M., & Le Clainche, S. (2021). Higher Order Dynamic Mode Decomposition. Higher Order Dynamic Mode Decomposition and Its Applications, 29–83. https://doi.org/10.1016/b978-0-12-819743-1.00009-4

[3] Le Clainche, S., Vega, J. M., & Soria, J. (2017). Higher order dynamic mode decomposition of noisy experimental data: The flow structure of a zero-net-mass-flux jet. Experimental Thermal and Fluid Science, 88, 336–353. https://doi.org/10.1016/j.expthermflusci.2017.06.011

[4] Groun, N., Villalba-Orero, M., Lara-Pezzi, E., Valero, E., Garicano-Mena, J., & Le Clainche, S. (2022). Higher Order Dynamic Mode decomposition: From fluid dynamics to heart disease analysis. Computers in Biology and Medicine, 144, 105384. https://doi.org/10.1016/j.compbiomed.2022.105384

Structured solutions of the reduced biquaternion matrix equations with applications

Sk. Safique Ahmad[1], *Neha Bhadala*[2]

[1] *Department of Mathematics, IIT Indore, India*
E-mail: safique@iiti.ac.in
[2] *Department of Mathematics, IIT Indore, India*
E-mail: phd1901141004@iiti.ac.in

Abstract This paper presents a framework for computing the structure constrained least squares solutions to the generalized reduced biquaternion matrix equations. We have looked at three different matrix equations, a linear matrix equation in one unknown L-structure, a linear matrix equation in several unknown L-structures, and the general coupled linear matrix equations in one unknown L-structure. Also, the framework can be adapted to a variety of applications. Firstly, we find the least squares Toeplitz solutions for the reduced biquaternion matrix equation $(AXB, CXD) = (E, F)$. Then, we derive a purely imaginary solution to the reduced biquaternion matrix equation $AX = E$ and investigate its application to color image restoration. Finally, we have utilized our framework to find the structure constrained least squares solutions to the complex matrix equations. As an illustration, we have obtained the least squares Hankel solutions to the complex matrix equation $AXB + CYD = E$. Moreover, we have used our results to solve partially described inverse eigenvalue problems, partially described inverse generalized eigenvalue problems, and partially described inverse quadratic eigenvalue problems. Our study concludes with algorithms and numerical examples.

Acknowledgements: Research work of second author is funded by PMRF (Prime Minister's Research Fellowship) India.

References

[1] Guiping Xu, Musheng Wei and Daosheng Zheng. On solutions of matrix equation $AXB + CYD = F$. *Linear Algebra Appl.*, 279: 93–109 (1998).

[2] F. Zhang, Musheng Wei, Ying Li and Jianli Zhao. Special least squares solutions of the quaternion matrix equation $AX = B$ with applications. *Appl. Math. Comput.*, 270: 425–433 (2015).

[3] Shi-Fang Yuan and Qing-Wen Wang. L-structured quaternion matrices and quaternion linear matrix equations. *Linear Multilinear Algebra*, 64: 321–339 (2016).

Quantum walk-based ranking algorithms for directed networks

Paola Boito[1], Roberto Grena[2]

[1] Department of Mathematics, University of Pisa, Italy
E-mail: paola.boito@unipi.it
[2] C. R. ENEA Casaccia, via Anguillarese 301, 00123 Roma, Italy
E-mail: roberto.grena@enea.it

Abstract

The recently renewed surge of interest in quantum computation has motivated (among others) applications to network analysis, where the perspective of quantum advantage holds promises for efficient treatment of large-scale problems. One facet of this line of research is the use of quantum walks – as opposed to classical random walks – in the definition and analysis of centrality measures for graphs.

In this work we focus on unitary, continuous-time quantum walks (CTQW) applied to directed graphs. Recall that the time evolution of a CTQW on a graph is described by the Schrödinger equation

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle, \tag{1}$$

where $|\psi(t)\rangle$ is the state of the system at time t, and H is the Hamiltonian operator. The evolution operator takes the form $U(t) = \exp(-itH)$. Note that the underlying Hilbert space has dimension equal to the number of nodes, in contrast to usual setups for discrete-time quantum walks, which require a larger dimension.

In the directed case, usual choices of Hamiltonian matrices that characterize the walk dynamics (e.g., the adjacency matrix) are typically not symmetric and therefore do not directly yield unitary walk operators. We address this difficulty by re-casting classical ranking algorithms, such as HITS and PageRank, as eigenvector problems for symmetric matrices, and using these symmetric matrices as Hamiltonians for CTQWs, in order to obtain a unitary evolution operator.

Note that CTQWs exhibit a dependence on the initial state $|\psi(0)\rangle$. Therefore, the choice of $|\psi(0)\rangle$ plays a crucial role in a quantum walk-based ranking algorithm. Here we experiment with two options: a vector with uniform occupation and a vector weighted w.r.t. in- or out-degrees (for authority and hub centrality, respectively).

The four new quantum ranking algorithms presented here have been extensively tested and compared to classical HITS and PageRank. Numerical results

show that, despite some variation in behavior, all the methods are effective in finding the first and top ten nodes in larger-sized graphs.

Acknowledgements: This work was partially supported by the project PRA_2020_92 "Quantum computing, technologies and application" funded by the University of Pisa.

References

[1] P. Boito and R. Grena. Ranking nodes in directed networks via continuous-time quantum walks. *ArXiv preprint* arXiv:2210.13379 (2022).

[2] J. A. Izaac et al. Centrality measure based on continuous-time quantum walks and experimental realization. *Phys. Rev. A* 95, 032318 (2017).

[3] L. Rossi, A. Torsello, and E. R. Hancock, Node Centrality for Continuous-Time Quantum Walks, in *S+SSPR 2014: Proceedings of the Joint IAPR International Workshop on Structural, Syntactic, and Statistical Pattern Recognition*, Lecture Notes in Computer Science Vol. 8621 (Springer 2014) pp. 103-112.

Cyclic matrices, polynomial interpolation, and Sylvester equation over division rings

Vladimir Bolotnikov

Department of Mathematics, William&Mary, USA
E-mail: vladi@math.wm.edu

Abstract

As is well known, any complex cyclic matrix A is similar to the unique companion matrix associated with the minimal polynomial of A. On the other hand, a cyclic matrix over a division ring F is similar to a companion matrix of a polynomial which is defined up to polynomial similarity. We will discuss more rigid canonical forms by embedding a, autor2-article given cyclic matrix over a division ring F into a controllable or an observable pair. Using the characterization of ideals in $F[z]$ in terms of controllable and observable pairs we will consider ideal interpolation schemes in $F[z]$ which merge into a polynomial interpolation problem containing both left and right interpolation conditions. The solvability criterion for such a problem is given in terms of certain Sylvester equation, which also will be discussed in some detail. The talk is based on the papers [1, 2].

Acknowledgements: Work (partially) supported by Simons Foundation grant 524539

References

[1] V. Bolotnikov, *Cyclic matrices and polynomial interpolation over division rings*, Linear Algebra Appl. **646** (2022), 132–174.

[2] V. Bolotnikov, *Lagrange interpolation over division rings*, Comm. Algebra **48** (2020), no. 9, 4065–4084.

Geometry of Sub-Algebras of $\mathcal{H}ol(\Gamma \cup \mathrm{Int}(\Gamma))$ and Zeros of Holomorphic Functions

Babhrubahan Bose[1], *Saikat Roy*[2], *Dembalya Sain*[3]

1 Department of Mathematics, Indian Institute of Science, Bengaluru 560012, Karnataka, India
Email: babhrubahanb@iisc.ac.in
[2] Department Of Mathematics, Indian Institute of Technology Bombay, Mumbai 400076, Maharashtra, India
E-mail: saikatroy.cu@gmail.com
[3] Department of Mathematics, Indian Institute of Information Technology, Raichur 584135, Karnataka, India
E-mail: saindebmalya@gmail.com

Abstract

We study $\mathcal{H}ol(\Gamma \cup \mathrm{Int}(\Gamma))$, the normed algebra of all holomorphic functions defined on some simply connected neighborhood of a simple closed curve Γ in $\mathbb{C}$, equipped with the supremum norm on Γ. We explore the geometry of nowhere vanishing, point separating sub-algebras of $\mathcal{H}ol(\Gamma \cup \mathrm{Int}(\Gamma))$. We characterize the extreme points and the exposed points of the unit balls of the said sub-algebras. We also characterize the smoothness of an element in these sub-algebras by using Birkhoff-James orthogonality techniques. As a culmination of our study, we assimilate the geometry of the aforesaid sub-algebras with some classical concepts of complex analysis and establish a connection between Birkhoff-James orthogonality and zeros of holomorphic functions.

References

[1] B. Bose, S. Roy, D. Sain, *"Geometry of sub-algebras of $\mathcal{H}ol(\Gamma \cup \mathrm{Int}(\Gamma))$ and zeros of holomorphic functions"*, Banach J. Math. Anal., 17, 23 (2023). https://doi.org/10.1007/s43037-022-00242-1

Computational aspects related to Serre's reduction of underdetermined linear functional systems

Mohamed S. Boudellioua[1]

[1] *Department of Mathematics, Sultan Qaboos University, Muscat, Oman*
E-mail: boudell1@squ.edu.om

Abstract

In this paper we present two algorithms for the reduction of a class of rectangular multivariate polynomial matrices to their Smith forms, using the Maple packages QuillenSuslin and OreModules. The particular resulting Smith form corresponds to the simplification of an underdetermined linear system of functional equations to one containing a single equation in one unknown function. This reduction is known as Serre's reduction of functional systems.

1 Introduction

Polynomial matrices over $\mathbb{R}[s]$, $s \equiv d/dt$ are used to represent linear systems of ordinary differential equations, see for example [11]. The ring $\mathbb{R}[s]$ is a principal ideal domain with the Euclidean division property and matrices over such a ring are equivalent to their Smith normal form. However for more general linear functional systems e.g. partial differential systems or delay-differential systems, the resulting system matrices are multivariate. Polynomial rings in more than one indeterminate are not principal ideal rings and matrices over these rings are in general not equivalent to their Smith forms. Despite its importance in single variable matrix theory, the Smith normal form in the multivariate case has received relatively little attention. A few exceptions are [7], [6], [8], [10], and [3]. The computations involved in the reduction of a given square matrix to its equivalent Smith form have been set out in [1] using Maple. The main motivation behind the reduction of a multivariate polynomial matrix to its Smith form is to be able to reduce the associated system of linear functional equations to a one containing fewer equations and unknowns. The reduction involved must of

course preserve relevant system properties. The reduced equivalent representation simplifies in general the study of such systems. The results on the reduction to Smith form obtained by the authors mentioned before deal with the case of square matrices. In this paper we consider the case of rectangular matrices associated with underdetermined functional systems. The class of systems which will be dealt with are those equivalent to one involving a single equation in one unknown. In what follows let $D = K[x_1, \ldots, x_n]$ denote a commutative multivariate polynomial ring with indeterminates $x_1, \ldots, x_n$ over an arbitrary but fixed field K.

2 Definitions

Definition 1 *Let $T \in D^{q\times p}, p > q$, the Smith form of T is given by*

$$S = \begin{pmatrix} diag\{\Phi_i\} & 0 \end{pmatrix} \tag{1}$$

where

$$\Phi_i = \begin{cases} \alpha_i/\alpha_{i-1}, & 1 \leq i \leq r \\ 0, & r < i \leq q, \end{cases} \tag{2}$$

r is the normal rank of T, $\alpha_0 \equiv 1$, α_i is the gcd of all the $i \times i$ minors of T and Φ_i's satisfy the divisibility property

$$\Phi_1 | \Phi_2 | \ldots | \Phi_r. \tag{3}$$

Definition 2 *The general linear group $GL_p(D)$ is defined by*

$$GL_p(D) = \left\{ M \in D^{p\times p} \;\middle|\; \exists N \in D^{p\times p} : MN = NM = I_p \right\} \tag{4}$$

An element $M \in GL_p(D)$ is called a unimodular matrix. It follows that M is unimodular if and only if $|M| \in \mathbb{K}\backslash\{0\}$.

Definition 3 *Let T_1 and T_2 denote two matrices in $D^{q\times p}$ then T_1 and T_2 are said to be (unimodular) equivalent if there exist two matrices $M \in GL_q(D)$ and $N \in GL_p(D)$ such that*

$$T_2 = MT_1N \tag{5}$$

Unimodular equivalence has been shown to exhibit fundamental algebraic properties amongst its invariants. In particular, it preserves the zero structure of the original matrix which is captured by the determinantal ideals of the matrix. In fact for the case when $D = K[x_1]$, it is well known that every matrix with elements in D is equivalent to its Smith form. However this result is not valid for the case when $D = K[x_1, \ldots, x_n], n > 1$.

3 Reduction to Smith form by unimodular equivalence

The aim of the reduction is to simplify linear functional systems in the sense of finding an equivalent presentation which contains only one equation in one unknown. This generally makes it easier to study the structural properties of the linear functional system and in some cases can be used to compute its closed-form solutions. This reduction also finds applications in numerical analysis. The objective of the equivalence transformation applied on the matrix is to produce an identity matrix of appropriate size at the top left corner of the original matrix.

Theorem 1 (Section 5 of [5]) *Let $D = K[x_1, \ldots, x_n]$ be a commutative polynomial ring over a field K and $R \in D^{q\times p}$ a full row rank matrix. Then the following two assertions are equivalent:*

1. *The ideal $I_q(R)$ generated by the $q \times q$ minors of R is principal, i.e. can be generated by the greatest common divisor Φ of these minors.*
2. *There exist $R' \in D^{q\times p}, R'' \in D^{q\times q}$, and $N \in GL_p(D)$ such that:*

$$R = R''R', \qquad \det(R'') = \Phi, \qquad R'N = \begin{pmatrix} I_q & 0 \end{pmatrix} \tag{6}$$

Theorem 2 *[2] Let $D = K[z_1, \ldots, z_n]$ and $T \in D^{q\times p}$, $p > q$ with full row rank, then T is equivalent to the Smith form*

$$S = \begin{pmatrix} I_{q-1} & 0 & 0 \\ 0 & \Phi_q & 0 \end{pmatrix} \tag{7}$$

where $\Phi_q \in D$ is the gcd of the $q \times q$ minors of T, if and only if there exist a vector $U \in D^q$ which admits a left inverse in D such that the matrix $\begin{pmatrix} T & U \end{pmatrix}$ has a right inverse over D and the ideal generated by the $q \times q$ minors of T is principal.

Proof. Let $T \in D^{q\times p}$ and suppose that there exist a vector $U \in D^q$ which admits a left inverse in D satisfying the given condition and that the ideal generated by the $q \times q$ minors of T is principal. Then since U admits a left inverse in D, there exists a matrix $M_1 \in GL_q(D)$ such that $M_1U = E_q$, where E_q is the q-th column of I_q. It follows that

$$M_1 \begin{pmatrix} T & U \end{pmatrix} = \begin{pmatrix} T_1 & 0 \\ T_2 & 1 \end{pmatrix} \tag{8}$$

where $T_1 \in D^{(q-1)\times p}$ and $T_2 \in D^{1\times p}$ are given by

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = M_1T \tag{9}$$

Now since the matrix on the RHS of (8) has admits a right inverse over D, it follows that T_1 also admits a right inverse over D, i.e. there exists a matrix $N_1 \in GL_p(D)$ such that

$$T_1 N_1 = \begin{pmatrix} I_{q-1} & 0 \end{pmatrix} \tag{10}$$

Then,

$$\begin{pmatrix} T_1 & 0 \\ T_2 & 1 \end{pmatrix} \begin{pmatrix} N_1 & 0 \\ 0 & 1 \end{pmatrix} = \left(\begin{array}{cc|c} I_{q-1} & 0 & 0 \\ T_3 & T_4 & 1 \end{array}\right) \tag{11}$$

where $T_3 \in D^{1\times(q-1)}, T_4 \in D^{1\times(p-q+1)}$ and

$$\begin{pmatrix} T_3 & T_4 \end{pmatrix} = T_2 N_1 \tag{12}$$

It follows that

$$M_1 T N_1 = \begin{pmatrix} I_{q-1} & 0 \\ T_3 & T_4 \end{pmatrix} \tag{13}$$

Premultiplying the matrix $M_1 T N_1$ in (13) by the unimodular matrix

$$M_2 = \begin{pmatrix} I_{q-1} & 0 \\ -T_3 & 1 \end{pmatrix} \tag{14}$$

yields the matrix

$$M_2 M_1 T N_1 = \begin{pmatrix} I_{q-1} & 0 \\ 0 & T_4 \end{pmatrix} \tag{15}$$

Now since the ideal generated by the $q \times q$ minors of T is principal, by virtue of the Lin-Bose Theorem 1, there exists a matrix $N_2 \in GL_p(D)$ with

$$N_2 = \begin{pmatrix} I_q & 0 \\ 0 & \bar{N} \end{pmatrix} \tag{16}$$

such that

$$M_2 M_1 T N_1 N_2 = \begin{pmatrix} I_{q-1} & 0 & 0 \\ 0 & \Phi_q & 0 \end{pmatrix} \tag{17}$$

where $T_4 \bar{N} = \begin{pmatrix} \Phi_q & 0 \end{pmatrix}$.

Conversely assume that $T \in D^{q\times p}$ is equivalent to the Smith form

$$S = \begin{pmatrix} I_{q-1} & 0 & 0 \\ 0 & \Phi_q & 0 \end{pmatrix}. \tag{18}$$

where $\Phi_q \in D$ is the gcd of all the i^{th} order minors of T. It follows that there exist unimodular matrices $M \in GL_q(D)$ and $N \in GL_p(D)$ such that $S = MTN$.

Now consider the vector $U = M^{-1}E_q$ where E_q is the q^{th} column of I_n, then

$$\begin{aligned} M\begin{pmatrix} T & U \end{pmatrix}\begin{pmatrix} N & 0 \\ 0 & 1 \end{pmatrix} &= M\begin{pmatrix} T & M^{-1}E_q \end{pmatrix}\begin{pmatrix} N & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} MTN & E_q \end{pmatrix} \\ &= \begin{pmatrix} I_{q-1} & 0 & 0 & 0 \\ 0 & \Phi_q & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} I_q & 0 \end{pmatrix} \end{aligned} \tag{19}$$

i.e. the matrix $\begin{pmatrix} T & U \end{pmatrix}$ has a right inverse over D. Clearly the ideal of the $q \times q$ minors of S is generated by the unique polynomial $\Phi_q \in D$ and therefore the ideal generated by $q \times q$ minors of T is principal. ■

Example 1 *Consider the system of linear delay-differential equations*

$$T\psi(t) = 0 \tag{20}$$

where $\psi(t) = \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \\ \psi_3(t) \\ \psi_4(t) \end{pmatrix}$ *and the system matrix* T *is given by*

$$T = \begin{pmatrix} 2d\sigma^2+\sigma^3+\sigma^2+1 & d\sigma^2-d\sigma+d & 2d\sigma+\sigma^2 & d\sigma^2+d\sigma+d+\sigma^2 \\ 2d\sigma+\sigma^2+\sigma & d\sigma-d & 2d+\sigma & d\sigma+d+\sigma \\ 2d^2\sigma+d\sigma^2+d\sigma+\sigma & d^2\sigma-d^2-1 & 2d^2+d\sigma+1 & d^2\sigma+d^2+d\sigma \end{pmatrix} \tag{21}$$

where $D = \mathbb{R}[d, \sigma]$, $d\,f(t) = \dot{f}(t)$, $\sigma f(t) = f(t-h)$ *and* $h \in \mathbb{R}^+$. *Consider* $U = (\sigma \quad 1 \quad d)^T \in D^3$ *and* $P = (T \quad U) \in D^{3\times 5}$. *Using the package* OreModules *in Maple, (see [4]), we can check that* P *admits a right inverse over* D. *Also using Gröbner bases, we can verify that the ideal of the* 3×3 *minors of* T *is generated by the polynomial* $d + \sigma$. *It follows that the system in (20,21) is equivalent to the following simple delay-differential equation:*

$$\dot{x}(t) + x(t-h) = 0 \tag{22}$$

Algorithm 1 (OreModules) >

```
> libname:="OreModules",libname:with(OreModules):with(
    LinearAlgebra):

> A:=DefineOreAlgebra(diff=[d,t],dual_shift=[sigma,s],polynom=[t,
    s]):

> T:=Matrix(3,4,[2*d*sigma^2+sigma^3+sigma^2+1,d*sigma^2-d*sigma+
    d,2*d*sigma+sigma^2,d*sigma^2+d*sigma+d+sigma^2,2*d*sigma+
    sigma^2+sigma,d*sigma-d,2*d+sigma,d*sigma+d+sigma,2*d^2*sigma
    +d*sigma^2+d*sigma+sigma,d^2*sigma-d^2-1,2*d^2+d*sigma+1,d^2*
    sigma+d^2+d*sigma]);U:=<sigma,1,d>;
```

$$T := \begin{bmatrix} 2\,d\sigma^2+\sigma^3+\sigma^2+1 & d\sigma^2-d\sigma+d & 2\,d\sigma+\sigma^2 & d\sigma^2+d\sigma+\sigma^2+d \\ 2\,d\sigma+\sigma^2+\sigma & d\sigma-d & 2\,d+\sigma & d\sigma+d+\sigma \\ 2\,d^2\sigma+d\sigma^2+d\sigma+\sigma & d^2\sigma-d^2-1 & 2\,d^2+d\sigma+1 & d^2\sigma+d^2+d\sigma \end{bmatrix}$$

$$U := \begin{bmatrix} \sigma \\ 1 \\ d \end{bmatrix}$$

Constructing the matrix $P = (T|-U)$,

```
> P:=<T|-U>;
```

$$P := \begin{bmatrix} 2\,d\sigma^2+\sigma^3+\sigma^2+1 & d\sigma^2-d\sigma+d & 2\,d\sigma+\sigma^2 & d\sigma^2+d\sigma+\sigma^2+d & -\sigma \\ 2\,d\sigma+\sigma^2+\sigma & d\sigma-d & 2\,d+\sigma & d\sigma+d+\sigma & -1 \\ 2\,d^2\sigma+d\sigma^2+d\sigma+\sigma & d^2\sigma-d^2-1 & 2\,d^2+d\sigma+1 & d^2\sigma+d^2+d\sigma & -d \end{bmatrix}$$

Computing the right inverse of P,

```
> Prightinv:=RightInverse(P,A);
```

$$Prightinv := \begin{bmatrix} 1 & -\sigma & 0 \\ 0 & 0 & 0 \\ -\sigma & \sigma^2-d & 1 \\ 0 & 0 & 0 \\ \sigma & -2\,d^2-d\sigma-\sigma^2-1 & 2\,d+\sigma \end{bmatrix}$$

Computing a minimal parametrization Q *of* P,

```
> Q:=convert(MinimalParametrizations(P,A)[1],Matrix);
```

$$Q := \begin{bmatrix} -d & 0 \\ 0 & 1 \\ d\sigma & 1 \\ 1 & -1 \\ d+\sigma & 0 \end{bmatrix}$$

Computing the left inverse of Q,

```
> LeftInverse(Q,A);
```

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Computing the matrix $PQ = [Prightinverse|Q]$ *and checking its determinant,*

```
> PQ:=<Prightinv|Q>;DetPQ:=Determinant(PQ);
```

$$PQ := \begin{bmatrix} 1 & -\sigma & 0 & -d & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -\sigma & \sigma^2 - d & 1 & d\sigma & 1 \\ 0 & 0 & 0 & 1 & -1 \\ \sigma & -2\,d^2 - d\sigma - \sigma^2 - 1 & 2\,d + \sigma & d + \sigma & 0 \end{bmatrix}$$

$$DetPQ := 1$$

Extarcting the matrix $Q2$ *and* $Q1$ *from* Q,

```
> Q2:=SubMatrix(Q,5..5,1..2);
```

$$Q2 := \begin{bmatrix} d + \sigma & 0 \end{bmatrix}$$

```
> Q1:=SubMatrix(Q,1..4,1..2);
```

$$Q1 := \begin{bmatrix} -d & 0 \\ 0 & 1 \\ d\sigma & 1 \\ 1 & -1 \end{bmatrix}$$

Checking the left inverse of $Q1$,

```
> LeftInverse(Q1,A);
```

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Computing the SyzygyModule F *of* $Q1$,

```
> F:=SyzygyModule(Q1,A);
```

$$F := \begin{bmatrix} \sigma & -1 & 1 & 0 \\ 1 & d & 0 & d \end{bmatrix}$$

Computing $Q3$, *the right inverse of* F,

```
> Q3:=convert(RightInverse(F,A),Matrix);
```

$$Q3 := \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & -\sigma \\ 0 & 0 \end{bmatrix}$$

Constructing the unimodular matrix $N = [Q3|Q1]$ *and checking its determinant,*

```
> N:=<Q3|Q1>;detN:=Determinant(N);
```

$$N := \begin{bmatrix} 0 & 1 & -d & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -\sigma & d\sigma & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$detN := -1$$

Computing the unimodualr matrix $M = [T.Q3|U]^{-1}$ *and checking its determinant,*

```
> X:=simplify(<T.Q3|U>);DetX:=Determinant(X);
```

$$X := \begin{bmatrix} \sigma\,(2\,d+\sigma) & \sigma^2+1 & \sigma \\ 2\,d+\sigma & \sigma & 1 \\ 2\,d^2+d\sigma+1 & d\sigma & d \end{bmatrix}$$

$$DetX := 1$$

```
> M:=MatrixInverse(X);
```

$$M := \begin{bmatrix} 0 & -d & 1 \\ 1 & -\sigma & 0 \\ -\sigma & 2\,d^2+d\sigma+\sigma^2+1 & -2\,d-\sigma \end{bmatrix}$$

Finally checking that the product $M.T.N$ *yields the Smith form of* T*,*

```
> SmT:=simplify(M.T.N);
```

$$SmT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & d+\sigma & 0 \end{bmatrix}$$

Algorithm 2 (QuillenSuslin) >

```
>libname:="QuillenSuslin",libname:libname:="Involutive",libname:
    with(QuillenSuslin):with(LinearAlgebra):
```

```
> T:=Matrix(3,4,[2*d*sigma^2+sigma^3+sigma^2+1,d*sigma^2-d*sigma+
    d,2*d*sigma+sigma^2,d*sigma^2+d*sigma+d+sigma^2,2*d*sigma+
    sigma^2+sigma,d*sigma-d,2*d+sigma,d*sigma+d+sigma,2*d^2*sigma
    +d*sigma^2+d*sigma+sigma,d^2*sigma-d^2-1,2*d^2+d*sigma+1,d^2*
    sigma+d^2+d*sigma]);U:=<<sigma,1,d>>;vars:=d,sigma;
```

$$T := \begin{bmatrix} 2\,d\sigma^2+\sigma^3+\sigma^2+1 & d\sigma^2-d\sigma+d & 2\,d\sigma+\sigma^2 & d\sigma^2+d\sigma+\sigma^2+d \\ 2\,d\sigma+\sigma^2+\sigma & d\sigma-d & 2\,d+\sigma & d\sigma+d+\sigma \\ 2\,d^2\sigma+d\sigma^2+d\sigma+\sigma & d^2\sigma-d^2-1 & 2\,d^2+d\sigma+1 & d^2\sigma+d^2+d\sigma \end{bmatrix}$$

$$U := \begin{bmatrix} \sigma \\ 1 \\ d \end{bmatrix}$$

$$vars := d, \sigma$$

```
> p:=RowDimension(T);q:=ColumnDimension(T);
```

$$p := 3$$
$$q := 4$$

Checking that the ideal generated by $p \times p$ minors of P is principal,

```
> for i from 1 to 4 do m[i]:=Determinant(DeleteColumn(T,i)) od;
    with(Groebner):Basis([seq(m[i],i=1..q)],plex(vars));
```

$$m_1 := 0$$
$$m_2 := -\sigma - d$$
$$m_3 := \sigma + d$$
$$m_4 := \sigma + d$$
$$[\sigma + d]$$

Construction the matrix $P = [T|U]$,

```
> P:=<T|U>;
```

$$P := \begin{bmatrix} 2\,d\sigma^2+\sigma^3+\sigma^2+1 & d\sigma^2-d\sigma+d & 2\,d\sigma+\sigma^2 & d\sigma^2+d\sigma+\sigma^2+d & \sigma \\ 2\,d\sigma+\sigma^2+\sigma & d\sigma-d & 2\,d+\sigma & d\sigma+d+\sigma & 1 \\ 2\,d^2\sigma+d\sigma^2+d\sigma+\sigma & d^2\sigma-d^2-1 & 2\,d^2+d\sigma+1 & d^2\sigma+d^2+d\sigma & d \end{bmatrix}$$

Checking that P admits a right inverse,

```
> IsUnimod(P,[vars],true);
```

$$true$$

Computing unimodular matrix $M1$ such that $M1.U = E_p$, where E_p is the p-th column of I_p and checking that,

```
> M1:=RowOperation(IdentityMatrix(p),[1,p]).Transpose(QSAlgorithm
    (Transpose(U),[vars],true));Check:=M1U=M1.U;
```

$$M1 := \begin{bmatrix} 0 & -d & 1 \\ 1 & -\sigma & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$Check := M1U = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Computing the product $M1.P$,

```
> M1P:=simplify(M1.P);
```

$$M1P := \begin{bmatrix} \sigma & -1 & 1 & 0 & 0 \\ 1 & d & 0 & d & 0 \\ \sigma\,(2\,d+\sigma+1) & d\,(\sigma-1) & 2\,d+\sigma & (\sigma+1)\,d+\sigma & 1 \end{bmatrix}$$

Extracting the submatrix $T1$ *from* $M1P$,

```
> T1:=SubMatrix(M1P,1..p-1,1..q);
```

$$T1 := \begin{bmatrix} \sigma & -1 & 1 & 0 \\ 1 & d & 0 & d \end{bmatrix}$$

Computing the unimodular matrix $N1$ *such that* $T1.N1 = [I_{p-1}|\ 0]$,

```
> N1:=QSAlgorithm(T1,[vars],true);Check:=T1N=T1.N1;
```

$$N1 := \begin{bmatrix} d & 1 & -d & -d \\ -1 & 0 & 1 & 0 \\ -d\sigma & -\sigma & d\sigma+1 & d\sigma \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Check := T1N1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Checking the product $M1.T.N1$,

```
> M1TN1:=simplify(M1.T.N1);
```

$$M1TN1 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ d & \sigma & d+\sigma & d+\sigma \end{bmatrix}$$

Constructing the elementary row matrix $M2$ *to further reduce tha matrix* $M1TN1$,

```
> M2:=<<IdentityMatrix(p-1)|ZeroMatrix(p-1,1)>,<-SubMatrix(M1TN1,
    p..p,1..p-1)|1>>;
```

$$M2 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -d & -\sigma & 1 \end{bmatrix}$$

```
> M2M1TN1:=simplify(M2.M1TN1);
```

$$M2M1TN1 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & d+\sigma & d+\sigma \end{bmatrix}$$

Checking that the ideal generated by the elements of the last row is principal,

```
> with(Groebner):Phi:=Basis([seq(M2M1TN1[p,i],i=p..q)],plex(vars)
    );
```

$$\Phi := [d+\sigma]$$

```
> # Lin-Bose Theorems
```

Extracting the matrix T_LB *from* $M2M1TN1$,

```
> T_LB:=SubMatrix(M2M1TN1,p,1..q);
```

$$T\_LB := \begin{bmatrix} 0 & 0 & d+\sigma & d+\sigma \end{bmatrix}$$

Applying the Lin-Bose factorization theorem on T_LB,

```
> LinBose1(T_LB,[vars],true);# Factorization
```

$$[\begin{bmatrix} d+\sigma \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}]$$

Applying the Lin-Bose completion theorem on T_LB,

```
> LinBose2(T_LB,[vars],true);# Complete T_LB to a Matrix with det
    =Phi=d+sigma
```

$$\begin{bmatrix} 0 & 0 & d+\sigma & d+\sigma \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

```
> # Lin-Bose Conjecture on T_LB
```

Computing the unimodular matrix $N2$ *such that* $T\_LB.N2 = [0|\Phi]$,

```
> N2:=ColumnOperation(QSAlgorithm(T_LB/Phi[1],[vars],true),[1,q])
    ;
```

$$N2 := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

```
> Check:=T_LB.N2;
```

$$Check := \begin{bmatrix} 0 & 0 & 0 & d+\sigma \end{bmatrix}$$

Computing an column elementary matrix $N3$ that rearranges the columns,

```
> N3:=ColumnOperation(IdentityMatrix(q),[1,p]).ColumnOperation(
    IdentityMatrix(q),[p,q]);
```

$$N3 := \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The resulting unimodual matrices M and N are thus obtained by:

```
> M:=M2.M1;N:=N1.N2.N3;
```

$$M := \begin{bmatrix} 0 & -d & 1 \\ 1 & -\sigma & 0 \\ -\sigma & d^2+\sigma^2+1 & -d \end{bmatrix}$$

$$N := \begin{bmatrix} d & 1 & -d & 0 \\ -1 & 0 & 1 & -1 \\ -d\sigma & -\sigma & d\sigma+1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally it is verified that the product $M.T.N$ indeed yields the Smith form of T,

```
> SmT:=simplify(M.T.N);
```

$$SmT := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & d+\sigma & 0 \end{bmatrix}$$

Conclusion

We have presented two Maple based algorithms for the computation of the Smith form for a class of rectangular multivariate polynomial matrices. This class of matrices is associated with underdetermined linear functional systems which are amenable to be reduced to equivalent representations involving a single equation in one unknown function. This reduction will in general simplify the solution of such systems.

Acknowledgements: Work supported by Sultan Qaboos University.

References

[1] M. S. Boudellioua. Computation of the Smith form for multivariate polynomial matrices using Maple. *American J. of Computational Mathematics*, 2(1):21–26 (2012).

[2] M. S. Boudellioua. Further Results on the Equivalence to Smith Form of Multivariate Polynomial Matrices. *Control and Cybernetics*, 42(2):543–551 (2013).

[3] M.S. Boudellioua and A. Quadrat. Serre's reduction of linear functional systems. *Mathematics in Computer Science*, 4(2):289–312 (2010).

[4] F. Chyzak, A. Quadrat, and D. Robertz. OreModules: A symbolic package for the study of multidimensional linear systems. In J. Chiasson and J.-J. Loiseau, editors, *Applications of Time-Delay Systems*, LNCIS 352, pages 233–264. Springer, 2007.

[5] A. Fabiańska and A. Quadrat. Applications of the Quillen-Suslin theorem in multidimensional systems theory. In H. Park and G. Regensburger, editors, *Gröbner Bases in Control Theory and Signal Processing*, Radon Series on Computation and Applied Mathematics 3, pages 23–106. de Gruyter publisher, 2007.

[6] M.G. Frost and M.S. Boudellioua. Some further results concerning matrices with elements in a polynomial ring. *Int. J. Control*, 43(5):1543–1555 (1986).

[7] M.G. Frost and C. Storey. Equivalence of a matrix over $\mathbb{R}[s, z]$ with its Smith form. *Int. J. Control*, 28(5):665–671 (1979).

[8] E.B. Lee and S.H. Zak. Smith forms over $\mathbb{R}[z_1, z_2]$. *IEEE Trans. Autom. Control*, 28(1):115–118 (1983).

[9] Z. Lin and N.K. Bose. A generalization of Serre's conjecture and related issues. *Linear Algebra and its Applications*, 338:125–138 (2001).

[10] Z. Lin, M.S. Boudellioua, and L. Xu. On the equivalence and factorization of multivariate polynomial matrices. In *Proceedings of the 2006 IEEE international symposium of circuits and systems*, pages 4911–4914, Island of Kos (Greece), 2006.

[11] H. H. Rosenbrock. *State space and multivariable theory.* Nelson-Wiley, London, New York, 1970.

Locating Eigenvalues of Unicyclic Graphs

Rodrigo O. Braga[1]

[1] *Institute of Mathematics and Statistics, Universidade Federal do Rio Grande do Sul, Brazil*
E-mail: rbraga@ufrgs.br

Abstract

Any real symmetric matrix $M = (m_{ij})$ of order n may be associated with a simple graph G with vertex set $[n] = \{1, \ldots, n\}$ such that distinct vertices i and j are adjacent if and only if $m_{ij} \neq 0$. In [1], Jacobs and Trevisan presented a linear time algorithm on trees to solve a problem that became known as *eigenvalue location* for matrices associated with graphs. An algorithm *locates* eigenvalues of a symmetric matrix $M(G)$ associated with graphs G in a class $\mathcal{C}$ if, for any graph $G \in \mathcal{C}$ and any given real interval I, it finds the number of eigenvalues of $M(G)$ in the interval I. The algorithm in [1] was specifically devised for the adjacency matrix of trees, but that approach could be extended in a natural way to arbitrary symmetric matrices associated with trees [2]. In this talk, we present a linear algorithm (appeared in [3]) that locates the eigenvalues of any symmetric matrix $M(G)$ of a connected unicyclic graph G. As an application, we apply this algorithm to study the largest eigenvalue of the Laplacian matrix of a *lollipop graph*, which is a unicyclic graph of order $n + k$ formed by adding an edge between a cycle C_k of order k and a path P_n of order n.

Acknowledgements: Work partially supported by CNPq, Brazil.

References

[1] D. P. Jacobs, V. Trevisan. Locating the eigenvalues of trees. *Linear Algebra Appl.*, 434 (1): 81–88 (2011).

[2] R. O. Braga, V. M. Rodrigues. Locating eigenvalues of perturbed Laplacian matrices of trees. *Tend. Mat. Apl. Comput.*, 18(3):479–491 (2017).

[3] R. O. Braga, V. M. Rodrigues, R. O. Silva. Locating eigenvalues of a symmetric matrix whose graph is unicyclic. *Trends in Computational and Applied Mathematics*, 22 (4): 659–674 (2021).

Combined matrices of diagonally equipotent matrices

Rafael Bru[1], *Maria T. Gassó*[2], *Máximo Santana*[3]

[1] *Inst. Matemtica Multidisciplinar, Universitat Politcnica de Valncia, Spain*
E-mail: rbru@imm.upv.es
[2] *Inst. Matemtica Multidisciplinar, Universitat Politcnica de Valncia, Spain*
E-mail: mgaso@imm.upv.es
[3] *Inst. Matemticas, Universidad Autonóma de Santo Domingo, Dominican Republic.*
E-mail: author3@

Abstract

Let $\mathcal{C}(A)$ be the combined matrix of an invertible matrix A. In this work, we study the combined matrix of a nonsingular matrix which is an H–matrix whose comparison matrix is singular. In particular, we focus on diagonally equipotent matrices. Related to these results we give some properties on the diagonal dominance of these matrices and their comparison matrix.

Acknowledgements: Work supported by the Generalitat Valenciana grant CIAICO/2021/162 and by the Dominican Republic FONDOCYT grant 2020–2021–1D1–121.

The geometry of numerical ranges over finite fields

Kristin A. Camenga[1], *Brandon Collins*[2], *Gage Hoefer*[3], *Jonny Quezada*[4], *Patrick X. Rault*[5], *James Willson*[6], *Rebekah B. Johnson Yates*[7]

[1] *Department of Mathematics, Juniata College, U.S.A.*
E-mail: camenga@juniata.edu
[2] *Department of Mathematics, University of Nebraska at Omaha, U.S.A.*
E-mail: brandoncollins@unomaha.edu
[3] *Department of Mathematics, University of Nebraska at Omaha, U.S.A.*
E-mail: ghoefer@unomaha.edu
[4] *Department of Mathematics, University of Nebraska at Omaha, U.S.A.*
E-mail: jonnyquezada@unomaha.edu
[5] *Department of Mathematics, Idaho State University, U.S.A.*
E-mail: patrickrault@isu.edu
[6] *Department of Mathematics, University of Nebraska at Omaha, U.S.A.*
E-mail: jwillson@unomaha.edu
[7] *Department of Mathematics, Houghton College, U.S.A.*
E-mail: rebekah.yates@houghton.edu

Abstract

The numerical range of a matrix has been studied over the complex numbers for over a hundred years; interesting properties include that the classical numerical range is a convex, compact set, invariant under unitary similarity, and containing the eigenvalues of the matrix. Numerical ranges over finite fields were first studied by a group of undergraduates [4], which classified the numerical ranges of certain types of matrices over finite fields with certain prime numbers of elements. Soon afterward, this classification was generalized to all finite fields $\mathbb{F}_{q^2}$ where q is a prime power [1, 2].

The new setting requires us to revisit classical results and suggests new questions: what can we say about the geometry of the numerical range, especially when there is no notion of convexity? what challenges arise since we can now have vectors v where $v^*v = 0$? since the setting is finite, what opportunities are there to enumerate and compare sets? We present progress toward answering these questions, including a classification for the shape of the numerical range of 2×2 matrices with certain properties, counting the number of preimages of numerical range elements for these matrices, and infinite sets of higher dimensional matrices that do not have the entire field $\mathbb{F}_{q^2}$ as the numerical range.

Acknowledgements: Work (partially) supported by American Institute of Mathematics REUF continuation program (NSF-DMS 1620073) and the University of Nebraska at Omaha Kerrigan Research Minigrants Program.

References

[1] E. Ballico *On the numerical range of matrices over a finite field*, Linear Algebra and its Applications **502** (2017), 162-171.

[2] E. Ballico *Corrigendum to "On the numerical range of matrices over a finite field" [Linear Algebra Appl. 502 (2017), 162-171*, Linear Algebra and its Applications **556** (2018), 421-427.

[3] K. A. Camenga, B. Collins, G. Hoefer, J. Quezada, P. X. Rault, J. Willson, R. Yates, *On the geometry of numerical ranges over finite fields*, Linear Algebra and its Applications **628** (2021), 182-201.

[4] J. I. Coons, D. Knowles, J. Jenkins, R. Luke, and P. X. Rault, *Numerical ranges over finite fields*, Linear Algebra and its Applications **501** (2016), 37-47.

Simplifying the compensation criteria for the real nonnegative inverse eigenvalue problem

Alberto BOROBIA[1], *Roberto CANOGAR*[1]

[1] *Dpt. of Mathematics, Universidad Nacional de Educación a Distancia (UNED), Spain*
E-mails: aborobia@mat.uned.es, rcanogar@mat.uned.es

Abstract

The nonnegative inverse eigenvalue problem is the problem of characterizing all possible spectra $\Lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ of entrywise nonnegative matrices. If Λ is the spectrum of a non-negative matrix, then Λ is said to be a *realizable list*.

There are three well know methods or rules to convert realizable lists into realizable lists:

Rule 1: If $(\rho, \lambda_2, \lambda_3, \ldots, \lambda_n)$ is realizable, where ρ is the Perron eigenvalue and λ_2 is real, then for all $\epsilon \geq 0$, the following set is also realizable:

$$(\rho + \epsilon, \lambda_2 \pm \epsilon, \lambda_3, \ldots, \lambda_n).$$

Rule 2: If $(\rho, \lambda_2, \lambda_3, \ldots, \lambda_n)$ is realizable, where ρ is the Perron eigenvalue, then for all $\epsilon \geq 0$, the following set is also realizable:

$$(\rho + \epsilon, \lambda_2, \lambda_3, \ldots, \lambda_n).$$

Rule 3: Let Λ_1 and Λ_2 be realizable sets. Then $\Lambda_1 \cup \Lambda_2$ is also realizable.

Rule 1 and 2 were proved by Guo [4], and Rule 3 is trivial since the spectrum of a block diagonal matrix is the union of the spectra of the diagonal blocks.

The compensation criteria for the real nonnegative inverse eigenvalue problem was described in [1] as a procedure that starts with a trivial spectra and builds up realizable lists by applying Rule 1-3. So the set of all possible realizable lists obtained in this way is defined recursively:

Definition. *[1] A list of real numbers* $(\lambda_1, \lambda_2, \cdots, \lambda_n)$ *is called* C-realizable *if it may be obtained by starting with the* n *trivially realizable lists* $(0), (0), \cdots, (0)$ *and then using Rule 1, 2 and 3 any number of times in any order. We also use the term* realizability by compensation *to refer to this method.*

We will show that using a simplified version of Rule 1 we obtain the same set of realizable lists. The new simplified Rule is the following:

Rule 1*: If $(\rho, \lambda_2, \lambda_3, \ldots, \lambda_n)$ is realizable, where ρ is the Perron eigenvalue and $\lambda_2 \leq 0$ is real, then for all $\epsilon \geq 0$, the following set is also realizable:

$$(\rho + \epsilon, \lambda_2 - \epsilon, \lambda_3, \ldots, \lambda_n).$$

It is important to note that there are two simplifications of Rule 1: first we do not use the transformation $(\rho + \epsilon, \lambda_2 + \epsilon, \lambda_3, \ldots, \lambda_n)$, and second we only apply the Rule for $\lambda_2 \leq 0$.

Clearly, applying the more restrictive Rule 1* instead of Rule 1 we obtain a subset of C-realizable lists. This new set is called *C*-realizable*. We are ready to state our main result:

Theorem. *The set of C*-realizable lists and the set of C-realizable lists is the same.*

Acknowledgements: Work funded by the *Agencia Estatal de Investigación* of Spain through grant PID2019-106362GB-I00/AEI/10.13039/501100011033 .

References

[1] A. Borobia, J. Moro, R.L. Soto, A unified view on compensation criteria in the real nonnegative inverse eigenvalue problem, Linear Algebra Appl. 428 (2008) 2574–2584.

[2] A. Borobia, R. Canogar. The real nonnegative inverse eigenvalue problem is NP-hard. Linear Algebra Appl., 522 (2017) 127–139.

[3] R. Ellard, E. Šmigoc. Connecting sufficient conditions for the symmetric nonnegative inverse eigenvalue problem. Linear Algebra Appl. 498 (2016) 521–552.

[4] W. Guo. Eigenvalues of nonnegative matrices. Linear Algebra Appl. 266 (1997) 261?270.

[5] C. Marijuán, J. Moro. A characterization of trace-zero sets realizable by compensation in the SNIEP. Linear Algebra Appl., 615 (2021) 42–76.

Obtaining the Jordan structure of a totally nonnegative matrix from the Jordan structures of an upper block echelon matrix

Begoña Cantó[1], *Rafael Cantó*[1], *Ana M. Urbano*[1]

[1] *Instituto de Matemática Multidisciplinar, Universitat Politècnica de València, Spain*
E-mail: bcanto@mat.upv.es, rcanto@mat.upv.es, amurbano@mat.upv.es

Abstract

A totally nonnegative matrix (TN) is a matrix with all its minors nonnegative. These matrices have been studied by several authors and, concretely, in [1] it is proved that the number of Jordan canonical forms of an irreducible TN matrix $A \in \mathbb{R}^{n\times n}$ associated with a triple (n, r, p) is given by the combinatorial number $P^p_{n-r}(n-p)$, where r is the rank of A, and p its principal rank. That is, the number of partitions of $n-p$ into exactly $n-r$ parts with the largest part at most p. Then, the matrix A has $n-r$ zero-Jordan blocks whose sizes are given by the Segre characteristic of A relative to its zero eigenvalue.

A sequence of integers $\alpha = \{h_1, h_2, \ldots, h_p\} \in \mathcal{Q}_{p,n}$ is called the sequence of the first p-indices of A if for $j = 2, \ldots, p$, we have $det(A[h_1, h_2, \ldots, h_{j-1}, h_j]) \neq 0$ and $det(A[h_1, h_2, \ldots, h_{j-1}, t]) = 0, \quad h_{j-1} < t < h_j$. It is known that some properties that irreducible TN matrices satisfy without prescribed p-indices, are not satisfied when they are prescribed. If the sequence of the first p-indices is prescribed, then the number of the zero-Jordan structures admissible for a realizable triple (n, r, p) is less than or equal to this number when the sequence is not prescribed.

A method to construct an irreducible TN matrix A is given by the product $A = LU$, where L is a lower block triangular matrix and $U \in \mathbb{R}^{n\times n}$ is an upper block echelon TN matrix associated with the realizable triple (n, r, p), with the same zero-Jordan structure and the same sequence of the first p-indices of A (see [2]). We recall that a matrix is an upper echelon matrix if the first nonzero entry in each row (leading entry) is to the right of the leading entry in the row above it and all zero rows are at the bottom. A matrix is upper block echelon if each nonzero block, starting from the left, is to the right of the nonzero blocks below and the zero blocks are at the bottom. A matrix is a lower (block) echelon matrix if its transpose is an upper (block) echelon matrix.

In [2], an upper block echelon TN matrix U can be transformed by similarity and permutation similarity, into a matrix $T = XUX^{-1}$, where T is an upper block triangular matrix such that the size of its blocks depends on the prescribed sequence of the first p-indices of matrix U. Now, in this work we obtain all

possible zero-Jordan structure admissible of a matrix T and its relationship with the zero-Jordan structure admissible of the upper block echelon TN matrix U.

Acknowledgements: Work (partially) supported by the Generalitat Valenciana under the grant CIAICO/2021/162.

References

[1] B.Cantó, R. Cantó, A.M. Urbano. All Jordan canonical forms of irreducible totally nonnegative matrices *Linear Multilinear Algebra*, 69(13): 2389–2409 (2021).

[2] B. Cantó, R. Cantó, A.M. Urbano. Jordan structures of irreducible totally nonnegative matrices with a prescribed sequence of the first p-indices *RACSAM REV R ACAD A*, 116(84): 1–27 (2022).

The range of combined matrices and doubly stochastic matrices

Macarena Boix[1], *Begoña Cantó*[2], *Maria T. Gassó*[2]

[1] *Departamento de Matemática Aplicada, Universitat Politècnica de València, Spain*
E-mail: mboix@mat.upv.es
[2] *Instituto de Matemática Multidisciplinar, Universitat Politècnica de València, Spain*
E-mail: bcanto@mat.upv.es, mgasso@mat.upv.es

Abstract

The combined matrix of a nonsingular matrix A or RGA is the matrix $C(A) = A \circ A^{-T}$ where $\circ$ means the Hadamard (entrywise) product. If the elements of $C(A) = U$, $0 \leq u_{ij} \leq 1$, $i, j = 1, \ldots, n$ then U is a doubly stochastic matrix.

In control theory combined matrices are called Relative Gain Array (RGA) and were introduced in 1966 by Bristol. These matrices are useful in many processes where they are used in order to know their behaviour. In these cases, the RGA matrices are usually small, $n = 2, 3, 4$, see for example [1].

Other problems on combined matrices are studied by several authors. In [2], Johnson and Shapiro raise the following question: if U is the combined matrix of A is there some nice description of the set of all matrices such that they have the same combined matrix U? It seems that this question is hard to solve and in this work we answer it for matrices of order 3 when U is a double stochastic matrix.

Acknowledgements: This research was partially supported by the Generalitat Valenciana under the grant CIAICO/2021/162.

References

[1] M. Hovd, S. Skogestad. Sequencial Desig of Descentralized Controllers. *Automatica*, 30(10): 1601–1607 (1994).

[2] C.R. Johnson, H. Shapiro. Mathematical aspects of the relative gain array. *SIAM J. Alg. Disc. Math.*, 7(4): 627–644 (1986).

Linear algebra in the category of linear systems

Miguel V. Carriegos[1]

[1] *Departamento de Matemáticas, Universidad de León, Spain*
E-mail: miguel.carriegos@unileon.es

Abstract

Maxwell's description [7] of flyball governor and the feedback actions to control steam engines put linear systems and linear feedback actions as main topic in control engineering. Kalman and Brunovsky's decomposition theorems [1] completed description of linear systems up to feedback actions.

A feedback morphism is a linear map between linear systems that preserves the dynamics. Classical feedback equivalences arise precisely as invertible feedback morphisms. Thus feedback classification of linear systems is in fact the search for $\mathcal{S}^{\text{iso}}$ where $\mathcal{S}$ is the category of systems and feedback morphisms [5].

This talk is intended to introduce linear algebra results in the category $\mathcal{S}_{\mathbb{K}}$ of linear systems over an arbitrary field $\mathbb{K}$. Kernels and cokernels of feedback morphisms between reachable systems are computed effectively. Hence we prove that full subcategory $\mathcal{A}_{\mathbb{K}}$ of reachable linear systems is pre-abelian [3]. The category fails to be abelian and hence we don't have Schur's lemma [6]. Nevertheless it is interesting to study exact structures [2] on $\mathcal{A}_{\mathbb{K}}$ in order to obtain (co)product decompositions of linear systems.

Acknowledgements: Work (partially) supported by CAFE research group.

References

[1] P.A. Brunovsky. A classification of linear controllable systems, *Kibernetika*, 3: 173–188 (1970).

[2] T. Brüstle, S. Hassoun, D. Langford, S. Roy. Reduction of exact structures. *J. Pure App. Algebra*, 224(4): 106212 (2019).

[3] T. Bühler. Exact categories. *Expo. Math.*, 28(1): 1–69 (2010).

[4] C. Busque, A. Facchini. Abelian categories in which linear algebra is possible. *Lin. Multilin. Algebra* 41(2):137–156 (1996).

[5] M.V. Carriegos, A.L Muñoz Castañeda. On the K-theory of feedback actions on linear systems. *Lin. Algebra App.*, 440: 233–242 (2014).

[6] H. Enomoto. Schur's lemma for exact categories implies abelian. *J. Algebra*, 584: 260–269 (2021).

[7] J.C. Maxwell. On governors. *Proc. Royal Soc. London*, 16, 270–283 (1868).

Total graphs of gain graphs

Matteo Cavaleri [1], Alfredo Donno [2]

[1] Università degli Studi Niccolò Cusano, Italy
E-mail: matteo.cavaleri@unicusano.it
[2] Università degli Studi Niccolò Cusano, Italy
E-mail: alfredo.donno@unicusano.it

Abstract

The *total graph* of a graph $\Gamma = (V, E)$ is the graph whose vertex set is $V \cup E$ and whose adjacencies are inherited from adjacency and incidence relations in Γ. In particular, it contains as induced subgraphs both a copy of Γ and of the *line graph* of Γ. A *gain graph* (Γ, ψ) is a pair consisting of an underlying graph Γ and a map ψ, called gain function, from the set of oriented edges to a group G, called *gain group*, with the property that to opposite orientations correspond inverse elements of G.

We give a definition for the total graph of a gain graph (Γ, ψ), that is a gain graph constructed through G-phases, similarly to what was done in [2, 3] for the line graph of a gain graph. This construction is well defined, in the sense that switching isomorphic gain graphs have switching isomorphic total graphs. Moreover, we characterize the sets (orbits) of G-phases in relation to the gain functions that they induce on the total graph.

Our construction is consistent with those of the total graph of a signed graph [4], that in fact can be regarded as a gain graph with $G = \{\pm 1\}$. In analogy with the signed case, we investigate the spectrum of the total graph of a gain graph over an arbitrary group G. This is possible thanks to the group representation approach to the spectrum of a gain graph [1].

References

[1] M. Cavaleri, D. D'Angeli, A. Donno. A group representation approach to balance of gain graphs. *J. Algebraic Combin.*, 54: 265–293 (2021).

[2] M. Cavaleri, D. D'Angeli, A. Donno. Gain-line graphs via G-phases and group representations. *Linear Algebra Appl.*, 613: 241–270 (2021).

[3] M. Cavaleri, D. D'Angeli, A. Donno. Characterizations of line graphs in signed and gain graphs. *European J. Combin.*, 102: Paper No. 103479 (2022).

[4] F. Belardo, Z. Stanić, T. Zaslavsky. Total graph of a signed graph. *Ars Math. Contemp.*, 23: no. 1, Paper No. 2 (2023).

Problems related to data analysis in non-Euclidean spaces: iterative filtering for signals defined on the sphere.

Barbarino Giovanni[1], *Cavassi Roberto*[2], *Cicone Antonio*[2]

[1] *Aalto University, Finland*
E-mail: giovanni.barbarino@aalto.fi
[2] *DISIM, University of l'Aquila, Italy*
E-mail: roberto.cavassi@graduate.univaq.it
E-mail: antonio.cicone@univaq.it

Abstract

Since real-life data are non-stationary, it would be better to study them through non-stationary techniques, and 'Fast Iterative Filtering' has proven to be an interesting and useful method to achieve this goal, especially in classic 1D or 2D cases [1]. But some problems arise in non-Euclidean settings since the filtering relies on convolution.

After developing a continuous operator we analysed its discretisation through the Generalised Locally Toeplitz (GLT) sequences of matrices [2]. Using some property from the GLT theory we studied the convergence of this procedure [3].

In this talk, after a brief review on the topic, we will describe some problems related to this setting and what we have obtained so far to overcome them. We conclude our talk with a few examples of applictions of this method to real life signals.

References

[1] A. Cicone, H. Zhou *Multidimensional Iterative Filtering method for the decomposition of high-dimensional non-stationary signals*. Cambridge Core in Numerical Mathematics: Theory, Methods and Applications, Volume 10, Issue 2, Pages 278-298, 2017.

[2] C. Garoni, S. Serra-Capizzano. *Generalized Locally Toeplitz Sequences: Theory and Applications*. 2017

[3] G. Barbarino, A. Cicone, *Conjectures on spectral properties of ALIF algorithm*. Linear Algebra and its Applications, Volume 647, Pages 127-152, 2022.

Monodromy group of decomposable Blaschke products of degree 2^n

Asuman Güven Aksoy[1], *Francesca Arici*[2], *M. Eugenia Celorrio*[3], *Pamela Gorkin*[4]

[1] *Department of Mathematical Sciences, Claremont McKenna College, Claremont, CA, 91711, USA.*
E-mail: asumanguvenaksoy@gmail.com
[2] *Mathematical Institute, Leiden University, P.O. Box 9512, 2300 RA Leiden, The Netherlands.*
E-mail: f.arici@math.leidenuniv.nl
[3] *Department of Mathematics and Statistics, Fylde College, Lancaster University, Lancaster, LA1 4YF, United Kingdom.*
E-mail: m.celorrioramirez@lancaster.ac.uk
[4] *Department of Mathematics, Bucknell University, 377 Olin Science Building, Lewisburg, PA 17837, USA.*
E-mail: pgorkin@bucknell.edu

Abstract

In this talk we shall give an overview of the main results obtained in [1] regarding to the monodromy group of Blaschke products of degree 2^n.

We shall go over some examples of Blaschke products that can facilitate the understanding of the monodromy group. Finally, we shall see a sketch proof of the result that states that for a regularized Blaschke product B that can be decomposed into n degree-2 Blaschke products, then the monodromy group associated with B is the wreath product of n cyclic groups of order 2.

References

[1] A.G. Aksoy, F. Arici, M.E. Celorrio, P. Gorkin. Decomposable Blaschke Products of Degree 2^n. N

On almost semimonotone matrices and the linear complementarity problem

Bharat Pratap Chauhan[1], *Dipti Dubey*[2]

[1] *Department of Mathematics, Shiv Nadar Institution of Eminence, Delhi-NCR, India*
E-mail: bc102@snu.edu.in
[2] *Department of Mathematics, Shiv Nadar Institution of Eminence, Delhi-NCR, India*
E-mail: dipti.dubey@snu.edu.in

Abstract

The linear complementarity problem, a unified framework for studying various optimization problems, has a strong connection with semimonotone matrices, and thus with almost (strictly) semimonotone matrices. The class of semimonotone matrices is a generalization of the class of copositive matrices, which contains nonnegative matrices. In this paper, we revisit the class of almost (strictly) semimonotone matrices and partially address the conjecture made by Wendler [Special Matrices 7 (2019) 291–303]. We disprove the second part of the conjecture by providing a counter example. The main result of this paper shows that Wendler's conjecture is true under the symmetry assumption. We show that a symmetric almost semimonotone matrix is an almost $\mathbf{P_0}$-matrix. We explore some interesting matrix theoretic properties of almost (strictly) semimonotone matrices and also present results pertaining to the existence and multiplicity of solutions to the linear complementarity problem associated with an almost semimonotone matrix.

Acknowledgements: The first author is thankful to the Council of Scientific & Industrial Research (CSIR), India, Junior Research Fellowship scheme for financial support for this research. The research of second author is supported MATRICS grant (MTR/2021/000849) of Science & Engineering Research Board (SERB), Department of Science and Technology, Government of India.

References

[1] B. P. Chauhan and D. Dubey, On almost semimonotone matrices and the linear complementarity problem, *Linear Algebra and its Applications*, 661: 35–50 (2023).

[2] R. Cottle, J. Pang, and R. Stone, The Linear Complementarity Problem, *Society for Industrial and Applied Mathematics*, 2009.

[3] D. Dubey, S.K. Neogy, On hidden **Z**-matrices and the linear complementarity problem, *Linear Algebra and its Applications*, 496: 81–100 (2016).

[4] M. Wendler, The almost semimonotone matrices, *Special Matrices*, 7: 291–303 (2019).

Matrix Nearness Problems with Rank-Structured Positive Definite Matrices

Lujing Chen[1], *Frederik H. Pedersen*[1], *Martin S. Andersen*[1]

[1] *Department of Applied Mathematics and Computer Science, Technical University of Denmark, Denmark*
E-mail: {lujc,fhape,mskan}@dtu.dk

Abstract

We consider the problem of computing a rank-structured approximation to a symmetric positive definite matrix. Of particular interest are matrices with off-diagonal low-rank structure or hierarchical off-diagonal low-rank structure [2]. The latter type of matrices can be factorized in quasi-linear time and have applications to, e.g., preconditioning [3] and Gaussian process regression [1]. We formulate the approximation problem as a matrix nearness problem with rank constraints and derive optimal approximations for several special cases based on different notions of nearness. We then use these to construct an efficient, greedy approximation scheme for computing hierarchical off-diagonal low-rank approximations that preserve positive definiteness. Finally, to illustrate the advantages and limitations of the methodology, we present some numerical results from different application areas within data science.

Acknowledgements: This work is supported by the Novo Nordisk Foundation (grant no. NNF20OC0061894).

References

[1] M. S. Andersen and T. Chen. Smoothing splines and rank structured matrices: Revisiting the spline kernel. *SIAM Journal on Matrix Analysis and Applications*, 41(2):389–412, 2020.

[2] W. Hackbusch. *Hierarchical Matrices: Algorithms and Analysis*. Springer, Berlin, Heidelberg, 2015.

[3] J. Xia. Effective and robust preconditioning of general SPD matrices via structured incomplete factorization. *SIAM Journal on Matrix Analysis and Applications*, 38(4):1298–1322, January 2017.

Properties of the shell of a square matrix and shell extremal eigenvalues

Christos Chorianopoulos[1], *Panayiotis Psarrakos*[2]

[1] *Dpt. of Electrical and Electronics Engineering, University of West Attica, Greece*
E-mail: cchorian@uniwa.gr
[2] *Dpt. of Mathematics, National Technical University of Athens, Greece*
E-mail: ppsarr@math.ntua.gr

Abstract

Given a square matrix A, the curve $\Gamma(A) = \{x + iy, x, y \in \Re : f(x, y) = 0\}$, where $f(x, y)$ is specific cubic polynomial in x, y, is called the shell of A and it gives interesting eigenvalue localization results. Moreover, it shares (at least one) common boundary point with the standard numerical range $F(A)$ of A and satisfies several properties similar to those of $F(A)$.

In one of its forms, $\Gamma(A)$ consists of a simple unbounded open curve and a closed branch in the form of a loop which surrounds a unique simple eigenvalue. If for some θ the curve $\Gamma(e^{i\theta}A)$ surrounds an eigenvalue $e^{i\theta}\lambda_0$, then, we call λ_0 of A a shell–extremal eigenvalue. Some geometrical aspects of the loop (maximum distance between boundary points and radius of curvature) are proposed as measures of the non–normality of this specific eigenvalue. These results are applied to several classes of matrices.

References

[1] M. Adam and M. Tsatsomeros, An eigenvalue inequality and spectrum localization for complex matrices, *Electron. J. Linear Algebra*, **15** (2006) 239–250.

[2] Ch. Chorianopoulos P. Psarrakos, On the Shell and the Shell–Extremal Eigenvalues of a Square Matrix. *Submitted*

[3] Ch. Chorianopoulos, Shell–Extremal Eigenvalues of a Tridiagonal Toeplitz Matrix. *In progress*

The (multivariate) Pascal matrix

Helena Cobo[1]

[1] *Dpt. Matemática aplicada I, Universidad de Sevilla, Spain*
E-mail: helenacobo@gmail.com

Abstract

A result in [1] relating the multivariate Pascal matrix with a generalized Stirling matrix, is crucial in [2] to solve some linear systems which appear when computing a b-function of certain ideals in the Weyl algebra. In this talk we will discuss this together with some other applications of the Pascal matrix in its symmetric version.

References

[1] H. Cobo. The Pascal matrix in the multivariate Riordan group. Linear Algebra Appl., **635** (2022), 1-23.

[2] H. Cobo. Generalized factorial Stirling matrices and weighted b-functions of holonomic ideals. (49 pp.). Preprint 2023.

Resolution of the circulant nut graph order–degree existence problem

Ivan Damnjanović[1,2]

[1] *Faculty of Electronic Engineering, University of Niš, Serbia*
E-mail: ivan.damnjanovic@elfak.ni.ac.rs
[2] *Diffine LLC, San Diego, California, USA*
E-mail: ivan@diffine.com

Abstract

A circulant nut graph is a non-trivial simple graph such that its adjacency matrix is a circulant matrix whose null space is spanned by a single vector without zero elements. Regarding these graphs, the order–degree existence problem can be thought of as the mathematical problem of determining all the possible pairs (n, d) for which there exists a d-regular circulant nut graph of order n. This problem was initiated by Bašić et al. [Art Discret. Appl. Math. **5(2)** (2021) #P2.01] and the first major results were obtained by Damnjanović and Stevanović [Linear Algebra Appl. **633** (2022) 127–151], who proved that for each odd $t \geq 3$ such that $t \not\equiv_{10} 1$ and $t \not\equiv_{18} 15$, there exists a $4t$-regular circulant nut graph of order n for each even $n \geq 4t + 4$. Afterwards, Damnjanović [arXiv:2210.08334 (2022)] improved these results by showing that there necessarily exists a $4t$-regular circulant nut graph of order n whenever t is odd, n is even, and $n \geq 4t + 4$ holds, or whenever t is even, n is such that $n \equiv_4 2$, and $n \geq 4t + 6$ holds. Finally, the aforementioned results were extended once again by Damnjanović, thus yielding a complete resolution of the circulant nut graph order–degree existence problem. In other words, all the possible pairs (n, d) for which there exists a d-regular circulant nut graph of order n are now determined.

Acknowledgements: Work supported by Diffine LLC.

References

[1] I. Sciriha, *On the construction of graphs of nullity one*, Discrete Math. **181(1–3)** (1998) 193–211.

[2] P. W. Fowler, B. T. Pickup, T. Z. Todorova, M. Borg, I. Sciriha, *Omni-conducting and omni-insulating molecules*, J. Chem. Phys. **140** (2014) 054115.

[3] K. Coolsaet, P. W. Fowler, J. Goedgebeur, *Generation and properties of nut graphs*, MATCH Commun. Math. Comput. Chem. **80** (2018) 423–444.

[4] P. W. Fowler, T. Pisanski, N. Bašić, *Charting the space of chemical nut graphs*, MATCH Commun. Math. Comput. Chem. **86(3)** (2021) 519–538.

[5] J. B. Gauci, T. Pisanski, I. Sciriha, *Existence of regular nut graphs and the Fowler construction*, Appl. Anal. Discrete Math. (2021), doi:10.2298/AADM190517028G.

[6] P. W. Fowler, J. B. Gauci, J. Goedgebeur, T. Pisanski, I. Sciriha, *Existence of regular nut graphs for degree at most* 11, Discuss. Math. Graph Theory **40** (2020) 533–557.

[7] N. Bašić, M. Knor, R. Škrekovski, *On* 12*-regular nut graphs*, Art Discret. Appl. Math. **5(2)** (2021) #P2.01.

[8] M. Filaseta, A. Schinzel, *On testing the divisibility of lacunary polynomials by cyclotomic polynomials*, Math. Comput. **73(246)** (2003) 957–965.

[9] I. Damnjanović, D. Stevanović, *On circulant nut graphs*, Linear Algebra Appl. **633** (2022) 127–151.

[10] I. Damnjanović, *Two families of circulant nut graphs*, submitted (2022), https://arxiv.org/abs/2210.08334.

[11] I. Damnjanović, *Complete resolution of the circulant nut graph order–degree existence problem*, submitted (2022), https://arxiv.org/abs/2212.03026.

New perturbation bounds for eigenvalues of quadratic eigenvalue problem for efficient damping optimization

Ranjan Kumar Das[1], *Zoran Tomljanović*[2], *Krešimir Veselić*[3]

[1] *Department of Mathematics, J. J. Strossmayer University of Osijek, Croatia*
E-mail: ranjankdiitg@gmail.com
[2] *Department of Mathematics, J. J. Strossmayer University of Osijek, Croatia*
E-mail: ztomljan@mathos.hr
[3] *Fakultät für Mathematik und Informatik, Fernuniversität Hagen, Germany*
E-mail: kresimir.veselic@fernuni-hagen.de

Abstract

Parameter dependent quadratic eigenvalue problems (PQEP)

$$(\lambda^2(\mathbf{v})M + \lambda(\mathbf{v})D(\mathbf{v}) + K)x(\mathbf{v}) = 0, \tag{1}$$

where M and K are $n \times n$ Hermitian positive definite matrices and $D(\mathbf{v})$ is an $n \times n$ Hermitian positive semidefinite matrix which depends on a damping (viscosity) parameter vector $\mathbf{v} \in \mathbb{R}^k_+$, arise in many applications. A perturbation bound for approximations of eigenvalues of (1) is derived in [2] by using the dimension reduction method and standard Gerschgorin theorem. In order to improve damping optimization, we find new perturbation bounds for the eigenvalues of PQEP by extending the results of [2]. The quality and advantages of the new bounds are illustrated in numerical experiments. We believe that the new bounds are helpful for the efficient determination of optimal positions of eigenvalues.

Acknowledgements: This work is supported by the Croatian Science Foundation project: Vibration Reduction in Mechanical Systems (IP-2019-04-6774, VIMS).

References

[1] K. Veselic, *Damped Oscillations of Linear Systems.* Springer Lecture Notes in Mathematics, Springer-Verlag, Berlin, 2011.

[2] N. Truhar, Z. Tomljanović, M. Puvača, *Approximation of damped quadratic eigenvalue problem by dimension reduction.* Applied mathematics and computation 347: 40-53 (2019).

[3] I. Nakić, Z. Tomljanović, N. Truhar, *Optimal Direct Velocity Feedback.* Applied mathematics and computation 225: 590-600 (2013).

Wiener Index and Eccentricity after Edge contraction

Joyentanuj Das[1], *Ritabrata Jana*[2]

[1] *Department of Applied Mathematics, National Sun Yat-sen University, Gushan District, Kaohsiung City, 804, Taiwan (ROC)*
E-mail: joyentanuj@gmail.com
[2] *School of Mathematics, IISER Thiruvananthapuram, Maruthamala P.O., Vithura, Thiruvananthapuram, India.*
E-mail: ritabrata20@iisertvm.ac.in

Abstract

For a connected graph G, the Wiener index, denoted by $W(G)$, is the sum of the distance of all pairs of distinct vertices and the eccentricity, denoted by $\varepsilon(G)$, is the sum of the eccentricity of individual vertices. In [1], the authors posed a conjecture that states that given a graph G with at least three vertices, the difference between $W(G)$ and $\varepsilon(G)$ decreases when an edge is contracted and proved that the conjecture is true when e is a bridge. In this talk, we will prove that the conjecture is true for any connected graph G with at least three vertices irrespective of the nature of the edge chosen. We will also mention some of the ongoing work in this area.

References

[1] H. Darabi, Y. Alizadeh, S. Klavzar and K.C. Das, On the Relation Between Wiener Index and Eccentricity of a Graph, J. Comb. Optim. 41, no. 4, 817–829, (2021).

[2] J. Das and R. Jana, Proof of a Conjecture on Wiener Index and Eccentricity of a graph due to edge contraction, Discrete Applied Mathematics 307, 19-21, (2022).

On the max k-cut problem and the smallest signless Laplacian eigenvalue of a graph

Jorge Alencar [1], *Leonardo de Lima*[2], *Vladimir Nikiforov* [3]

[1] *Department of Mathematics, Instituto Federal de Educação, Ciência e Tecnologia do Triângulo Mineiro, Brazil*
E-mail: jorgealencar@iftm.edu.br
[2] *Departamento de Administração Geral e Aplicada, Federal University of Paraná, Brazil*
E-mail: leonardo.delima@ufpr.br
[3] *Department of Mathematical Sciences, University of Memphis, United States*
E-mail: vnikifrv@memphis.edu

Abstract

In this talk, we present an extension of the graph parameter max k-cut to square matrices and prove a general sharp upper bound, which implies upper bounds on the max k-cut of a graph using the smallest signless Laplacian eigenvalue, the smallest adjacency eigenvalue, and the largest Laplacian eigenvalue of the graph. In addition, we construct infinite families of extremal graphs for the obtained upper bounds.

Acknowledgements: Work (partially) supported by CNPq grant number 315739/2021–5

References

[1] J. Alencar, L. de Lima. On graphs with adjacency and signless Laplacian matrix eigenvectors entries in $\{-1, +1\}$, *Linear Algebra and its Applications* **614** (2021) 301–315.

[2] M. A. de Freitas, N.M.M. de Abreu, R.R. Del-Vecchio, S. Jourkiewicz, Infinite families of Q-integral graphs, *Linear Algebra and its Applications* **432** (2010) 2352–2360.

[3] V. Nikiforov, Max k-cut and the smallest eigenvalue, *Linear Algebra and its Applications* **504** (2016) 462–467.

[4] E.R. van Dam, R. Sotirov, New bounds for the max-k-cut and chromatic number of a graph, *Linear Algebra and its Applications* **488** (2016) 216–234.

Linear dynamical systems for constructing observable convolutional codes with good decodable properties

N. DeCastro-García[1] *and A.L. Muñoz Castañeda* [2]

[1] *Dpt. Mathematics, Universidad de León, Spain*
E-mail: ncasg@unileon.es
[2] *Dpt. Mathematics, Universidad de León, Spain*
E-mail: amunc@unileon.es

Abstract

Convolutional codes are error-detecting and correcting codes applied to transmit, detect, and correct the information sent through a channel. In this talk, we focus on the linear dynamical systems that describe a convolutional code as a free submodule $\mathcal{C} \subset R[z]^n$ of rank k. In that case, this associated linear dynamical system is known as an *input/state/output* (I/S/O) representation and it is useful because they allow us to use the structural algebraic properties of linear systems to work in coding theory

A fundamental issue in convolutional code theory is finding methods to construct convolutional codes with good properties, such as non-propagation of errors (observability), or performing well when a decoding algorithm is applied. On the other hand, another desirable property of a convolutional code is that it has a good distance. In this case, the code will have an optimal recovery rate.

This talk aims to give several algebraic ways to construct observable convolutional codes with good decodable properties considering reachable and observable linear systems with specific properties in the matrices that form them. Also, we are able to get good distances in the obtained codes. We will study convolutional codes over finite fields and over certaing commutative rings.

Quantum Hitting Time According to a Given Distribution

Paola Boito[1], *Gianna M. Del Corso*[2]

[1] *Department of Mathematics, University of Pisa, Italy*
E-mail: paola.boito@unipi.it
[2] *Department of Computer Science, University of Pisa, Italy*
E-mail: gianna.delcorso@unipi.it

Abstract

The hitting time of a random walk on a graph G is the expected number of steps required to reach a marked node starting from a given node or a given distribution. Hitting time finds a crucial application in search problems, where it tells us how many steps are needed to detect a marked node. Other applications are in the analysis of complex networks [1], in the link prediction problem [2], or in the clustering problem[3].

In a quantum framework, random walks are replaced by quantum walks, which exhibit peculiar properties. In particular, quantum walks typically tend to diffuse faster on a graph than classical random walks. One way in which this remark can be made more precise is through the definition of a quantum notion of hitting time. We focus in this talk on quantum hitting time for discrete-time quantum walks.

Usually, quantum hitting time is defined in terms of the stationary distribution π associated with the given graph [4, 5], while the classical hitting time can be defined in terms of a generic distribution σ. We generalize the notion of quantum hitting time in terms of a generic distribution emphasizing nalogies and differences with the case where π is used. We provide conditions for the quadratic speedup of quantum hitting time over the classical counterpart and we report the results of numerical experiments on several examples of graphs both directed and undirected and for several different distributions.

Acknowledgements: Project partially supported by the National Recovery and Resilience Plan (NRRP), CN1 (Spoke10) NextGenerationEU and by GNCS-INdAM.

References

[1] Xia F, Liu J, Nie H, et al. Random walks: A review of algorithms and applications. IEEE Transactions on Emerging Topics in Computational Intelligence. 2019;4(2):95–107.

[2] Liben-Nowell D, Kleinberg J. The link prediction problem for social networks. In: Proceedings of the twelfth international conference on Information and knowledge management; 2003. p. 556–559.

[3] Chen M, Liu J, Tang X. Clustering via random walk hitting time on directed graphs. In: AAAI; Vol. 8; 2008. p. 616–621.

[4] Szegedy M. Quantum speed-up of Markov chain based algorithms. In: 45th Annual IEEE Symposium on Foundations of Computer science; IEEE; 2004. p. 32–41.

[5] Magniez F, Nayak A, Roland J, et al. Search via quantum walk. SIAM Journal on Computing. 2011;40(1):142–164.

Rational approximation for the recovery of short exponential sums

N. Derevianko[1], G. Plonka[2], M. Petz[3]

[1] Göttingen University / Institute for Numerical and Applied Mathematics, Germany
E-mail: n.derevianko@math.uni-goettingen.de
[2] Göttingen University / Institute for Numerical and Applied Mathematics, Germany
E-mail: plonka@math.uni-goettingen.de
[3] Göttingen University / Institute for Numerical and Applied Mathematics, Germany
E-mail: m.petz@math.uni-goettingen.de

Abstract

We introduce a new method - ESPIRA [1] (Estimation of Signal Parameters based on Iterative Rational Approximation) - for the recovery of complex exponential sums

$$f(t) = \sum_{j=1}^{M} \gamma_j \mathrm{e}^{\lambda_j t},$$

that are determined by a finite number of parameters: the order M, weights $\gamma_j \in \mathbb{C} \setminus \{0\}$ and nodes $z_j = \mathrm{e}^{\lambda_j} \in \mathbb{C}$ for $j = 1, ..., M$. Our new recovery procedure is based on the observation that Fourier coefficients of exponential sums have a special rational structure. To reconstruct this structure in a stable way we use the AAA algorithm for rational approximation recently proposed by Nakatsukasa et al. [2]. During the talk we will present results regarding application of the AAA algorithm to this special recovery problem. We need at least $2M + 1$ Fourier coefficients for the recovery of the exponential sum $f(t)$. We show that the Fourier coefficients can be also replaced by DFT coefficients which makes the algorithm more suitable for applications. Furthermore we show that ESPIRA can be interpreted as a matrix pencil method applied to Loewner matrices, special construction of which via an adaptive selection of index sets stabilizes the matrix pencil method (MPM). During the talk we will demonstrate that ESPIRA strongly outperforms Prony-type methods such as ESPRIT and MPM for noisy data and for signal approximation by short exponential sums.

References

[1] N. Derevianko, G. Plonka, and M. Petz, From ESPRIT to ESPIRA: Estimation of Signal Parameters by Iterative Rational Approximation, *IMA Journal of Numerical Analysis*, drab108 (2022), https://doi.org/10.1093/imanum/drab108.

[2] Y. Nakatsukasa, O. Sète, and L. N. Trefethen, The AAA Algorithm for rational approximation, *SIAM J. Sci. Comput.*, **40**(3) (2018) A1494–A1522.

Quasiseparable representations of Green matrices

Paola Boito,[1], *Yuli Eidelman*,[2]

[1] *Department of Matematics, University of Pisa, Italy*
E-mail: paola.boito@unipi.it
[2] *Department of Mathematics, Tel Aviv University, Israel*
E-mail: eideyu@tauex.tau.ac.il

Abstract

The Green matrices is a class of rank structured matrices. By the classical Asplund theorem inverse to a band matrrix is a Green matrix. Such a matrix in accordance with the developed in [2] theory admits a quasiseparable representation in the corresponding part. Based on the quasiseparable structure we derive new inversion algorithms for band matrices. The performance of our algorithms is illustrated by the results of numerical tests. The relations obtained for quasiseparable generators allow also to obtain estimates for decreasing of the offdiagonal entries of the Green matrices which are inverses of the strongly diagonally dominant band matrices in terms of the entries of the last ones.

References

[1] E. Asplund. Inverses of matrices a_{ij} which satisfies $a_{ij} = 0$ for $j > i + p$. *Math. Scand.*, 7: 57–60 (1960).

[2] Y. Eidelman, I. Gohberg, I. Haimovici *Separable type representations of matrices and fast algorithms. Volume 1. Basics. Completion problems. Multiplication and inversion algorithms*. Operator Theory: Advances and Applications, Birkhäuser, 2013.

Dispersion Entropy for Graph Signals

John Stewart Fabila-Carrasco[1,*], *Chao Tan*[2], *Javier Escudero*[1]

[1] *School of Engineering, Institute for Digital Communications, University of Edinburgh, UK.*
[*] *E-mail:* John.Fabila@ed.ac.uk
[2] *School of Electrical and Information Engineering, Tianjin University, China*

Abstract

Entropy metrics are one of the most important tool to assess the irregularity and non-linear behaviour of data. Dispersion Entropy is a non-linear measure of irregularity used for analysing complex time series [1]. The availability of data in various fields (including social science, data science or biology) defined on complex networks has increased the interest of extending entropy metrics from univariate time series to irregular domains or graphs [2].

This talk will introduce the concept of Dispersion Entropy for Graph Signals (DE_G), and describe the key steps involved in the algorithm, highlighting how the topological relationships between the graph and signals are leveraged to extend the classical univariate algorithm to graph signals. DE_G is a novel technique for analysing graph signals that is computationally efficient, robust to noise, and capable of capturing dynamic patterns in data defined on graphs.

The effectiveness of DE_G is demonstrated through several synthetic and real-world data examples, including MIX processing on Random Geometric Graphs and small-world networks. The talk will also explore the relationships between DE_G and the combinatorial Laplacian and its spectrum.

Acknowledgements: Work supported by Leverhulme Trust through a Research Project under Grant RPG-2020-158.

References

[1] Rostaghi, M. and Azami, H., *Dispersion entropy: A measure for time-series analysis.* IEEE Signal Processing Letters, 23(5), pp.610-614. 2016.

[2] Fabila-Carrasco, J.S., Tan, C. and Escudero, J., *Permutation entropy for graph signals.* IEEE Transactions on Signal and Information Processing over Networks, 8, pp.288-300. 2022.

Solving linear systems of the form $(A + \gamma UU^T)\,\mathbf{x} = \mathbf{b}$

Michele Benzi[1], *Chiara Faccio*[1]

[1] *Scuola Normale Superiore, Pisa, Italy*
E-mail: michele.benzi@sns.it chiara.faccio@sns.it

Abstract

I will discuss the iterative solution of large linear systems of equations in which the coefficient matrix is the sum of two terms, a sparse matrix A and a possibly dense, rank deficient matrix of the form γUU^T, where $\gamma > 0$ is a parameter which in some applications may be taken to be 1. The matrix A itself can be singular, but I assume that the symmetric part of A is positive semidefinite and that $A + \gamma UU^T$ is nonsingular. Linear systems of this form arise frequently in fields like optimization, fluid mechanics, computational statistics, finance, and others. I will investigate preconditioning strategies based on an alternating splitting approach combined with the use of the Sherman-Morrison-Woodbury matrix identity. The performance of the proposed approach is demonstrated by means of numerical experiments on linear systems from different application areas.

References

[1] M. Benzi, C. Faccio. Solving linear systems of the form $(A+\gamma UU^T)\,\mathbf{x} = \mathbf{b}$ by preconditioned iterative methods. *arXiv preprint* arXiv:2206.10444 (2022). *To appear in SIAM Journal on Scientific Computing.*

How perturbations propagate along the solutions of linear ordinary differential equations: a relative error analysis

Asma Farooq1[1], *Stefano Maset*[2]

[1] *Dpt Mathematics, Gran Sasso Science Institute, Italy*
E-mail: asma.farooq@gssi.it
[2] *Dpt. Mathematics and Geoscience , University of Trieste, Italy*
E-mail: maset@units.it

Abstract

In this talk, we are going to present how perturbations in the co-efficient matrix A propagate along the solutions of n-dimensional linear ordinary differential equations

$$\begin{cases} y'(t) = Ay(t), \ t \geq 0, \\ y(0) = y_0. \end{cases}$$

In other words we are considering the conditioning of the problem

$$(y_0, A) \mapsto e^{tA} y_0$$

and an asymptotic analysis of condition numbers, as $t \to +\infty$, will be given. The analysis is accomplished for the case where A is normal matrix.

We remark that conditioning of such problems attained less attention in literature. At the best of our knowledge there are only two papers [1] and [2] on this topic. These papers present computational aspects of the condition number. On the other hand our study is more on theoretical aspects of the condition number. It studies how this condition number depends on the time t and the initial data y_0. Also the asymptotic behavior of condition number as $t \mapsto +\infty$ is part of our study.

References

[1] A. Al-Mohy and N. Higham. Computing the action of the matrix exponential, with an application to exponential integrators. SIAM Journal on Scientific Computing, 33 (2011) no. 2, 488-511.

[2] E. Deadman. Estimating the condition number of $f(A)b$. Numerical Algorithms 70 (2015), 287–308..

Recovering the Spectrum of a Graph Having Most of its Eigenvalues Shared by a Vertex Deleted Subgraph

Alexander Farrugia[1]

[1] *Department of Mathematics, Ġ.F. Abela University of Malta Junior College, Msida, Malta.*
E-mail: alex.farrugia@um.edu.mt

Abstract

Let G be a simple graph and $\{1, 2, \ldots, n\}$ be its vertex set. The polynomial reconstruction problem asks the question: given a deck $\mathcal{P}(G)$ containing the n characteristic polynomials of the vertex deleted subgraphs $G-1, G-2, \ldots, G-n$ of G, can $\phi(G, x)$, the characteristic polynomial of G, be reconstructed uniquely? To date, this long-standing problem has only been solved in the affirmative for some specific classes of graphs. We prove that if there exists a vertex v such that more than half of the eigenvalues of G are shared with those of $G - v$, then this fact is recognizable from $\mathcal{P}(G)$, which allows the reconstruction of $\phi(G, x)$. To accomplish this, we make use of determinants of certain walk matrices of G. Our main result is used, in particular, to prove that the reconstruction of the characteristic polynomial from $\mathcal{P}(G)$ is possible for a large subclass of disconnected graphs, strengthening a result by Sciriha and Formosa.

References

[1] A. Farrugia. On Pseudo Walk Matrices. *Discrete Math. Lett.* 1: 8–15 (2019).

[2] I. Sciriha, J. Formosa. On polynomial reconstruction of disconnected graphs. *Util. Math.* 64: 33–44 (2003).

Is there a Kemeny's constant for second-order random walks?

Dario Fasino[1]

[1] *Dept. Mathematics, Computer Science and Physics, University of Udine, Italy*
E-mail: dario.fasino@uniud.it

Abstract

Kemeny's constant for the random walk on a (possibly directed) graph is the expected number of timesteps the walker takes to travel between any two nodes sampled using the stationary distribution. That constant provides valuable information on the navigability of the graph and depends on the eigenvalues of the transition matrix of the associated Markov chain.

In a second-order random walk, transition probabilities depend on two past states and are encoded into a stochastic tensor. Such a process can be turned into a Markov chain by 'lifting' the state space from the graph nodes to the directed edges. This procedure naturally produces a Kemeny's constant for the lifted Markov chain as done, for example, in [1] for non-backtracking walks on regular graphs. However, that constant does not immediately yield new knowledge on the second-order walk in the original network.

The main goal of this talk is to show that the average travel times for the lifted chain can be 'pulled back' in some sense to the original graph, producing sound definitions for the average travel times of the second-order walker [2]. For the eager ones who want a spoiler, the answer to the question in the title is: well, almost so.

Acknowledgements: Work partially supported by INdAM-GNCS, Italy

References

[1] J. Breen, N. Faught, C. Glover, M. Kempton, A. Knudson, A. Oveson. Kemeny's constant for non-backtracking random walks. *ArXiv*, 2203.12049 (2022).

[2] D. Fasino, A. Tonetto, F. Tudisco. Hitting times for second-order random walks. *European Journal of Applied Mathematics* (First View), 1–25 (2022).

Number of non-isomorphic graphs obtained from a tree by switches

Rosário Fernandes[1]

[1] *Center for Mathematics and Applications (NOVA Math) and Department of Mathematics, NOVA SST, Portugal*
E-mail: mrff@fct.unl.pt

Abstract

Let G be a simple graph and x, y, z, w be 4 vertices of G. A switch in G is the replacement of the edges $\{x, y\}$ and $\{z, w\}$ of G by the edges $\{x, z\}$ and $\{y, w\}$, given that $\{x, z\}$ and $\{y, w\}$ were not present in G originally. Two simple graphs have the same degree sequence if and only if there is a sequence of switches that transforms one into another. The number $S(G)$ is the number of non-isomorphic graphs that have the same degree sequence as G, in other words, that are obtained from G by switches. The $S(G)$-switch-graph class is the class whose elements, the $S(G)$-switch-graphs, are the simple graphs H with $S(H) = S(G)$. In this talk we describe the trees that are k-switch-graphs, for $1 \leq k \leq 5$.

Acknowledgements: Work supported by national funds through the FCT - Fundação para a Ciência e a Tecnologia, I.P., under the scope of the projects UIDB/00297/2020 and UIDP/00297/2020 (Center for Mathematics and Applications)

References

[1] R.A. Brualdi, R. Fernandes, S. Furtado. On the Bruhat order of labeled graphs, *Discrete Applied Mathematics*, 258: 49–64 (2019).

[2] R. Fernandes. On certain trees with the same degree sequence. *Discrete Applied Mathematics*, 321: 138–146 (2022).

Symmetrization Techniques in Image Deblurring

Marco Donatelli[1], *Paola Ferrari*[1], *Silvia Gazzola*[2]

[1] *Department of Science and High Technology, University of Insubria, Como, Italy*
E-mail: marco.donatelli@uninsubria.it, paola.ferrari@uninsubria.it
[2] *Department of Mathematical Sciences, University of Bath, Bath, UK*
E-mail: S.Gazzola@bath.ac.uk

Abstract

This talk focuses on preconditioning techniques that enhance the performance of iterative regularization methods in image deblurring. The preconditioners are applied to problems with different point spread functions (PSFs) and boundary conditions [1]. More precisely, we first consider the anti-identity preconditioner [3], which symmetrizes the coefficient matrix associated to problems with zero boundary conditions, allowing the use of MINRES as a regularization method. When considering more sophisticated boundary conditions and strongly nonsymmetric PSFs, the anti-identity preconditioner improves the performance of GMRES. We present both stationary and iteration-dependent regularizing circulant preconditioners that speed up the iterations when applied in connection with the anti-identity matrix and both standard and flexible Krylov subspaces [2]. A theoretical result about the clustering of the eigenvalues of the preconditioned matrices is proved in a special case [1]. The results of many numerical experiments are illustrated to show the effectiveness of the new preconditiong techniques, including when considering the deblurring of sparse images.

Acknowledgements: Work (partially) supported by *Gruppo Nazionale per il Calcolo Scientifico (GNCS-INdAM).*

References

[1] M. Donatelli, P. Ferrari, S. Gazzola. Symmetrization Techniques in Image Deblurring. https://arxiv.org/abs/2212.05879

[2] S. Gazzola, J. G. Nagy, M. Sabaté Landman. Iteratively reweighted FGMRES and FLSQR for sparse reconstruction. *SIAM J. Sci. Comput.*, 43(5): S47–S69 (2021).

[3] A. J. Wathen, J. Pestana. A preconditioned MINRES method for nonsymmetric Toeplitz matrices. *SIAM J. Matrix Anal. Appl.*, 36(1): 273–288 (2015).

Bi-additive Models and Symmetry

Sandra S. Ferreira[1,2], Dário Ferreira[1,2], Patrícia Antunes[2]

[1] Department of Mathematics, University of Beira Interior, Portugal
[2] Center of Mathematics and Applications, University of Beira Interior, Portugal
E-mail: sandraf@ubi.pt, dario@ubi.pt, antunes.patricia03@gmail.com

Abstract

In this talk we will consider additive models

$$\mathbf{Y} = \mathbf{X}\beta + \sum_{i=1}^{w} \mathbf{X}_i \mathbf{Z}_i,$$

where the $\mathbf{Y}$ has n observations, $\mathbf{X}, \mathbf{X}_1, ..., \mathbf{X}_w$ are given matrices and β is a vector with fixed coefficients. The random vectors $\mathbf{Z}_1, ..., \mathbf{Z}_w$ are independent, with $c_1, ..., c_w$ *i.i.d.* (independent and identically distributed) components with r-*th* order cumulants $\chi_{r,i}$, $r = 1, 2, 3, 4, ...,$ $i = 1, ..., w$. We assume that $\chi_{1,i} = \chi_{3,i} = 0$, $i = 1, ..., w$, that is the distributions of components $Z_{i,l}$, $l = 1, ..., c_i, i = 1, ..., w$, are cumulant symmetric, which means that they have null odd-order moments relative to the origin. We will show that from the symmetry of the distributions of the components of $\mathbf{Z}_i, i = 1, ..., w$ results the symmetry of the distributions of their linear combinations. We also present the adjustment of our models to estimate the parameters distributions.

Acknowledgements: Work (partially) supported by the Portuguese Foundation for Science and Technology, FCT, through the projects UIDP/MAT/00212/2020 and UIDP/MAT/00297/2020.

References

[1] P. Antunes, S. S. Ferreira, D. Ferreira, C. Nunes, J. T. Mexia. *Estimation in additive models and ANOVA-like applications*. J. Appl. Stat., 47: 1–10 (2020).

[2] N. Balakrishnan, N. L. Johnson, S. Kotz. *A note on relationships between moments, central moments, and cumulants from multivariate distributions*. Stat Probab Lett. 39(1): 49–54 (1998).

[3] C. C. Craig. *On A Property of the Semi-Invariants of Thiele*. Ann. Math. Stat., 2 (2): 154–164 (1931).

[4] A. I. Khuri, T. Mathew, B. K. Sinha. *Statistical Tests for Mixed Linear Models*. Wiley Interscience Publication, John Wiley & Sons, Inc, 1998.

Symbol-Based Convergence Analysis in (Block) Multigrid Methods for saddle-point problems

Matthias Bolten[1], *Marco Donatelli*[2], *Paola Ferrari*[2], *Isabella Furci*[1]

[1] *School of Mathematics and Natural Sciences, University of Wuppertal, Wuppertal, Germany*
E-mail: bolten@uni-wuppertal.de, furci@uni-wuppertal.de
[2] *Department of Science and High Technology, University of Insubria, Como, Italy*
E-mail: marco.donatelli@uninsubria.it, paola.ferrari@uninsubria.it

Abstract

The main focus of the talk is on efficient multigrid methods for large linear systems with a particular saddle-point structure. In these problems the system matrix is symmetric, but indefinite, so the variational convergence theory that is usually used to prove multigrid convergence cannot be directly applied [6]. In most cases more powerful smoothers are used to take into account the special coupling, represented by the off-diagonal blocks [3, 4]. Alternatively, instead of altering the smoother, recently [5] a different algebraic approach that analyzes properly preconditioned saddle-point problems has been presented.

In the present talk we analyze saddle-point problems where the blocks are both circulant and Toeplitz-like within this framework. Such structured matrix sequences are associated to a function called symbol. We are able to derive sufficient conditions for the multigrid convergence in terms of the associated symbols, which are also useful for tuning the multigrid parameters [2].

Moreover, we present how it is possible to extend the analysis to the block setting, that is, to structured matrices generated by a matrix-valued function. The illustrative example is the linear system stemming from the Finite Element approximation of the Stokes problem [1].

Finally, we present several numerical tests to show the efficiency of the approach also with a comparison with the state-of-the-art strategies.

Acknowledgements: Work (partially) supported by *Gruppo Nazionale per il Calcolo Scientifico (GNCS-INdAM).*

References

[1] M. Bolten, M. Donatelli, P. Ferrari, and I. Furci. Symbol-based convergence analysis in block multigrid methods with Applications for Stokes Problems. *(submitted).*

[2] M. Bolten, M. Donatelli, P. Ferrari, and I. Furci. Symbol-based convergence analysis in multigrid methods for saddle point problems. *(submitted)*.

[3] D. Braess and R. Sarazin. An efficient smoother for the Stokes problem. *Appl. Numer. Math.*, 23(1): 3–19 (1997).

[4] J.-F. Maitre, F. Musy, and P. Nigon. A fast solver for the Stokes equations using multigrid with a Uzawa smoother. *Advances in multigrid methods (Oberwolfach, 1984), Notes Numer. Fluid Mech.*, 11, 77–83. Friedr. Vieweg, Braunschweig, 1985.

[5] Y. Notay. A new algebraic multigrid approach for Stokes problems. *Numer. Math.*, 132(1): 51–84 (2016).

[6] J. W. Ruge and K. Stüben. Algebraic multigrid. *Multigrid Methods, S. McCormick, ed., Frontiers Appl. Math. 3*, SIAM, Philadelphia, (1987), 73–130.

Generating efficient vectors for pairwise comparison matrices

Susana Furtado[1], *Charles Johnson*[2]

[1] *Faculdade de Economia, Universidade do Porto/ CEAFEL, Portugal*
E-mail: sbf@fep.up.pt
[2] *Department of Mathematics, College of William and Mary, United States*
E-mail: crjohn@wm.edu

Abstract

In this talk we focus upon the pairwise comparison (PC) matrix (also called reciprocal matrix) component of the often discussed Analytic Hierarchy Process and its approximation by a consistent matrix formed from an efficient vector.

We give a method of inductively generating efficient vectors for any given PC matrix.

It is known that the entry-wise geometric mean of all columns is efficient for any PC matrix. The relationship between any set of columns of the PC matrix and efficient vectors is explored.

Acknowledgements: Work partially supported by FCT- Fundação para a Ciência e Tecnologia, under project UIDB/04721/2020 (Susana Furtado) and National Science Foundation grant DMS-0751964 (Charles Johnson).

Spectral geometric mean, geometric mean, and Kantorovich constant

Hamid Reza Moradi[1], *Shigeru Furuichi*[2], *Mohammad Sababheh*[3]

[1] *Department of Mathematics, Mashhad Branch, Islamic Azad University, Mashhad, Iran*
E-mail: hrmoradi@mshdiau.ac.ir
[2] *Department of Information Science, College of Humanities and Sciences, Nihon University, Tokyo, Japan.*
E-mail: furuichi.shigeru@nihon-u.ac.jp
[3] *Department of Basic Sciences, Princess Sumaya University for Technology, Amman, Jordan.*
E-mail: sababheh@psut.edu.jo

Abstract

The weighted geometric mean is defined by $A\sharp_t B := A^{\frac{1}{2}}\left(A^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)^t A^{\frac{1}{2}}$ for positive operators A, B and $0 \leq t \leq 1$. The weighted spectral geometric mean was defined in [1] by

$$A\natural_t B = \left(A^{-1}\sharp B\right)^t A\left(A^{-1}\sharp B\right)^t, 0 \leq t \leq 1.$$

In this talk, we give two different operator inequalities between the weighted spectral geometric mean $A\natural_t B$ and the weighted geometric mean $A\sharp_t B$. We study the mathematical properties for the generalized Kantorovich constant for $0 < m < M$ and $t \in \mathbb{R}$:

$$K(m, M, t) = \frac{(mM^t - Mm^t)}{(t-1)(M-m)}\left(\frac{t-1}{t}\frac{M^t - m^t}{mM^t - Mm^t}\right)^t.$$

Employing the shown properties, we give the ordering of two inequalities.

In addition, we give some inequalities such as Ando type inequality, Kantorovich type inequality, and Ando–Hiai type inequality with the weighted spectral geometric mean $A\natural_t B$ and the generalized Kantorovich constant $K(m, M, t)$.

Our talk is mainly based on the results in [2]. We will also show new results related to the weighted spectral geometric mean $A\natural_t B$ and/or the weighted geometric mean $A\sharp_t B$.

Acknowledgements: The author (S.F.) was partially supported by JSPS KAKENHI Grant Number 21K03341.

References

[1] H. Lee and Y. Lim, *Metric and spectral geometric means on symmetric cones*, Kyungpook Math. J., **47**(1) (2007), 133–150.

[2] H.R.Moradi, S.Furuichi, and M.Sababheh, *Operator spectral geometric versus geometric mean*, arXiv:2111.03256.

Frames and Finite-rank Integral Representations of Positive Operator-Valued Measures

Jean-Pierre Gabardo[1], *Deguang Han*[2]

[1] *Department of Mathematics & Statistics, McMaster University, Hamilton, Ontario, Canada*
E-mail: gabardo@mcmaster.ca
[2] *Department of Mathematics, University of Central Florida, Orlando, Florida, USA*
E-mail: deguang.han@ucf.edu@

Abstract

Discrete and continuous frames can be considered as positive operator-valued measures (POVMs) that have integral representations using rank-one operators. However, not every POVM has an integral representation. One goal of this paper is to examine the POVMs that have finite-rank integral representations. More precisely, we present an necessary and sufficient condition under which a positive operator-valued measure $F : \Omega \to B(H)$ has an integral representation of the form

$$F(E) = \sum_{k=1}^{m} \int_E G_k(\omega) \otimes G_k(\omega) d\mu(\omega)$$

for some weakly measurable maps G_k $(1 \leq k \leq m)$ from a measurable space Ω to a Hilbert space $\mathcal{H}$ and some some positive measure μ on Ω. Similar characterizations are also obtained for projection-valued measures. In particular, we show that an integral representable probability POVM can be dilated to a integral representable projection-valued measure if and only if the corresponding measure is purely atomic.

References

[1] J. Diestel, H. Jarchow and A. Tonge, Absolutely summing operators. Cambridge Studies in Advanced Mathematics, 43. Cambridge University Press, Cambridge, 1995.

[2] M. Ehler and K. Okoudjou, Probabilistic Frames: An Overview, in Finite frames, 415–436, Appl. Numer. Harmon. Anal., Birkhäuser/Springer, New York, 2013.

[3] V. Gupta, P. Mandayam and V. S. Sunder, The functional analysis of quantum information theory. A collection of notes based on lectures by Gilles Pisier, K. R. Parthasarathy, Vern Paulsen and Andreas Winter. Lecture Notes in Physics, 902. Springer, Cham, 2015.

[4] P. R. Halmos, A Hilbert space problem book. Second edition. Graduate Texts in Mathematics, 19. Encyclopedia of Mathematics and its Applications, 17. Springer-Verlag, New York-Berlin, 1982.

[5] D. Han, Frame representations and Parseval duals with applications to Gabor frames, *Trans. Amer. Math. Soc.*, 360 (2008), no. 6, 3307–3326.

[6] D. Han, P. Li, B. Meng and W. Tang, Operator valued frames and structured quantum channels, *Sci. China Math.*, 54 (2011), no. 11, 2361–2372.

[7] D. Han and D. Larson, Frames, bases and group representations, *Memoirs Amer. Math. Soc.,* 697 (2000).

[8] D. Han, D. Larson, B. Liu and R. Liu, Operator-Valued Measures, Dilations, and the Theory of Frames, *Memoirs Amer. Math. Soc.,* Vol. 229, No. 1075 (2014).

[9] D. Han, D. Larson and R. Liu, Dilations for systems of imprimitivity acting on Banach spaces, *J. Funct. Anal.,* 266 (12), 2014, 6914?6937.

[10] D. Han, D. Larson and R. Liu, Dilations of operator-valued measures with bounded p-variations and framings on Banach spaces , *J. Funct. Anal.,* 274 (2018), 1466–1490.

[11] T. Heinosaari, L. Mazzarella and M. Wolf, Quantum tomography and prior information, *Commun. Math. Phys.,* 318 (2013), 355–374.

[12] T. Hytönen, J. Pellonpää and K. Ylinen, Positive sesquilinear form measures and generalized eigenvalue expansions, *J. Math. Anal. Appl.,* 336(2007), 1287–1304.

[13] M. Maslouhia and S. Loukilib, Probabilistic tight frames and representation of Positive Operator-Valued Measures, *Appl. Comput. Harmon. Anal.,* 2018, to appear.

[14] B. Moran, S. Howard and D. Cochran, Positive-operator-valued measures: a general setting for frames, in Excursions in Harmonic Analysis. Volume 2, 49–64, Appl. Numer. Harmon. Anal., Birkhäuser/Springer, New York, 2013.

Decompositions of matrices into torsion matrices and zero-square matrices

Peter V. Danchev[1], *Esther García*[2], *Miguel Gómez Lozano*[3]

[1] *Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria*
E-mail: danchev@math.bas.bg
[2] *Dpt. Matemática Aplicada, Ciencia e Ingeniería de Materiales y Tecnología Electrónica, Universidad Rey Juan Carlos, Spain*
E-mail: esther.garcia@urjc.es
[3] *Dpt. Álgebra, Geometría y Topología, Universidad de Málaga, Spain*
E-mail: miggl@uma.es

Abstract

Recall that a square matrix A is called a torsion matrix if there exists some natural number n such that A^n is the identity matrix. We study when a square matrix over a field can be decomposed as the sum of a torsion matrix and a nilpotent matrix of order at most two. We present several examples that show that the decomposition does not hold in general, and we give necessary and sufficient conditions to get this decomposition for nilpotent matrices.

Acknowledgements: The first-named author (Peter V. Danchev) was supported in part by the Bulgarian National Science Fund under Grant KP-06 No. 32/1 of December 07, 2019, the second-name author (Esther García) was partially supported by Ayuda Puente 2022, URJC, and the three authors were partially supported by the Junta de Andalucía FQM264.

References

[1] P. Danchev, E. García, M. Gómez Lozano. Decompositions of matrices into potent and square-zero matrices. *Internat. J. Algebra Comput.*, 32(2):251–263 (2022).

[2] P. Danchev, E. García, M. Gómez Lozano. Decompositions of matrices into invertible and square-zero matrices. arXiv:2301.06106 [math.RA].

Self-dual polyhedral cones and their slack matrices

João Gouveia[1], *Bruno F. Lourenço*[2]

[1] *CMUC, Department of Mathematics, University of Coimbra, Portugal*
E-mail: jgouveia1@mat.uc.pt
[2] *Dept. of Statistical Inference and Mathematics, Institute of Statistical Mathematics, Japan*
E-mail: bruno@ism.ac.jp

Abstract

We analyze self-dual polyhedral cones and prove several properties about their slack matrices. In particular, we show that self-duality is equivalent to the existence of a positive semidefinite (PSD) slack. Beyond that, we show that if the underlying cone is irreducible, then the corresponding PSD slacks are not only doubly nonnegative matrices (DNN) but are extreme rays of the cone of DNN matrices, which correspond to a family of extreme rays not previously described. More surprisingly, we show that, unless the cone is simplicial, PSD slacks not only fail to be completely positive matrices but they also lie outside the cone of completely positive semidefinite matrices. Finally, we show how one can use semidefinite programming to probe the existence of self-dual cones with given combinatorics.

Acknowledgements: The first author was supported by the Centre for Mathematics of the University of Coimbra (UIDB/00324/2020, funded by the Portuguese Government through FCT/MCTES. The second author was supported partly by the JSPS Grant-in-Aid for Early-Career Scientists JP19K20217 and the Grant-in-Aid for Scientific Research (B) JP21H03398.

References

[1] J. Gouveia and B. F. Lourenço. Self-dual polyhedral cones and their slack matrices. *arXiv:2207.11747*

Challenges and opportunities in solving Navier-Stokes equations in patient-specific left heart model

M. S. Nagargoje[1], *E. Lazpita-Suinaga*[1], *J. Garicano-Mena*[1,2], *S. Le Clainche*[1,2]

[1] *ETSI Aeronáutica y del Espacio - Universidad Politécnica de Madrid, 28040 Madrid, Spain.*
E-mail: mahesh.nagargoje@upm.es, e.lazpita@alumnos.upm.es
[2] *Center for Computational Simulation (CCS), 28660 Boadilla del Monte, Spain*
E-mail: soledad.leclainche@upm.es, jesus.garicano.mena@upm.es

Abstract

The solution of the incompressible Navier-Stokes (NS) equations is crucial across many scientific and technological fields, namely aerodynamics, water or oil transport, and biological flow modelling. Cardiovascular blood flow modeling is becoming crucial due to the increased risk of cardiovascular diseases. In this contribution, we address the challenges and opportunities brought forward by the numerical simulation of left ventricular (LV) blood flow, which is now becoming feasible due to advances in numerical methods and computing power. It is accepted that many cardiovascular diseases occur due to abnormal functioning of the heart, which lead to severe complications and mortalities. Computational fluid dynamics (CFD) has the potential to assist in early diagnosis and treatment of heart diseases by analysis of blood flow patterns in patient-specific heart models.

The NS equations are very difficult to solve analytically due to their highly non-linear, non-homogenous second-order nature and the non-trivial coupling of momentum and mass conservation. Using the finite volume method, we discretize the NS partial differential equations into a grid of smaller control volumes. The NS equations are integrated over these grids by enforcing the conservation of mass and momentum principles. Gradients of flow variables are calculated at cell faces by using central difference, upwind schemes, or higher order schemes. Appropriate boundary conditions are incorporated at grid faces. The discrete algebraic equations, which are non-linear due to convective terms, are solved typically using SIMPLE method to disentagle the pressure-momenta coupling. The resulting linear equations are solved using the iterative Gauss-Seidel method until a convergence criterion is fulfilled.

More specifically, in this work, we have simulated a patient-based left ventricle (LV) model to understand the flow patterns in healthy left ventricles. Forming an asymmetric vortex in a healthy LV shows an efficient way of blood transport from LV to various body parts. The vortex patterns in the LV have been analyzed using velocity streamlines, wall shear stress, and vorticity. We

conclude that understanding vortex dynamics in LV and various vortex indexes can be used as an early diagnosis tool and improvement of heart disease treatment. Further, vortex indexes can be used to analyze the outcomes of various heart surgeries.

Keywords: Navier-Stokes equations, computational fluid dynamics (CFD), vortex dynamics, Left ventricle (LV).

Acknowledgements: This work is supported by Grant No. TED2021-129774B-C21 (DigitHEART).

Parallel High-Resolution Compact PFFT-Type Algorithms vs. Low-Dimensional Eigenvectors Solvers for 3D Subsurface Scattering Problems.

Yury A. Gryazin[1], *Ron L. Gonzales*[2]

[1] *Department of Mathematics & Statistics, Idaho State University, USA*
E-mail: yurygryazin@isu.edu
[2] *Department of Mathematics & Statistics, Idaho State University, USA*
E-mail: ronaldgonzales@isu.edu

Abstract

In this talk, we present two types of efficient parallel high-resolution algorithms for the solution of a subsurface electromagnetic scattering problem. The first method is based on a partial FFT-type approach (PFFT) where the direct solution of the 3D Helmholtz equation with Sommerfeld-like boundary conditions relies on a sequence of FFT solutions with Dirichlet, Neumann, or periodic boundary conditions. The key part of the second algorithm is the solution of two one-dimensional eigenvalue problems that ensure the fast and scalable direct solution of the 3D layered Helmholtz equation.

The 3D Helmholtz equation is discretized by high-order compact finite-difference schemes. The resulting layered systems of finite-difference equations are solved by two proposed direct methods. The systems that include the three-dimensioned subsurface inclusions are solved by iterative preconditioned Krylov subspace-based methods. The PFFT-based preconditioner and low-dimensional eigenvectors solvers are used for efficient implementation of the developed iterative approach.

The complexity and scalability of the methods are analyzed on scattering problems with realistic ranges of parameters in soil and mine-like targets.

References

[1] Y.A. Gryazin, M.V. Klibanov, T.R. Lucas. GMRES computation of high frequency electrical field propagation in land mine detection. *J. Comput. Phys.*, 158: 98–115 (2000).

[2] J.Toivanen, M. Wolfmayr, A fast Fourier transform based direct solver for the Helmholtz problem *Numer. Linear Algebra Appl.*, 27: 272–287 (2020).

[3] R. Gonzales, Y. Gryazin, Y.T. Lee, Parallel FFT Algorithms for High-Order Approximations on Three-Dimensional Compact Stencils *Parallel Computing, 103,Article 102757*,(2021).

[4] R. Gonzales, Y. Gryazin, Parallel high-resolution compact partial FFT-type direct algorithms for subsurface scattering problems *Proc. SPIE 12116, Chemical, Biological, Radiological, Nuclear, and Explosives (CBRNE) Sensing XXIII, Article 1211612, doi: 10.1117/12.2617140*, (2022).

Orbit-injective Covariant Quantum Channels

Deguang Han[1]

[1] *Department of Mathematics, University of Central Florida, USA*
E-mail: deguang.han@ucf.edu

Abstract

We examine the quantum channels that preserve and also separate the orbits of pure states under the action of a group unitary representation π. Such a quantum channel will be called π-orbit injective. We prove that for finite group and complex Hilbert space cases, such a channel necessarily separates all the pure states. However, this is no longer true for quantum channels acting on real Hilbert spaces, or quantum channels acting on complex Hilbert spaces with (infinite) compact group representations. In both cases, we obtain necessary and/or sufficient conditions under which the quantum channel is orbit injective. These conditions are given in terms of the so called property (H) of characters (more generally, irreducible representations) of the group, and characterizations of property (H) are presented for real and complex valued multiplicative characters.

Acknowledgements: Work (partially) supported by the NSF grant DMS-2105038

Developing an efficient aeronautical design tool using modal decomposition and deep learning for fluid dynamics analysis

A. Hetherington[1],
A. Corrochano[1], *R. Abadía-Heredia*[1], *E. Lazpita*[1], *S. Le Clainche*[1]

[1] *E.T.S.I. Aeronáutica y del Espacio, Universidad Politécnica de Madrid, Spain*
E-mail: ashton.ian@upm.es

Abstract

The need for environmentally friendly solutions has led to a significant focus on developing more efficient and less polluting aeronautical designs. However, traditional methods for testing these designs using simulations and experiments can be expensive and time-consuming. The developed approach is based on modal decomposition, deep learning [1, 2], and algebraic principles, including solving eigenvalue problems, rotations, and translations using singular value decomposition (SVD) and higher-order dynamic mode decomposition (HODMD) [3]. By leveraging these techniques, it is possible to identify the physical properties associated with fluid dynamics problems, specially turbulent flows, reconstruct corrupted data from sensors, and generate reduced-order models for accelerated computational fluid dynamics simulations. This approach offers a cost-effective solution to traditional design and testing methods and has demonstrated promising results on datasets, including a three-dimensional cylinder and two concentric jets. With its potential for future industrial applications, this method will be capable of providing environmentally friendly solutions.

References

[1] Abadía-Heredia, R., Lopez-Martin, M., Carro, B., Arribas, J.I., Pérez, J.M., Le Clainche, S., *A predictive hybrid reduced order model based on proper orthogonal decomposition combined with deep learning architectures*, Exp. Syst. Appl., 187, 115910, 2022.

[2] Muñoz, E., Dave, H., D'Alessio, G., Parente, A., Le Clainche, S., *Extraction and analysis of flow features in planar synthetic jets using different machine learning techniques*, preprint in ARXIV, 2023.

[3] Le Clainche, S., Vega, J.M., Soria, J., *Higher order dynamic mode decomposition of noisy experimental data: The flow structure of a zero-net-mass-flux jet*, Exp. Thermal and Fluid Sci.,Vol. 88, 336-353, 2017.

Canonical forms for strictly regular matrix polynomials

Richard Hollister[1], *D. Steven Mackey*[2]

[1] *Department of Mathematics, University at Buffalo, USA*
E-mail: rahollis@buffalo.edu
[2] *Department of Mathematics, Western Michigan University, USA*
E-mail: steve.mackey@wmich.edu

Abstract

Canonical forms are an integral part of matrix theory. From the Jordan canonical form for square matrices under similarity and the Kronecker canonical form for matrix pencils under strict equivalence to the Smith canonical form for matrix polynomials under unimodular equivalence, the discovery of new canonical forms can lead to significant advances in new theory. We propose a new canonical form for strictly regular matrix polynomials under unimodular equivalence that not only has many of the features of existing canonical forms, but also has the property that the degree of the matrix polynomial is preserved.

In the last several decades, much work has been done to try to emulate the Kronecker canonical form for higher degree matrix polynomials, particularly the quadratic case [**?**, 2, 3, 4]. In [1], the authors construct a Kronecker-like canonical form for quadratic matrix polynomials, but the result is somewhat complicated. It is suspected that the complexity of Kronecker-like canonical forms will grow untractably with the degree of the matrix polynomial. In this talk, we present a new strategy for constructing canonical forms for matrix polynomials of arbitrary degrees, and the resulting canonical form for strictly regular polynomials.

References

[1] F. De Terán, F. Dopico, D. S. Mackey. A quasi-canonical form for quadratic matrix polynomials, Part II: the singular case. *In preparation.*

[2] F. De Terán, F. Dopico, D. S. Mackey, V. Perović. Quadratic realizability of palindromic matrix polynomials. *Linear Algebra Appl.*, 567: 202–262, 2019.

[3] D. S. Mackey, V. Perović. Quadratic realizability of palindromic matrix polynomials: the real case. *Linear and Multilinear Algebra*, DOI:10.1080/03081087.2022.2044443.

[4] D. S. Mackey, F. Tisseur. The Hermitian quadratic realizability problem. *In preparation.*

On a matrix perspective of Sobolev-type inner products and higher-order recurrence relations

Carlos Hermoso[1], *Edmundo J. Huertas*[2], *Alberto Lastra*[3], *Francisco Marcellán*[4]

[1] *Dpto. de Física y Matemáticas, Universidad de Alcalá, 28805 Alcalá de Henares, Madrid, Spain*
E-mail: carlos.hermoso@uah.es
[2] *Dpto. de Física y Matemáticas, Universidad de Alcalá, 28805 Alcalá de Henares, Madrid, Spain*
E-mail: edmundo.huertas@uah.es
[3] *Dpto. de Física y Matemáticas, Universidad de Alcalá, 28805 Alcalá de Henares, Madrid, Spain*
E-mail: alberto.lastra@uah.es
[4] *Dpto. de Matemáticas, Universidad Carlos III de Madrid, 28911 Leganés, Madrid, Spain*
E-mail: pacomarc@ing.uc3m.es

Abstract

It is well known in the literature that Sobolev-type orthogonal polynomials on the real line satisfy higher-order recurrence relations, and these can be expressed as a (2N + 1)–banded symmetric semi-infinite matrix. In this talk, we analyze the connection between these (2N + 1)–banded matrices, and the Jacobi matrices associated with the three-term recurrence relation satisfied by the standard 2-iterated Christoffel sequence of orthonormal polynomials with respect to certain positive Borel orthogonality measure $d\mu(x)$.

Acknowledgements: The work of CH, EJH and AL is supported by Dirección General de Investigación e Innovación, Consejería de Educación e Investigación of the Comunidad de Madrid (Spain), and Universidad de Alcalá under grants CM/JIN/2019-010 and CM/JIN/2021-014, Proyectos de I+D para Jóvenes Investigadores de la Universidad de Alcalá 2019 and 2021, respectively. The work of FM has been supported by FEDER/Ministerio de Ciencia e Innovación-Agencia Estatal de Investigación of Spain, grant PGC2018-096504-B-C33, and the Madrid Government (Comunidad de Madrid-Spain) under the Multiannual Agreement with UC3M in the line of Excellence of University Professors, grant EPUC3M23 in the context of the V PRICIT (Regional Programme of Research and Technological Innovation).

References

[1] C. Hermoso, E. J. Huertas, A. Lastra, and F. Marcellán, Higher-order recurrence relations, Sobolev-type inner products and matrix factorizations, *Numer. Algorithms*, 92: 665–692 (2023).

Approximating manifold-valued functions

Simon Jacobsson[1]

[1] *Department of Computer Science, KU Leuven, Belgium*
E-mail: simon.jacobsson@kuleuven.be

Abstract

Many functions that are worth approximating map into Riemannian manifolds, spaces equipped with a notion of distance derived from an inner product on their tangent space. We present a construction using Riemann normal coordinates for approximating such functions. Our construction extends approximation techniques for functions between linear spaces, like Adaptive Cross Approximation or Chebyshev interpolation, in such a way that we are able to upper bound the max error in terms of a lower bound on the manifold's sectional curvature. Furthermore, when the sectional curvature is nonnegative, e.g. as for compact Lie groups, the mean error is also bounded.

Of special interest are manifolds that are not naturally embedded into a vector space or whose codimension is large. The Segre manifold of rank 1 tensors is an example of such a manifold where we are able to apply our construction. Approximating functions that map to the Segre manifold has applications in Model Order Reduction and tensor completion.[1]

In some cases, the condition number of approximating functions that map into manifolds is guaranteed to be exponential in the radius of the domain. We classify and discuss those cases.

This is joint work with Raf Vandebril (KU Leuven), Joeri Van der Veken (KU Leuven), and Nick Vannieuwenhoven (KU Leuven).

References

[1] Swijsen, L. and Van der Veken, J. and Vannieuwenhoven, N. *Tensor completion using geodesics on Segre manifolds*. Numerical Linear Algebra with Applications, 2022, 10.1002/nla.2446.

Fast multiplication, determinants, inverses and eigendecomositions of arrowhead and diagonal-plus-rank-one matrices over associative fields

Nevena Jakovčević Stor[1], *Ivan Slapničar*[1], *Thaniporn Chaysri*[1]

[1] *Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, University of Split, Croatia*
E-mail: nevena@fesb.hr
E-mail: ivan.slapnicar@fesb.hr
E-mail: thaniporn.chaysri@fesb.hr

Abstract

We present a work considering arrowhead and diagonal-plus-rank-one matrices in $\mathbb{F}^{n\times n}$ where $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$. $\mathbb{H}$ is a non-commutative field of quaternions. All the presented formulas (matrix-vector multiplications, determinants, inverses) are unified in the sense that the same formula holds in both, commutative and noncommutative algebras. Each formula requires $O(n)$ arithmetic operations. Most of the formulas hold for block matrices, as well. We also present an algorithm for the eigendecomposition computation of arrowhead and diagonal-plus-rank-one matrices of quaternions. The code, written in the programing language Julia, along with examples, is available on GitHub. The code relies on the Julia's polymorphism feature.

Acknowledgements: This work has been fully supported by Croatian Science Foundation under the project IP-2020-02-2240.

Linear preservers of semipositive matrices

Sachindranath Jayaraman[1], *Vatsalkumar N. Mer*[2]

[1] *IISER Thiruvananthapuram, India*
E-mail: sachindranathj@iisertvm.ac.in, sachindranathj@gmail.com
[2] *Chungbuk National University, South Korea*
E-mail: vnm232657@gmail.com

Abstract

Given proper cones K_1 and K_2 in $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively, an $m \times n$ matrix A with real entries is said to be semipositive if there exists a $x \in K_1^\circ$ such that $Ax \in K_2^\circ$, where K° denotes the interior of a proper cone K. This set is denoted by $S(K_1, K_2)$. The purpose of this talk is to bring out the structure of an invertible linear map L on $M_{m,n}(\mathbb{R})$ that preserves the set $S(\mathbb{R}^n_+, \mathbb{R}^m_+)$. This talk is based on [1].

References

[1] S. Jayaraman and V. N. Mer. On linear preservers of semipositive matrices. *Electronic Journal of Linear Algebra*, 37: 88–112 (2021).

New quantum divergence and barycenter with the spectral geometric mean

Miran Jeong[1], *Sejong Kim*[2]

[1] *Department of Mathematics, Chungbuk National University, Korea*
E-mail: jmr4006@chungbuk.ac.kr
[2] *Department of Mathematics, Chungbuk National University, Korea*
E-mail: skim@chungbuk.ac.kr

Abstract

Many researchers [1, 2] studied the quantum divergences associated with Kubo-Ando means, which are given by

$$\Phi(A,B) = \mathrm{tr}[(1-t)A + tB - A\sigma_t B)],$$

where $A\sigma_t B$ is a Kubo-Ando mean with the weight $0 < t < 1$. On the other hand, there are many non Kubo-Ando means such as the generalized mean $Q_t(A,B) = ((1-t)A^p + tB^p)^{1/p}$ for $-1 \le p \le 1$, the log-Euclidean mean $\mathcal{LE}_t(A,B) = \exp((1-t)\log A + t\log B)$ and the Wasserstein mean $A \diamond_t B = (1-t)^2 A + t^2 B + t(1-t)[A(A^{-1}\#B) + (A^{-1}\#B)A]$ for $0 < t < 1$.

In this talk, we consider a new quantum divergence for $0 < t \le 1/2$

$$\Phi_t(A,B) = \mathrm{tr}[(1-t)A + tB - A\natural_t B]$$

associated with the spectral geometric mean $A\natural_t B = (A^{-1}\#B)^t A (A^{-1}\#B)^t$, which is a non Kubo-Ando mean. Note that for $t = 1/2$, $\Phi_{1/2}(A,B) = \mathrm{tr}\left(\frac{A+B}{2} - A\natural_{1/2}B\right)$ is same as the square of the Bures-Wasserstein distance. Then we study the barycenter of the quantum divergence Φ_t with some properties, minimizing the weighted sum of divergences.

Acknowledgements: This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. NRF-2022R1A2C4001306).

References

[1] R. Bhatia, S. Gaubert, and T. Jain, Matrix versions of the Hellinger distance, *Lett. Math. Phys.*, 109: 1777-1804 (2019).

[2] J. Pitrik and D. Virosztek, Quantum Hellinger distances revisited. *Lett. Math. Phys.* 110 : 2039–2052 (2020).

On an Analogue of a Property of Singular M-matrices, for the Lyapunov and the Stein Operators

A.M. Encinas [1], *Samir Mondal* [2], *K.C. Sivakumar*[3]

[1] *Department of Mathematics, Polytechnic University of Catalunya, Barcelona, Spain.*
E-mail: andres.marcos.encinas@upc.edu
[2] *Department of Mathematics, Indian Institute of Technology Madras, India.*
E-mail: ma19d750@smail.iitm.ac.in
[3] *Department of Mathematics, Indian Institute of Technology Madras, India.*
E-mail: kcskumar@iitm.ac.in

Abstract

A matrix $A \in \mathbb{R}^{n\times n}$ is called a Z-matrix, if all the off-diagonal entries of A are nonpositive. Any Z-matrix A has the representation $A = sI - B$, where $s \geq 0$ and B is an entrywise nonnegative matrix. If s is at least the spectral radius $\rho(B)$ of B, then A is called an *M-matrix*. Let $s = \rho(B)$, above. Then A is a singular M-matrix. It is well known that if, in addition, A is irreducible, then A has the following property: the only nonnegative vector that belongs to the range space of A is the zero vector. In this talk, a discussion of analogues of this result, for the Lyapunov and the Stein operators, on the Hilbert space of real symmetric matrices, will be presented.

On interpolation with finite Blaschke products

S. Kalmykov[1,2]

[1] *School of mathematical sciences, Shanghai Jiao Tong University, China*
E-mail: kalmykovsergei@sjtu.edu.cn
[2] *Keldysh Institute of Applied Mathematics of Russian Academy of Sciences, Russia*

Abstract

We denote the set of Blaschke products of degree m by

$$\mathbf{B}_m := \left\{ \gamma \prod_{j=1}^{m} \frac{z - a_j}{1 - \overline{a_j} z} \ : \ \gamma, a_1, \dots, a_m \in \mathbb{C}, |\gamma| = 1, |a_1|, \dots, |a_m| < 1 \right\}.$$

Here, $\mathbf{B}_0$ consists of constants (with modulus 1). We also write

$$\mathbf{B}_{\leq m} := \cup_{k=0}^{m} \mathbf{B}_k.$$

In this talk, we discuss an alternative proof of the Jones-Ruscheweyh theorem [1].

Theorem 1. *Let* $0 \leq \varphi_1 < \varphi_2 < \ldots < \varphi_m < 2\pi$ *and* $\psi_1, \psi_2, \dots, \psi_m \in [0, 2\pi)$. *Then there exists a Blaschke product* $B \in \mathbf{B}_{\leq m-1}$ *such that* $B(\exp i\varphi_j) = \exp i\psi_j$, $j = 1, 2, \dots, m$.

The approach is based on direct solution of the equations. To be more precise, the original equations are transformed into a system of polynomial equations with real coefficients. This leads to "geometric representation" of Blaschke products. Then, a Positivstellensatz by Prestel and Delzell [2] and a representation of positive polynomials in a special form due to Berr and Wörmann [3] together with a particular structure of the equations are used.

This is based on a joint work with Béla Nagy.

References

1. Jones W.B., Ruscheweyh S. *Blaschke product interpolation and its application to the design of digital filters* // Constr. Approx. 1987. V. 3(4). P. 405–409.

2. Prestel A., Delzell C.N. *Positive polynomials.* Berlin: Springer. Springer Monographs in Mathematics. 2001.

3. Berr R., Wörmannn T. *Positive polynomials and tame preorderings* // Math. Z. 2001. V. 236(4). P. 813–840.

Determinants of some special matrices

Yogesh Kapil

Department of Mathematics, Sant Longowal Institute of Engineering and Technology, Longowal-148106, Punjab, India
E-mail: yogesh_kapill@yahoo.com

Abstract

Let $p_1, p_2, \cdots, p_n$ be distinct positive real numbers and m be any integer. Every symmetric polynomial $f(x, y) \in C[x, y]$ induces a symmetric matrix $[f(p_i, p_j)]_{i,j=1}^{n}$. We obtain the determinants of such matrices with an aim to find the determinants of $P_m = [(p_i + p_j)^m]_{i,j=1}^{n}$ and $B_{2m} = \left[(p_i - p_j)^{2m}\right]_{i,j=1}^{n}$ for $m \in \mathbb{N}$ (where $\mathbb{N}$ is the set of natural numbers) in terms of the Schur polynomials. We also discuss and compute determinant of the matrix $K_m = \left[\frac{p_i^m + p_j^m}{p_i + p_j}\right]_{i,j=1}^{n}$ for any integer m in terms of the Schur and skew-Schur polynomials.

The Varchenko Determinant for Complexes of Oriented Matroids

Winfried Hochstättler[1], *Sophia Keip*[2], *Kolja Knauer*[3]

[1] *Fakultät für Mathematik und Informatik, FernUniversität in Hagen, Germany*
E-mail: winfried.hochstaettler@fernuni-hagen.de
[2] *Fakultät für Mathematik und Informatik, FernUniversität in Hagen, Germany*
E-mail: sophia.keip@fernuni-hagen.de
[3] *Departament de Matemàtiques i Informàtica, Universitat de Barcelona, Spain*
E-mail: kolja.knauer@ub.edu

Abstract

Let $\mathcal{A}$ be a real hyperplane arrangement and $L(\mathcal{A})$ the geometric lattice formed by the intersections of hyperplanes in $\mathcal{A}$. We call the fulldimensional cells of $\mathcal{A}$ topes. The *Varchenko Matrix* ist defined by $\mathcal{V}_{ij} = \prod_{e \in S(T_i,T_j)} w_e$, where the w_e are weights on the hyperplanes H_e of the arrangement and $S(T_i, T_j)$ is the set of hyperplanes that have to be crossed on a shortest path from a tope T_i to a tope T_j. Varchenko [1] gave an elegant factorization of the determinant of that matrix, considering the weights as variables:

$$\det(\mathcal{V}) = \prod_{F \in L(\mathcal{A})} \left(1 - w_F^2\right)^{m_F}$$

where $w_F = \prod_{F \subset H_e} w_e$ and m_F are positive integers depending only on the geometric lattice $L(\mathcal{A})$.

We generalize this theorem for a combinatorial structure called *complexes of oriented matroids*. They can be described by only two axioms which capture local symmetry and local convexity and are a generalization of *oriented matroids*. In this talk we will see how the Varchenko Matrix generalizes to complexes of oriented matroids and will give the general idea of the proof for the nice factorization formula.

References

[1] A. Varchenko, Bilinear form of real configuration of hyperplanes, Adv. Math. **97** (1993) 110–144.

[2] W. Hochstättler, V. Welker. "The Varchenko determinant for oriented matroids." Mathematische Zeitschrift 293.3 (2019): 1415-1430.

No cycle-spliced bipartite signed graph with nullity $\eta(\Sigma) = c(\Sigma)$

Suliman Khan

Department of Mathematics and Physics
University of Campania Luigi Vanvitelli
Caserta, Italy

E-mail: suliman.khan@unicampania.it
suliman5344@gmail.com

Abstract

Let $\Sigma = (G, \sigma)$ be a signed graph and $A(\Sigma)$ be its adjacency matrix. The nullity and cyclomatic number of Σ is denoted by $\eta(\Sigma)$ and $c(\Sigma)$, respectively. A connected signed graph Σ is said to be cycle-spliced bipartite if every block is an even cycle. In 2022, Wong, et al., showed for every cycle-spliced bipartite graph $0 \leq \eta(G) \leq c(G) + 1$. In this paper, we extend the results of Wong, et al., to signed graphs, and prove for every cycle-spliced bipartite signed graph $0 \leq \eta(\Sigma) \leq c(\Sigma) + 1$. Next, we prove that there is no cycle-spliced bipartite signed graph Σ of any order with $\eta(\Sigma) = c(\Sigma)$. We give a structural characterization of cycle-spliced bipartite signed graphs Σ with nullity $\eta(\Sigma) = c(\Sigma) - 1$. Nonsingular cycle-spliced bipartite signed graphs are characterized. For cycle-spliced signed graphs Σ having only odd cycles, we show that $\eta(\Sigma)$ is 0 or 1. Furthermore, we characterize nonsingular such graphs where every cycle has at most two cut vertices of Σ.

Joint work with Francesco Belardo, Department of Mathematics and Applications, University of Naples Federico-II, Italy (Email: fbelardo@unina.it)

References

[1] D. Cvetkovic, M. Doob, H. Sachs. *Spectra of graphs*. Johann Ambrosius Barth, Heidelberg, 1995.

[2] D. Wong, Q. Zhou, F. Tian. Nullity and singularity of a graph in which every block is a cycle, *Discr. Math.*, 345: 112851 (2022).

[3] Y. Lu, J. Wu. No signed graph with nullity $\eta(G, \sigma) = |V(G)| - 2m(G) + 2c(G) - 1$, *Linear Algebra Appl.*, 615: 175–193 (2021).

[4] Y.Z. Fan, Y. Wang, Y. Wang. A note on the nullity of unicyclic signed graphs, *Linear Algebra Appl.*, 438(3): 1193–1200 (2013).

Trifactorization of pattern symmetric nonnegative matrices

Damjana Kokol Bukovšek[1], *Helena Šmigoc*[2]

[1] *School of Economics and Business, University of Ljubljana, Slovenia*
E-mail: damjana.kokol.bukovsek@ef.uni-lj.si
[2] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: helena.smigoc.ucd.ie

Abstract

Let A be symmetric entrywise nonnegative matrix of order $n \times n$. A factorization $A = BCB^T$, where B is nonnegative matrix of order $n \times k$ and C is symmetric nonnegative matrix of order $k \times k$, is called *symmetric nonnegative trifactorization* of A. Minimal possible k in such factorization is called the *SNT-rank* of A. In the talk we will for the most part take aside the actual values of matrices and only consider their patterns. Our main focus will be the question, how small can SNT-rank be among all symmetric nonnegative matrices A with given zero-nonzero pattern. The pattern of a matrix can be described by a simple graph that allows loops. We will answer this question for trees and complete graphs without loops.

Acknowledgements: Damjana Kokol Bukovšek acknowledges financial support from the Slovenian Research Agency (research core funding No. P1-0222).

An interactive user-friendly software supporting research in graph theory

Kristina Kostić[1], *Tamara Koledin*[2], *Zoran Stanić*[3]

[1] *Faculty of Mathematics, University of Belgrade, Serbia*
E-mail: kristina.kostic@matf.bg.ac.rs
[2] *School of Electrical Engineering, University of Belgrade, Serbia*
E-mail: tamara@etf.rs
[3] *Faculty of Mathematics, University of Belgrade, Serbia*
E-mail: zstanic@matf.bg.ac.rs

Abstract

We present the facilities of a recently developed expert system designed for improving research procedures in the theory of simple graphs, weighted graphs (in particular, signed graphs) and oriented graphs. The system is a user-friendly web-based platform based on modern web development frameworks and technologies.

It is also

- interactive (in the sense that a user have a possibility to create a graph and immediately receive a number of its structural or spectral invariants or particular properties),
- upgradeable (in the sense that a user has a possibility to upgrade it by implementing the computation of new invariants or properties) and
- created under the terms of the GNU General Public License as published by the Free Software Foundation (in simple words, the License guarantees every user the freedom to run, study and modify the software).

Other functionalities include importing graphs from files and editing through a drawing interface.

This expert system is designed to substitute existing tools such as newGRAPH and similar software. It is developed under a national research project, partially supported by EU funds.

On Q-tensors

T. Parthasarathy[1], *G. Ravindran*[2], *Sunil Kumar*[3]

[1] *Chennai Mathematical Institute, India*
E-mail: pacha14@yahoo.com
[2] *SQC & OR, Indian Statistical Institute, India*
E-mail: gravi@hotmail.com
[3] *SQC & OR, Indian Statistical Institute, India*
E-mail: sunny061193@gmail.com

Abstract

A tensor is a multidimensional analog of a matrix. We call $\mathcal{A}$ to be a Q-tensor if the associated tensor complementarity problem, $TCP(q, \mathcal{A})$, has solution for every vector q. In this paper, we extend some properties of Q-matrices (which are prominently studied in the linear complementarity theory) to Q-tensors. In particular, we provide sufficient condition for a principal subtensor of a Q-tensor to be a Q-tensor. We also provide sufficient conditions for a tensor to be a Q-tensor using the Q-property of its principal subtensors. It is known that R-tensors are Q-tensors and converse is not true in general. In this paper, we give a condition for a Q-tensor to be an R-tensor. In addition, we prove a few results for positive (nonnegative) tensors. We illustrate our results with examples.

References

[1] L. Qi and Z. Luo: Tensor Analysis, Society for Industrial and Applied Mathematics, Philadelphia, PA (2017). doi: 10.1137/1.9781611974751.

[2] K.G. Murty: Linear Complementarity, Linear and Nonlinear Programming, Heldermann Verlag, Berlin (1988).

[3] T. Parthasarathy, G. Ravindran, and Sunil Kumar: On Semimonotone Matrices, R_0-Matrices and Q-Matrices, Journal of Opt. Theory and App., 195, 131-147, (2022).

[4] Z.H. Huang, Y.Y. Suo and J. Wang: On Q-Tensors, arXiv e-prints, arXiv:1509.03088 (2015), pp. 1-23.

On the numerical range of some structured matrices

Rute Lemos[1]

[1] *CIDMA and Department of Mathematics, University of Aveiro, Portugal*
E-mail: rute@ua.pt

Abstract

The numerical range of a matrix $A \in M_n$ is the subset of the complex plane denoted and defined by $W(A) = \{x^*Ax : x \in \mathbb{C}^n, x^*x = 1\}$. It is a convex set, as asserted by the famous Toeplitz-Hausdorff Theorem [2, 3], containing the spectrum of A. This concept has been intensively investigated, due to its theoretical interest and applications. The Elliptical Range Theorem characterizes $W(A)$ for $A \in M_2$ and the elliptic shape persists in certain cases [1], independently of the size of A. In this talk, the numerical range of some structured matrices, whose eigenvalues are integer numbers, and their boundary generating curves, are explored. Some of these boundary generating curves are oval shaped. Illustrative figures of the obtained results are presented.

Based on a joint work with Natália Bebiano (CMUC, University of Coimbra) and Graça Soares (CMAT-UTAD, University of Trás-os-Montes e Alto Douro).

Acknowledgements: Work supported by Portuguese funds through the Center for Research and Development in Mathematics and Applications (CIDMA) and the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), project UIDB/04106/2020.

References

[1] E. Brown and I. M. Spitkovsky, On matrices with elliptical numerical ranges, *Linear Multilin. Algebra*, 52: 177–193 (2004).

[2] F. Hausdorff, Der Wertvorrat einer Bilinearform. *Math. Z.*, 3: 314–316 (1919).

[3] O. Toeplitz, Das algebraische Analogon zu einem Satze von Fejér. *Math. Z.*, 2: 187–197 (1918).

Geometric Estimates of Kernel Matrix Eigenvalues

Mikhail Lepilov[1], *Yuanzhe Xi*[2], *Shifan Zhao*[3]

[1] *Department of Mathematics, Emory University, United States*
E-mail: mikhail.lepilov@emory.edu
[2] *Department of Mathematics, Emory University, United States*
E-mail: yuanzhe.xi@emory.edu
[3] *Department of Mathematics, Emory University, United States*
E-mail: szhao89@emory.edu

Abstract

Kernel matrices have appeared over the past few decades as intermediate structures when computing with "big data," such as during support vector machine classification or kernel ridge regression. Naive matrix algorithms quickly become too computationally intensive once such matrices reach moderate size; in fact, even explicitly forming such matrices is undesirable when the number of points is large. Hence, various low-rank approximations to such matrices become indispensable. If the underlying points come from the real world, however, it is *a priori* not often clear what the numerical rank of the resulting kernel matrix is for a given tolerance: existing methods like rank-revealing QR factorization or its randomized variants only apply in the case when the full matrix to be approximated has already been formed.[1] Instead, we may facilitate computation if we could approximate the spectral decay of the kernel matrix by that of the submatrix formed after sampling relatively few of the underlying points, using just the geometry of the points and the analytical properties of the kernel alone. In this work, we seek to characterize the eigenvalue decay approximations that result after using various point sampling schemes, both deterministic and randomized. This idea is motivated by the point sampling schemes explored previously in the context of constructing low-rank approximations, as in [2]. We explore these sampling methods and others while highlighting the connections between geometric (point-based) and algebraic (matrix-based) eigenvalue approximation techniques.

Acknowledgements: This material is based upon work supported by the National Science Foundation under Grant No. 2038118.

References

[1] P. G. Martinsson. Randomized methods for matrix computations. *IAS/Park City Mathematics Series*, 25: 187–229 (2019).

[2] D. Cai, E. Chow, and Y. Xi. Data-driven linear complexity low-rank approximation of general kernel matrices: a geometric approach. *pre-print*, (2022).

A geometric construction of isospectral graphs for the discrete magnetic Laplacian

Fernando Lledó [1]

[1] *Department of Mathematics, University Carlos III de Madrid and Instituto de Ciencias Matemáticas (ICMAT), SPAIN*
E-mail: flledo@math.uc3m.es

Abstract

Analysis on graphs studies the connections between geometrical or combinatorial properties of graphs and natural operators defined on them. In this talk, I will present a new geometrical construction leading to an infinite collection of families of discrete graphs [1, 2], where all the elements in each family are (finite) isospectral non-isomorphic graphs for the discrete magnetic Laplacian with normalised weights. The construction is based on the notion of (isospectral) frames which, together with the s-partition of a natural number r, define the isospectral families of graphs by contraction of distinguished vertices of the frames. The isospectral frames have high symmetry and we use a spectral preorder of graphs studied in [3, 4] to control the spectral spreading of the eigenvalues under elementary perturbations of the graph like vertex contraction and vertex virtualisation.

Acknowledgements: Work (partially) supported by Madrid Government under the Agreement with UC3M in the line of Research Funds for Beatriz Galindo Fellowships (C&QIG-BG-CM-UC3M) and in the context of the V PRICIT and the project 6G-INTEGRATION-3 (grant no. TSI-063000-2021-127), funded by UNICO program (under the Next Generation EU umbrella funds), Ministerio de Asuntos Económicos y Transición Digital of Spain.

References

[1] J.S. Fabila-Carrasco, F. Lledó and O. Post, A geometric construction of isospectral magnetic graphs, *preprint; https://arxiv.org/abs/2208.07280* (2022);

[2] J.S. Fabila-Carrasco, F. Lledó and O. Post, Isospectral graphs via spectral bracketing, *preprint https://arxiv.org/abs/2207.03924* (2022);

[3] J.S. Fabila-Carrasco, F. Lledó and O. Post, Spectral preorder and perturbations of discrete weighted graphs, *Math. Ann.*, 382: 1775-1823 (2022).

[4] J.S. Fabila-Carrasco, F. Lledó and O. Post,*Spectral gaps and discrete magnetic Laplacians*, *Lin. Alg. Appl.* 547: 183-216 (2018).

On the Directional Derivative of Kemeny's Constant

Connor Albright[1], *Kimberly P. Hadaway*[2], *Ari Holcombe Pomerance, Joel Jeffries, Kate J. Lorenzen*[3], *Abigail K. Nix*

[1] *Sonoma State University, USA*
E-mail: cpcalbright2010@gmail.com
[2] *Iowa State University, USA*
E-mail: kph3@iastate.edu
[3] *Macalester College, USA*
E-mail: aholcomb@macalester.edu
[3] *Iowa State University, USA*
E-mail: joeljeff@iastate.edu
[3] *Linfield University, USA*
E-mail: klorenzen@linfield.edu
[3] *Middlebury College, USA*
E-mail: anix@middlebury.edu

Abstract

In a connected graph, Kemeny's constant gives the expected time of a random walk from an arbitrary vertex x to reach a randomly-chosen vertex y. Because of this, Kemeny's constant can be interpreted as a measure of how well a graph is connected. It is generally unknown how the addition or removal of edges affects Kemeny's constant. Inspired by the directional derivative of the normalized Laplacian, we derive the directional derivative of Kemeny's constant for several graph families. In addition, we find sharp bounds for the directional derivative of an eigenvalue of the normalized Laplacian and bounds for the directional derivative of Kemeny's constant.

Acknowledgements: This work was conducted primarily at the 2022 Iowa State University Math REU which was supported through NSF Grant DMS-1950583.

References

[1] Askoy, Sinan G., Purvine, Emilie, and Young, Stephen J. "Directional Laplacian centrality for cyber situational awareness." *Pre-print on arXiv*, (2021). https://arxiv.org/2008.04357v2.

[2] Breen, Jane, Butler, Steve, Day, Nicklas, DeArmond, Colt, Lorenzen, Kate, Qian, Haoyang, and Riesen, Jacob. "Computing Kemeny's constant for a barbell graph." *Electronic Journal of Linear Algebra*, 35 (2019):583–598. https://doi:10.13001/ela.2019.5175.

[3] Breen, Jane and Kirkland, Steve. "A structured condition number for Kemeny's constant." *SIAM Journal on Matrix Analysis and Applications*, 40, no. 4 (2019):1555–1578, https://doi:10.1137/19M1240964.

[4] Grinstead, Charles M. and Snell, J. Laurie. *Introduction to Probability.* Rhode Island: American Math Society, 1997.

[5] Levene, Mark and Loizou, George. "Kemeny's constant and the random surfer." *American Math Monthly,* 109 no. 8 (2002): 741–745.

[6] Kemeny, John. G.; Snell, J. Laurie. *Finite Markov Chains.* Princeton, NJ: D. Van Nostrand, 1960.

[7] quid. "Can eigenvectors of a symmetric circulant matrix be real?" Last modified August 5, 2016.
https://math.stackexchange.com/q/1883244

Approximation of the smallest eigenvalue of large hermitian matrices dependent on parameters

N. Guglielmi[1], *M. Manucci*[2], *E. Mengi*[3]

[1] *Division of Mathematics, Gran Sasso Science Institute, Italy*
E-mail: nicola.guglielmi@gssi.it
[2] *Division of Mathematics, Gran Sasso Science Institute, Italy*
E-mail: mattia.manucci@gssi.it
[3] *Department of Mathematics, Koç University, Turkey*
E-mail: emengi@ku.edu.tr

Abstract

We investigate the numerical solution of the global optimization problem

$$\lambda_{LB} = \min_{\boldsymbol{\mu}\in\mathcal{D}} \lambda_{-1}(\mu), \tag{1}$$

where $\mathcal{D}$ is a compact subset of $\mathbb{R}^p$ and $\lambda_{-1}(\mu)$ denotes the smallest eigenvalue of a parametric dependent hermitian matrix

$$A(\mu) := \sum_{l=1}^{\kappa} f_l(\mu) A_l \tag{2}$$

where $A_l : \mathbb{C}^n \to \mathbb{C}^n$ and $f_l : \bar{\mathcal{D}} \to \mathbb{R}$ for $l = 1, ..., \kappa$ represent given hermitian matrices and real-analytic functions, respectively. Being able to solve in a fast and reliable way problem (1) is crucial in *projection Model Order Reduction*, in particular for the construction of reduced spaces through *greedy algorithms* [1].

In general problems of type (1) come with two main challenges: 1) they are nonconvex and 2) they have an elevate computational complexity since we are naturally interested in the case of large matrices (2), i.e. $n \gg 1$. To deal with these difficulties we develop an algorithm that, concerning 2), employs the *subspace* framework [2] which enables to significantly *reduce the computational complexity*; concerning 1) it relays on `EigOpt` [3], for $p = 1$ or $p = 2$, and it uses a gradient flow penalization method to treat the case $p > 2$. The proposed algorithm, under suitable assumptions, can be shown to be globally convergent.

We show through numerical test examples and comparisons that the proposed method is efficient and reliable in solving (1).

References

[1] J. Hesthaven, G. Rozza, and B. Stamm. *Certified Reduced Basis Methods for Parametrized Partial Differential Equations*. Jan. 2016.

[2] F. Kangal, K. Meerbergen, E. Mengi, and W. Michiels. A subspace method for large-scale eigenvalue optimization. *SIAM Journal on Matrix Analysis and Applications*, 39(1): 48–82 (2018).

[3] E. Mengi, E. A. Yildirim, and M. Kiliç. Numerical optimization of eigenvalues of hermitian matrix functions. *SIAM Journal on Matrix Analysis and Applications*, 35(2): 699–724 (2014).

Matrices similar to centrosymmetric matrices

Banjemín A. Itzá-Ortiz[1] *and* *Rubén A. Martínez-Avendaño*[2]

[1] *Universidad Autónoma del Estado de Hidalgo, Mexico*
E-mail: itza@uaeh.edu.mx
[2] *Instituto Tecnológico Autónomo de México, Mexico*
E-mail: ruben.martinez.avendano@gmail.com

Abstract

In this talk we give conditions that guarantee that a matrix is similar to a centrosymmetric matrix. Furthermore, we give conditions for a matrix to be similar to a matrix which has a centrosymmetric principal submatrix, and conditions under which a matrix can be dilated to a matrix similar to a centrosymmetric matrix.

Parallel-in-time solver for the all-at-once Runge-Kutta discretization

Luca Bergamaschi[1], *Santolo Leveque*[2], *Ángeles Martínez*[3], *and John W. Pearson*[4]

[1] *Dept. of Civil Environmental and Architectural Engineering, University of Padova, Italy*
E-mail: luca.bergamaschi@unipd.it
[2] *CRM Ennio De Giorgi, Scuola Normale Superiore, Italy*
E-mail: santolo.leveque@sns.it
[3] *Department of Mathematics and Geosciences, University of Trieste, Italy*
E-mail: amartinez@units.it
[4] *School of Mathematics, The University of Edinburgh, United Kingdom*
E-mail: j.pearson@ed.ac.uk

Abstract

Time-dependent PDEs arise quite often in many scientific areas, such as mechanics, biology, economics, or chemistry, just to name a few. Of late, researchers have devoted their effort in devising parallel-in-time methods for the numerical solution of time-dependent PDEs [1, 2, 3]. As opposed to the classical approach, in which an approximation of the solution at a time t is computed after solving for all the previous times, parallel-in-time methods approximate the solution of the problem for all times concurrently. This in turns adds a new dimension of parallelism and allows to speed-up the numerical solution on modern supercomputers.

In this talk, we present a fully parallelizable preconditioner for the all-at-once linear system arising when employing a Runge–Kutta method in time. The resulting system is solved iteratively for the numerical solution and for the stages. The proposed preconditioner results in a block-diagonal solve for all the stages at all the time-steps, and a Schur complement obtained by solving again systems for the stages. In order to solve for the system for the stages, we employ a new block-preconditioner based on the SVD of the Runge–Kutta coefficient matrix.

Parallel results on the Stokes equation show the robustness of the preconditioner with respect to the discretization parameters and to the number of stages, as well as very promising scalability and parallel efficiency indices.

Acknowledgements: Work (partially) supported by INDAM-GNCS Project CUP_E55F22000270001.

References

[1] R. D. Falgout, S. Friedhoff, T.V. Kolev, S. P. MacLachlan, and J. B. Schroder, *Parallel time integration with multigrid*, SIAM J. Sci. Comput., 36, pp. C635–C661, 2014.

[2] M. J. Gander, L. Halpern, J. Rannou, and J. Ryan, *A direct time parallel solver by diagonalization for the wave equation*, SIAM J. Sci. Comput., 41, pp. A220–A245, 2019.

[3] E. McDonald, J. Pestana, and A. Wathen, *Preconditioning and iterative solution of all-at-once systems for evolutionary partial differential equations*, SIAM J. Sci. Comput., 40, pp. A1012–A1033, 2018.

V-AISM, an Approximate Inverse LU Preconditioner

J. Cerdán[1], *J. Marín*[2], *J. Mas*[3], *R. Bru*[1]

[1] *Instituto de Matemática Multidisciplinar, Universitat Politècnica de València, Spain*
E-mail: jcerdan@imm.upv.es
[2] *Instituto de Matemática Multidisciplinar, Universitat Politècnica de València, Spain*
E-mail: jmarinma@imm.upv.es
[3] *Instituto de Matemática Multidisciplinar, Universitat Politècnica de València, Spain*
E-mail: jmasm@imm.upv.es
[4] *Instituto de Matemática Multidisciplinar, Universitat Politècnica de València, Spain*
E-mail: rbru@imm.upv.es

Abstract

To solve a large, sparse nonsymmetric linear system $Ax = b$, where A is a nonsingular matrix using iterative methods, the use of preconditioning techniques is fruitful. The right preconditioning technique consists of finding a matrix M for which the solution via an iterative method of the equivalent linear system $AM^{-1}y = b$, where $y = Mx$, is obtained more efficiently. Thus, the preconditioner M should approximate the matrix A in some sense. There are mainly two preconditioning techniques. One that computes the matrix M and another that computes its inverse. In this work we study factorized approximate inverse preconditioners that compute explicitly the preconditioner as an approximation of A^{-1}. Then, preconditioning is applied by matrix-vector products in each iteration of the Krylov method, whichis important for efficient parallel computations.

In this work, we use the Sherman–Morrison formula to obtain an approximate inverse LU preconditioner. The main difference with respect to the AISM preconditioner [1], which is also based on the Sherman-Morrison formula, is the way of applying recursively the inversion formula to obtain a new decomposition of A^{-1}. Then we use a compact representation of this decomposition to build our proposed preconditioner V–AISM.

The inverse of A may be computed considering a nonsingular matrix A_0 of the same size and two sets of vectors $\{x_k\}_{k=1}^n$ and $\{y_k\}_{k=1}^n$ such that

$$A = A_0 + \sum_{k=1}^{n} x_k y_k^T = A_0 + XY^T, \tag{1}$$

where $X = [x_1\ x_2\ \cdots\ x_n]$ and $Y = [y_1\ y_2\ \cdots\ y_n]$.

Defining $A_k = A_0 + \sum_{i=1}^{k} x_i y_i^T$ with $k = 1, \ldots, n$ we have

$$\begin{cases} A_k = A_{k-1} + x_k y_k^T \\ A_n = A. \end{cases}$$

Suppose that $r_1 = 1 + y_1^T A_0^{-1} x_1 \neq 0$. By the Sherman–Morrison formula [2, Eq. (2)] and [3], the matrix $A_1 = A_0 + x_1 y_1^T$ is nonsingular and

$$A_1^{-1} = A_0^{-1} - \frac{1}{r_1} A_0^{-1} x_1 y_1^T A_0^{-1} = A_0^{-1} \left(I - \frac{1}{r_1} x_1 w_1^T \right) = A_0^{-1} V_1$$

where $w_1^T = y_1^T A_0^{-1}$ and $V_1 = I - \frac{1}{r_1} x_1 w_1^T$. Following this process, assuming that $r_k = 1 + y_k^T A_{k-1}^{-1} x_k \neq 0$ for x_k and y_k, then

$$A_k^{-1} = A_{k-1}^{-1} - \frac{1}{r_k} A_{k-1}^{-1} x_k y_k^T A_{k-1}^{-1} = A_{k-1}^{-1} \left(I - \frac{1}{r_k} x_k w_k^T \right) = A_{k-1}^{-1} V_k,$$

where $w_k^T = y_k^T A_{k-1}^{-1}$ and $V_k = I - \frac{1}{r_k} x_k w_k^T$.

Then,

$$A^{-1} = A_n^{-1} = A_0^{-1} V_1 \cdots V_n. \tag{2}$$

It is worth to say that there is a main difference between the expressions of A^{-1} obtained in [1] and (2). Actually, to build the preconditioner AISM given in [1] the expression of A^{-1} is an additive decomposition obtained applying also the Sherman–Morrison formula. Here to construct the new preconditioner V–AISM we have a multiplicative representation of A. In fact, this decomposition depends explicitly on the matrices V_k.

To compute the preconditioner, as usual, entries are zeroed in the process if they are small enough. We prove that this process is breakdown-free for M- and H-matrices. Moreover, numerical experiments show that this new preconditioner is efficient and faster than AISM.

Acknowledgements: Work supported by Conselleria de Innovación, Universidades, Ciencia y Sociedad Digital, Generalitat Valenciana (CIAICO/2021/162)

References

[1] R. Bru, J. Cerdán, J. Marín and J. Mas. Preconditioning sparse nonsymmetric linear systems with the Sherman–Morrison formula. *SIAM J. on Sci. Comput.*, 25: 701–715 (2003).

[2] W. W. Hager. Updating the inverse of matrix. *SIAM Rev.*, 31(2): 221–239 (1989).

[3] J. Sherman and W. J. Morrison. Adjustment of an inverse matrix corresponding to a change in one element of a given matrix. *Ann. Math. Statist.*, 21, 124–127, 1950.

A defect-correction algorithm for quadratic matrix equations, with applications to quasi-Toeplitz matrices

Dario A. Bini[1], *Beatrice Meini*[2]

[1] *Dipartimento di Matematica, Università di Pisa, Italy*
E-mail: dario.bini@unipi.it
[2] *Dipartimento di Matematica, Università di Pisa, Italy*
E-mail: beatrice.meini@unipi.it

Abstract

A defect correction formula for the quadratic matrix equation $A_1X^2+A_0X+A_{-1}=0$ is presented. More specifically, assume that $\widetilde{G}$ is an approximation of the sought solution G. Then, by following the ideas of [2] and [3], we derive an equation for the defect $H=G-\widetilde{G}$ and express H in terms of an invariant subspace of a suitable pencil. This equation allows us to introduce a modification of the Structure-preserving Doubling Algorithm (SDA), that enables refining an initial approximation to the sought solution. This modification provides substantial advantages, in terms of convergence acceleration, in the solution of equations coming from stochastic models.

Finally, we show an application to the analysis of random walks in the quarter plane, where the matrix coefficients A_i, $i=-1,0,1$, as well as the sought solution G, are infinite matrices endowed of the quasi-Toeplitz structure (QT matrices). In this framework, there are situations where Cyclic Reduction and SDA fail to converge if applied in the customary way, whereas, under a suitable choice of the starting approximation $\widetilde{G}$, our modified version of SDA converges in a few iteration steps. Numerical experiments confirm the effectiveness of the proposed method.

More details can be found in [1].

References

[1] D. A. Bini, B. Meini, A defect-correction algorithm for quadratic matrix equations, with applications to quasi-Toeplitz matrices, 2022; arXiv:2212.09491.

[2] A. Bunse-Gerstner, V. Mehrmann, A symplectic QR like algorithm for the solution of the real algebraic Riccati equation. IEEE Trans Automat Control. 1986;31(12):1104–1113.

[3] V. Mehrmann, E. Tan, Defect correction methods for the solution of algebraic Riccati equations. IEEE Trans Automat Control. 1988;33(7):695–698.

Multi-variable Wasserstein means of positive definite operators

Sejong Kim[1], *Vatsalkumar N. Mer*[2]

[1] *Department of Mathematics, Chungbuk National University, Cheongju 28644, Korea*
E-mail: skim@chungbuk.ac.kr
[2] *Institute for Industrial and Applied Mathematics, Chungbuk National University, Cheongju 28644, Korea*
E-mail: vnm232657@gmail.com

Abstract

In this talk, I will discuss two forms of least squares mean: the Karcher mean and the Wasserstein mean on the cone of positive definite Hermitian matrices. Then, I will talk about the extension of Karcher mean in the infinite-dimensional setting of positive operators on a Hilbert space and its important properties. Lastly, the Wasserstein mean in the infinite-dimensional setting of positive operators on a Hilbert space and its attractive properties will be explored.

Acknowledgements: The work of S. Kim was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. NRF-2022R1A2C4001306).

On the minimal least eigenvalues of circulant graphs

Milan Bašić[1], *Andjela Todorović*[2]

[1] *Faculty of Sciences and Mathematics, University of Niš, Serbia*
E-mail: basic_milan@yahoo.com
[2] *Ecole Polytechnique, Institut Polytechnique de Paris, France*
E-mail: andjelatodorovich@gmail.com

Abstract

The integral circulant graph $\mathrm{ICG}_n(D)$ has the vertex set $Z_n = \{0, 1, 2, \ldots, n-1\}$ and vertices a and b are adjacent if $\gcd(a-b, n) \in D$, where $D \subseteq \{d : d \mid n,\ 1 \leq d < n\}$. In this paper we prove that the minimal value of the least eigenvalues (minimum least eigenvalue) of the integral circulant graphs $\mathrm{ICG}_n(D)$ of a given order n with its prime factorization $p_1^{\alpha_1} \cdots p_k^{\alpha_k}$, is equal to $-\frac{n}{p_1}$. We characterize the unique graph with minimum least eigenvalues among all integral circulant graphs of a given order n. Furthermore, it is shown that the minimum least eigenvalue of the connected integral circulant graphs $\mathrm{ICG}_n(D)$ of a given order n such that their complements are also connected, is equal to $-\frac{n}{p_1}+p_1^{\alpha_1-1}$. We determine all such graphs whose spectra contain minimum least eigenvalue. Finally, We calculate second minimal value of the least eigenvalues of integral cicrulant graphs of a given order n, which is $-\frac{n}{p_1}+1$, and characterize all graphs whose spectra contain that value.

Acknowledgements: The authors gratefully acknowledge support from the research project of the Ministry of Education, Science and Technological Development of the Republic of Serbia (number 451-03-68/2022-14/ 200124).

References

[1] M. Bašić, *Characterization of strongly regular integral circulant graphs by spectral approach*, Appl Anal Discrete Math. 16(2022), 288–306.

[2] W. So, *Integral circulant graphs*, Discrete Math. 306 (2006), 153–158.

[3] H. N. Ramaswamy, C. R. Veena, *On the Energy of Unitary Cayley Graphs*, Electron. J. Combin. 16 (2007) #N24.

[4] R. Xing, B. Zhou, *On the least eigenvalue of cacti with pendant vertices*, Linear Algebra Appl. 438 (2013) 2256–2273.

[5] M. Petrović, B. Borovićanin, T. M. Aleksić, *Bicyclic graphs for which the least eigenvalue is minimum*, Linear Algebra Appl. 430 (2009) 1328–1335.

Jordan Structure and Stability of Schur Canonical Form

Anastasiia Minenkova[1], *Evelyn Nitch-Griffin*[2], *Vadim Olshevsky*[3]

[1] *Dpt. Math&Stats, Mount Holyoke College, USA*
E-mail: aminenkova@mtholyoke.edu
[2] *Dpt. of Mathematics, University of Connecticut, USA*
E-mail: evelyn.nitch-griffin@uconn.edu
[3] *Dpt. of Mathematics, University of Connecticut, USA*
E-mail: olshevsky@gmail.com

Abstract

In the present talk we show the relation between the stability of Schur decomposition and the Jordan structure of the perturbed matrix.

References

[1] A. Minenkova, E. Nitch-Griffin, V. Olshevsky, *Gohberg-Kaashoek Numbers and Forward Stability of the Schur Canonical Form*, arXiv:2110.1534

[2] Backward Stability of the Schur Canonical Form, with Evelyn Nitch-Griffin and Vadim Olshevsky, arXiv:2108.02312

NFFT in Parameter Learning for Nonlocal Image Denoising Models

Andrés Miniguano-Trujillo[1], *John Pearson*[2]

[1] *Maxwell Institute for Mathematical Sciences, UK*
E-mail: Andres.Miniguano-Trujillo@ed.ac.uk
[2] *School of Mathematics, The University of Edinburgh, UK*
E-mail: j.pearson@ed.ac.uk

Abstract This work considers the numerical solution of a bilevel optimisation problem for the estimation of parameters in nonlocal image denoising models. The relevant parameters are the fidelity weight and a weight within the kernel of the nonlocal operator. Variational methods are used to characterise local minima via a first order optimality system. A finite element method is used to discretise the kernel and the associated linear systems. For the former, a nonequispaced fast Fourier transform [**?**] is used to efficiently compute the vectorial Gauss transform associated with the nonlocal kernel as in [2]. For the latter, we use a preconditioner based on the nonlocal matrix to speed up the iterations of the LGMRES Krylov method. We use a second-order trust-region algorithm for optimising the denoising parameters. Several experiments are provided to illustrate the efficiency of the method and contrast them against the dense-matrix approximation showcased in the previous work [3].

Acknowledgements: Work supported by MAC-MIGS CDT Scholarship under EPSRC grant EP/S023291/1.

References

[1] Jens Keiner, Stefan Kunis, Daniel Potts. *Using NNFT 3 - a software library for various nonequispaced fast Fourier transforms.* ACM Transactions on Mathematical Software 36, 1–30, 2009.

[2] Franziska Nestler, Martin Stoll, Theresa Wagner. *Learning in high-dimensional feature spaces using ANOVA-based fast matrix-vector multiplication.* Foundations of Data Science 4(3), 423-440, 2022.

[3] Marta D'Elia, Juan Carlos De Los Reyes, Andrés Miniguano-Trujillo. *Bilevel parameter learning for nonlocal image denoising models.* Journal of Mathematical Imaging and Vision 63, 753–775, 2021.

Topologically-induced suppression of explosive synchronization on graphs

Manuel Miranda[1], *Mattia Frasca*[23], *Ernesto Estrada*[1]

[1] *Institute of Cross-Disciplinary Physics and Complex Systems, IFISC (UIB-CSIC), 07122 Palma de Mallorca, Spain*
E-mail: mmiranda@ifisc.uib-csic.es
[2] *Department of Electrical, Electronics and Computer Science Engineering, University of Catania, Italy*
[3] *Istituto di Analisi dei Sistemi ed Informatica "A. Ruberti", Consiglio Nazionale delle Ricerche (IASI-CNR), Roma, Italy*

Abstract

The transition from a disordered state to the one in which all the nodes oscillates with the same phase typically occurs in a gradual way, which is characteristic of second order transitions. Therefore, the discovery of explosive synchronization on the networked Kuramoto model [1] when there's a correlation between degree (topological feature) and natural frequency (dynamical feature) marked a tipping point in this field.

For that purpose, we investigated how modifying the Kuramoto model by using degree-biased Laplacians [2] affects the explosive synchronization. In this talk I will report how explosive synchronization is modified by these kind of operators, changing the points at which the transition occurs [3]. Moreover, due to the heavy dependence between the operators and the network topology, we observed that the explosive synchronization happens on tree-like graphs, while it disappears for scale-free ones. Therefore, there is a transition between explosive synchronization in a branched acyclic system to normal one once cycles emerge in the system. This transition may represent a potential mechanism with which a neuronal system can synchronize explosively individual neurons, and returning to normal synchronization when the neuronal network is formed to avoid pathological states like epilepsy or chronic pain [3].

References

[1] J. Gómez-Gardenes, S. Gómez, A. Arenas, Y. Moreno. Explosive synchronization transitions in scale-free networks. *Physical review letters*, 106(12), 128701. (2011).

[2] M. Miranda, E. Estrada. Degree-biased advection–diffusion on undirected graphs/networks. *Mathematical Modelling of Natural Phenomena*, 17, 30. (2022).

[3] M. Miranda, M. Frasca, E. Estrada. Topologically-induced suppression of explosive synchronization. *arXiv preprint arXiv:2301.08000.* (2023).

Graph Degeneracy and Orthogonal Vector Representations

Lon Mitchell[1]

[1] *Mathematics & Statistics, Eastern Michigan University, USA*
E-mail: lmitch50@emich.edu

Abstract

We apply a technique of Sinkovic and van der Holst for constructing orthogonal vector representations of a graph whose complement is a k-tree to any graph whose complement has degeneracy k.

References

[1] John Sinkovic and Hein van der Holst. The minimum semidefinite rank of the complement of partial k-trees. Linear Algebra Appl., 434, 2011, 1468–1474.

Verified error bounds for all eigenvalues and basis of invariant subspaces of a real symmetric matrix

Shinya Miyajima

Faculty of Science and Engineering, Iwate University, Japan
E-mail: miyajima@iwate-u.ac.jp

Abstract

In this talk, we are concerned with the accuracy of computed eigenvalues and eigenvectors in the eigenvalue problem

$$Ax = \lambda x, \qquad A \in \mathbb{R}^{n\times n}, \quad \lambda \in \mathbb{R}, \quad x \in \mathbb{R}^n \setminus \{0\},$$

where A is symmetric, λ is an eigenvalue, and x is an eigenvector corresponding to λ. Let $k \leq n$, and $P \in \mathbb{R}^{n\times k}$ and $B \in \mathbb{R}^{k\times k}$ satisfy $AP = PB$, where B is not necessarily diagonal. The eigenvalues of B are those of A, and span(P) is called the invariant subspace of A corresponding to these eigenvalues. The eigenvectors of A corresponding to these eigenvalues are included in this subspace.

We consider computing verified error bounds for all numerically obtained (approximate) eigenvalues and eigenvectors, in which all the possible rounding and truncation errors have been taken into account. Algorithms for computing such error bounds have been proposed in [1, 2, 3]. The algorithms in [1, 3] are applicable even when A is non-symmetric, give error bounds for approximate eigenvectors when the corresponding eigenvalues are well-separated, and provide error bounds for approximate basis of invariant subspaces when the eigenvalues are closely clustered. Although the algorithm in [2] is applicable only when A is symmetric, this algorithm involves only four floating-point matrix multiplications and does not involve other procedures requiring cubic complexity. On the other hand, when the eigenvalues are closely clustered, this algorithm does not give error bounds for approximate eigenvectors and/or basis of invariant subspaces.

This talk has two purposes. The first purpose is to present a theory for computing error bounds for approximate basis of invariant subspaces when A is symmetric and the eigenvalues are closely clustered. The second purpose is to propose an algorithm for computing error bounds for all approximate eigenvalues, and approximate eigenvectors or basis of invariant subspaces. We develop this algorithm by combining the algorithm in [2] and the presented theory. Particular emphasis is put on the computational cost of the proposed algorithm. Additional procedures requiring cubic complexity are unnecessary for computing error bounds for approximate basis of invariant subspaces. As a consequence,

the proposed algorithm also involves only four floating-point matrix multiplications and does not involve other procedures requiring cubic complexity. We see from results of numerical experiments that the proposed algorithm was faster than the algorithms in [1, 3].

Acknowledgements: This work was partially supported by JSPS KAKENHI Grant Number JP21K03363.

References

[1] S. Miyajima, Fast enclosure for all eigenvalues and invariant subspaces in generalized eigenvalue problems, *SIAM J. Matrix Anal. Appl.*, 35(3): 1205–1225 (2014).

[2] S. Miyajima, T. Ogita, S. Oishi, Numerical verification for each eigenpair of symmetric matrix, *Trans. JSIAM*, 16(4): 535–552 (2006) (in Japanese).

[3] S. M. Rump, Verified error bounds for all eigenvalues and eigenvectors of a matrix, *SIAM J. Matrix Anal. Appl.*, 43(4): 1736–1754 (2022).

Existence of Characteristic like Vertices on Trees with Matrix Weights

Swetha Ganesh[1], *Sumit Mohanty*[2]

[1] *Department of Computer Science And Automation, IISc Bangalore, Bangalore, Karnataka-560012, India.*
E-mail: swethaganesh@iisc.ac.in
[2] *Humanities and Applied Sciences, IIM Ranchi, Suchana Bhawan, Audrey House Campus, Meur's Road, Ranchi, Jharkhand-834008, India.*
E-mail: sumitmath@gmail.com, sumit.mohanty@iimranchi.ac.in

Abstract

It is known that there is an alternative characterization of characteristic vertices for trees with positive weights on their edges via bottleneck matrices and Perron branches. In this talk, we will consider trees with matrix edge weights and discuss the existence of characteristic-like vertices in terms of bottleneck matrices and Perron branches. Furthermore, we also obtained a relation between characteristic-like vertices and the first non-zero Laplacian eigenvalue.

Acknowledgements: For this work Sumit Mohanty is partially supported by Department of Science and Technology, Government of India, through the project MATRICS (MTR/2017/000458).

On Strongly Infinitely Divisible Matrices

Samir Mondal[1], *K.C. Sivakumar*[2], *M.J. Tsatsomeros*[3]

[1] *Department of Mathematics, Indian Institute of Technology Madras, India*
E-mail: ma19d750@smail.iitm.ac.in
[2] *Department of Mathematics, Indian Institute of Technology Madras, India*
E-mail: kcskumar@iitm.ac.in
[3] *Department of Mathematics and Statistics, Washington State University, USA*
E-mail: tsat@wsu.edu

Abstract

Roots of a nonnegative matrix (if they exist,) may or may not be nonnegative. The matrix exponential $A = e^B$ of an (essentially) nonnegative matrix B is indeed a nonnegative invertible matrix all of whose nonnegative powers $A^t = e^{tB}$ $(t \geq 0)$ are, clearly nonnegative, too. The converse is also true, as shown recently by Van-Brunt [1]: If A is an invertible nonnegative matrix all of whose roots exist and are nonnegative, then there exists a nonnegative matrix B such that $A = e^B$. We refer to such an A as a strongly infinitely divisible matrix ($A \in$ **SIDM**). An inverse M-matrix is a particular example of an **SIDM**. Inverse M-matrices play an important role in this study.

In this talk, we discuss certain operations that leave **SIDM** invariant, examine submatrices of **SIDM**'s, discover an intimate connection of **SIDM**'s and their roots to P-matrices and eventually nonnegative matrices. Further, we will discuss Hadamard/Kronecker products of **SIDM**'s, monotonicity of roots of **SIDM**'s and eigenvalues of **SIDM**'s.

Acknowledgements:

Samir Mondal acknowledges funding received from the Prime Minister's Research Fellowship (PMRF), Ministry of Education, Government of India, for carrying out this work. He also thanks the Office of Global Engagement, IIT Madras, for financial assistance to visit Washington State University through their IIE programme, as well as the Department of Mathematics and Statistics, Washington State University, for partial financial assistance and for the excellent hospitality during his visit.

References

[1] A. Van-Brunt, *Infinitely divisible nonnegative matrices, M-matrices, and the embedding problem for finite state stationary Markov chains,* Linear Algebra Appl., **541** (2018), 163-176.

The combinatory under hyperinvariant subspaces

David Mingueza[1], *M.Eulàlia Montoro*[2], *Alicia Roca*[3]

[1] *Nestle, Barcelona, Spain*
E-mail: David.Mingueza@es.nestle.com
[2] *Departament de Matemàtiques i Informàtica, Universitat de Barcelona, Spain*
E-mail: eula.montoro@ub.edu
[3] *Departament de Matemàtica Aplicada, IMM, Universitat Politécnica de Valencia, Spain*
E-mail: aroca@mat.upv.es

Abstract

Let $f : \mathbb{C}^n \longrightarrow \mathbb{C}^n$ be a linear transformation defined over the complex field and $Hinv(f)$ the lattice of the hyperinvariant subspaces of f (that is, the set of linear transformations commuting with f).
We study the linear transformations whose lattices of hyperinvariant subspaces are isomorphic to $Hinv(f)$. Our work is inspired by [1]. The results are essentially combinatorial.

Acknowledgements: Work supported by the Spanish MICINN research project PID2019-104047GB-I00 and by PID2021-124827NB-I00.

References

[1] Pei Yuan Wu. Which linear transformations have isomorphic hyperinvariant subspace lattices?. *Linear Algebra and its Applications, 169: 163-178, (1992).*

Characterization of a sparse problem with stochastic coefficients to solve elliptic BVPs.

Jorge Morón-Vidal[1], *Francisco Manuel Bernal* [2]

[1] *Dpt. of Mathematics, Carlos III University of Madrid, Spain*
E-mail: jmoron@math.uc3m.es
[2] *Dpt. of Mathematics, Carlos III University of Madrid, Spain*
E-mail: fcoberna@math.uc3m.es

Abstract

PDDSparse is a new implementation of probabilistic domain decomposition aimed at computing the solution of elliptic PDEs with superior scalability. This promising performance is achievable by mixing the Feynman-Kac probabilistic representation of the BVP with a linear interpolator. As a result of this way of proceeding, the solution of the BVP on the interfaces is posed as the solution of a sparse linear system of equations with stochastic coefficients computed via Monte Carlo methods. In this talk we will give an insight into the heuristics that allow us to argue that this linear system is invertible. We will also portray how the closeness of the linear system's matrix structure to an M-matrix provides us with a polynomial preconditioner. Furthermore, resorting to the stochastic representation of the initial problem, a threshold to the matrix condition number and its dependence on the domain discretization will also be estimated. Numerical experiments supporting these investigations will be shown during the presentation.

References

[1] F. Bernal, J. Morón-Vidal and J. A. Acebrón *A hybrid probabilistic domain decomposition algorithm suited for very large-scale elliptic PDEs*. Submitted, 2022. Preprint available in *https://arxiv.org/abs/2301.05780*

Algebraic decoding for convolutional codes over modular integer rings

Ángel Luis Muñoz Castañeda [1], *Noemí de Castro García*[2], *Miguel V. Carriegos*[3]

[1] *Departamento de Matemáticas, Universidad de León, Spain*
E-mail: amunc@unileon.es
[2] *Departamento de Matemáticas, Universidad de León, Spain*
E-mail: ncasg@unileon.es
[3] *Departamento de Matemáticas, Universidad de León, Spain*
E-mail: miguel.carriegos@unileon.es

Abstract

In this talk, we will discuss a procedure to lift Rosenthal's decoding algorithm [3] for convolutional codes, C, over a finite prime field $\mathbb{F}_p$ to convolutional codes, $\mathcal{C}$, over the ring of modular integers $\mathbb{Z}/p^n\mathbb{Z}$. Viewed such convolutional code $\mathcal{C}$ as a linear dynamical system through (one of) its (equivalent) I/S/O representation [2], we are able to generalize the natural approach described by Babu and Zimmermann [1] for linear codes to the convolutional context.

Acknowledgements: Work (partially) supported by Ministerio de Ciencia e Innovación under Grant TED2021-131158A-I00.

References

[1] Babu, N. S. and Zimmermann, K.H., *Decoding of linear codes over Galois rings*. IEEE Transactions on Information Theory, vol. 47, no. 4, pp. 1599-1603, 2001.

[2] Muñoz Castañeda A. L., DeCastro-García N and Carriegos V. M., *On the State Approach Representations of Convolutional Codes over Rings of Modular Integers*. Mathematics, vol. 9, no. 22, pp. 2962–, 2021.

[3] Rosenthal, J. *An Algebraic Decoding Algorithm for Convolutional Codes*. In: Picci, G., Gilliam, D.S. (eds) Dynamical Systems, Control, Coding, Computer Vision. Progress in Systems and Control Theory, vol 25. Birkhäuser Basel, 1999.

A maximal residual two subspace projection algorithm for solving least squares problems

Ashif Mustafa[1], *Manideepa Saha*[2]

[1] *Department of Mathematics, National Institute of Technology Meghalaya, India*
E-mail: ashifmustafa@nitm.ac.in
[2] *Department of Mathematics, National Institute of Technology Meghalaya, India*
E-mail: manideepa.saha@nitm.ac.in

Abstract

Consider the linear system of equations $Ax = b$, where $A \in \mathbb{R}^{m,n}$, $b \in \mathbb{R}^m$ and $x \in \mathbb{R}^n$ is an unknown vector. Randomized Gauss-Seidel (RGS) method, which is also known as the randomized coordinate descent method, was first developed by Leventhal and Lewis [1], in which an improved approximation is obtained by executing an exact line search in the direction of randomly chosen e_j, the jth column of the identity matrix. Leventhal and Lewis showed that RGS method converges linearly in expectation to the least squres solution for overdetermined systems (both consistent and inconsistent). If a linear system is an underdetermined system ($m < n$), then one is often interested in finding the solution with least norm, which is given by $A^\dagger b$. In [2], Ma et al. proved that RGS may not converge to $A^\dagger b$ for an underdetermined system. They proposed an extension of RGS, known as the randomized extended Gauss-Seidel (REGS) method, which converges for both underdetermined and overdetermined (consistent/inconsistent) systems. Apart from the randomized selection rule, many greedy selection rules also exist to choose the search direction, some deterministic and some randomized, for the coordinate descent method [3, 4].

In this particular work, we present the coordinate descent method in the Petrov-Galerkin framework. The selection of the search subspace, which is of dimension 2, is done in a greedy manner. We name the proposed method as the 2-D maximal residual Gauss-Seidel (D2MRGS) method. Convergence is analysed for the stated method and numerical experiments are provided to demonstrate its efficiency. As compared to existing methods, the D2MRGS has two clear advantages additional to the observations that it is faster than the randomized versions. Firstly, in case of block methods one has to deal with pavings, which is not required in our method. Other than that, all calculations necessary for the execution of our method can be easily achieved algebraically, which is not usually the case with block versions using higher dimensions. Also, the convergence bound provided in our analysis is scalable in the sense that if higher dimensional subspaces are chosen, the bound gets as many times smaller. However, executions in such cases may become cumbersome.

For underdetermined systems, we perform an extension in the lines of the REGS. But in our case we perform it in a greedy manner similar to the D2MRGS. Again, we prove that our method converges to $A^{\dagger}b$ and put forward numerical examples showing the same.

References

[1] D. Leventhal, A. S. Lewis. Randomized methods for linear constraints: convergence rates and conditioning. *Math. Oper. Res.*, 35: 641-654 (2010).

[2] A. Ma, D. Needell, A. Ramdas. Convergence properties of the randomized extended Gauss-Seidel and Kaczmarz methods. *SIAM J. Matrix Anal. Appl.*, 36: 1590-1604 (2015).

[3] Z-Z. Bai, W-T. Wu. On greedy randomized coordinate descent methods for solving large linear least-squares problems. *Numer. Linear. Algebra. Appl.*, 26: e2237 (2019).

[4] Y-Q. Niu, B. Zheng. A new randomized GaussSeidel method for solving linear least-squares problems. *Appl. Math. Lett.*, 116: 107057 (2021).

Recent Progress in GMRES-Based Iterative Refinement for Weighted and Generalized Least-Squares Problems

Eda Oktay[1], *Erin Carson*[2]

[1] *Department of Numerical Mathematics, Charles University, Czech Republic*
Department of Mathematics, TU Chemnitz, Germany
E-mail: oktay@karlin.mff.cuni.cz
[2] *Department of Numerical Mathematics, Charles University, Czech Republic*
E-mail: carson@karlin.mff.cuni.cz

Abstract

With the recent emergence of mixed precision hardware, there has been a renewed interest in its use for solving numerical linear algebra problems fast and accurately. The solution of least squares (LS) problems $\min_x \|b - Ax\|_2$, where $A \in \mathbb{R}^{m\times n}$, arise in numerous application areas. Overdetermined standard least squares problems can be solved by using mixed precision within the iterative refinement method of Björck, which transforms the least squares problem into an $(m+n)\times(m+n)$ "augmented" system. It has recently been shown that mixed precision GMRES-based iterative refinement can also be used, in an approach termed GMRES-LSIR. In practice, we often encounter types of least squares problems beyond standard least squares, including weighted and generalized least squares, $\min_x \|D^{1/2}(b - Ax)\|_2$, where $D^{1/2}$ is a (diagonal) matrix of weights. In this talk, we discuss a mixed precision GMRES-LSIR algorithm for solving these problems.

Acknowledgements: Work supported by the Charles University Research Program No. UNCE/SCI/023, GAUK project No. 202722, and the Exascale Computing Project (17-SC-20-SC), a collaborative effort of the U.S. Department of Energy Office of Science and the National Nuclear Security Administration.

Partial Smoothness of the Numerical Radius at Matrices whose Fields of Values are Disks

Adrian S. Lewis[1], *Michael L. Overton*[2]

[1] *ORIE, Cornell University, U.S.A.*
E-mail: adrian.lewis@cornell.edu
[2] *Courant Institute of Mathematical Sciences, New York University, U.S.A.*
E-mail: mo1@nyu.edu

Abstract

Solutions to optimization problems involving the numerical radius often belong to the class of "disk matrices": those whose field of values is a circular disk in the complex plane centered at zero. We investigate this phenomenon using the variational-analytic idea of partial smoothness. We give conditions under which the set of disk matrices is locally a manifold $\mathcal{M}$, with respect to which the numerical radius r is partly smooth, implying that r is smooth when restricted to $\mathcal{M}$ but strictly nonsmooth when restricted to lines transversal to $\mathcal{M}$. Consequently, minimizers of the numerical radius of a parametrized matrix often lie in $\mathcal{M}$. Partial smoothness holds, in particular, at $n \times n$ matrices with exactly $n-1$ nonzeros, all on the superdiagonal. On the other hand, in the real 18-dimensional vector space of complex 3×3 matrices, the disk matrices comprise the closure of a semi-algebraic manifold $\mathcal{L}$ with dimension 12, and the numerical radius is partly smooth with respect to $\mathcal{L}$.

Factorization of completely positive matrices by alternating minimization

Roger Behling[◇], *Hugo Lara*[‡], *Harry Oviedo*[†]

[◇] *Escola de Matemática Aplicada, Fundação Getulio Vargas (FGV/EMAp), Rio de Janeiro, Brazil, and Universidade Federal de Santa Catarina, Blumenau, Brazil.*
E-mail: rogerbehling@gmail.com
[‡] *Universidade Federal de Santa Catarina, Blumenau, Brazil.*
E-mail: hugo.lara.urdaneta@ufsc.br
[†] *Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Santiago de Chile, Chile.*
E-mail: harry.oviedo@uai.cl

Abstract

In this work, we consider the completely positive matrix factorization problem where, given a symmetric matrix, one tries to find a Cholesky-type factorization with an entrywise nonnegative factor. To deal with this problem, we propose a proximal alternating minimization procedure. In each iteration, our method splits the original factorization problem into two optimization subproblems, the first one being a orthogonal procrustes problem, which is taken over the orthogoal group, and the second one over the set of entrywise positive matrices. We present both a convergence analysis of the method and favorable numerical results

Acknowledgements: H arry Oviedo was financially supported by the Faculty of Engineering and Sciences, Universidad Adolfo Ibáñez, through the FES startup package for scientific research. Roger Behling was supported by Fundação de Amparo á Pesquisa do Estado do Rio de Janeiro (FAPERJ), Grant E-26/201.345/2021

References

[1] Bomze I. M., Dickinson P. J. and Still G. *The structure of completely positive matrices according to their CP-rank and CP-plus-rank*. Linear algebra and its applications, vol. 482, p. 191-206. (2015)

[2] Boţ R. I. and Nguyen, D. K. *Factorization of completely positive matrices using iterative projected gradient steps*. Numerical Linear Algebra with Applications, vol. 28, no 6, p. e2391. (2021)

[3] Groetzner P., and Dür M. *A factorization method for completely positive matrices*. Linear Algebra and its Applications, vol. 591, p. 1-24. (2020)

On some extensions of the class of Q-matrices

K. C. Sivakumar[1], *Sushmitha P*[2], *Michael Tsatsomeros*[3]

[1] *Department of Mathematics, Indian Institute of Technology Madras, Chennai, India*
E-mail: kcskumar@iitm.ac.in
[2] *Mathematics Division, School of Advanced Sciences, Vellore Institute of Technology, Chennai, India*
E-mail: sushitulasi@gmail.com
[3] *Department of Mathematics and Statistics, Washington State University, USA*
E-mail: tsat@wsu.edu

Abstract

A real square matrix A is called a Q-matrix if LCP(A, q) has a solution for all $q \in \mathbb{R}^n$, i.e., for every vector q, there exists an $x \in \mathbb{R}^n$ such that $x \geq 0$, $Ax+q \geq 0$ and $x^T(Ax+q) = 0$. A well known result states that a Q-matrix with nonpositive off-diagonal entries is inverse nonnegative. In this talk, we shall look at properties of two classes of matrices that extend the inverse nonnegativity of the Q-matrices to the generalized inverse of a matrix. We shall also look at a new result for the class of Q-matrices.

References

[1] K. C. Sivakumar, *Nonnegative generalized inverses and certain subclasses of singular Q-matrices*, Linear Algebra Appl., **438** (2013) 4701-4708.

Quantitative version of Korovkin type theorems and its application to preconditioners

P.C. Vinaya[1]

[1] *Department of Mathematics, Cochin University of Science and Technology, India*
E-mail: vinayapc01@gmail.com

Abstract

The classical Korovkin theorem, due to P.P. Korovkin has many generalizations and analogues to different settings and applications to various branches of science (See [3]). A quantitative form of the Korovkin's theorem obtained by O. Shisha and B. Mond in 1968 [5] gives the rate of convergence of the approximation process utilizing the modulus of continuity. Recently, Yusuf Zeren, Migdad Ismailov and Cemil Karacam obtained a Korovkin-type theorem in the setting of Banach function spaces in [6]. An operator version of Korovkin's theorem is obtained by Dumitru Popa [4]. We proved quantitative versions of these results and applied our theorem to various examples in [2]. An important application of this result to the preconditioners of Toeplitz linear systems is also given there. In this talk, I plan to discuss these recent developments and other related problems concerning the convergence of preconditioned linear systems.

Acknowledgements: P.C. Vinaya is thankful to the University Grants Commission (UGC), India for financial support.

References

[1] Stefano Serra Capizzano, A Korovkin-type theory for finite Toeplitz operators via matrix algebras, *Numer. Math.* Volume **82,** issue 1, (1999), 117–142, DOI : 10.1007/s002110050413.

[2] V.B. Kiran Kumar, P.C. Vinaya, Operator version of Korovkin theorem; Degree of convergence and application to Preconditioners, J. Math. Anal. Appl. (2023), 127087

[3] P.P. Korovkin, Linear operators and approximation theory, *Gordon and Breach Publishers, Inc., New York; Hindustan Publishing Corp. (India), Delhi*, (1960).

[4] Dumitru Popa, An operator version of the Korovkin theorem, *J. Math. Anal. Appl.* Volume **515,**, (2022), DOI : 10.1016/j.jmaa.2022.126375.

[5] O. Shisha, B. Mond, The degree of convergence of sequences of linear positive operators, *Proc. Nat. Acad. Sci. U.S.A.* Volume **60,**, (1968), 1196–1200, DOI : 10.1073/pnas.60.4.1196.

[6] Yusuf Zeren, Migdad Ismailov and Cemil Karacam, *The analogs of the Korovkin theorems in banach function spaces*, Positivity , **26**(2022), n0.2, Paper No. 28

Learning Co-embedding for Multi-type Data based on Integrated Symmetric Nonnegative Matrix Factorization

Dongjin Choi[1], *Barry L. Drake*[2], *Haesun Park*[1]

[1] *Computational Science and Engineering, Georgia Institute of Technology, USA*
E-mail: jin.choi@gatech.edu, hpark@cc.gatech.edu
[2] *Georgia Tech Research Institute, USA*
E-mail: barry.drake@gtri.gatech.edu

Abstract

Embedding-based techniques for data representation learning have become increasingly important and have been applied to various data types such as text, knowledge graphs, and images. The embedding methods learn semantic information from the data and project it onto a low-dimensional vector space while preserving relational information. In many cases, even from a single data source, multiple types of data objects are present. Furthermore, with the recent advancements in supervised machine learning, human-annotated labels are often available for clustering and classification of these data. In this context, learning embeddings from multi-type data, where different types coexist, has been a difficult problem to solve. Latent Semantic Indexing (LSI) [5] was an early attempt to represent objects in vectors of the same length. Various recent methods have also been proposed to address this problem in specific contexts, such as Doc2vec [1] and PTE [2] for textual data and metapath2vec [3] for heterogeneous networks. However, they have limitations when it comes to learning semantic proximity between objects with different types because they fail to ensure that the learned spaces are in a common basis. This paper introduces a new method for learning co-embeddings from multi-type data using integrated symmetric nonnegative matrix factorization. The method integrates multi-type relational data into a symmetric collective data matrix and uses the SymNMF [4] formulation to represent co-embeddings of the data. Additionally, the proposed method incorporates semi-supervision with a constraint that imposes partial label information, in a similar manner as proposed in MEGA [9], thereby fully utilizing the available label information for clustering.

With the given collective symmetric nonnegative data matrix, the proposed method in this research decomposes the SymNMF formulation of a given symmetric nonnegative data matrix into a JointNMF [8] formulation, solving multiple symmetric and standard NMF problems with shared factor matrices simultaneously. It utilizes a block coordinate descent algorithm [6, 7] to solve nonnegative least squares problems and is evaluated on several benchmark datasets. The results demonstrate that the proposed method outperforms existing embedding methods for clustering and classification.

References

[1] Le, Q. Mikolov, T. Distributed representations of sentences and documents.

[2] Tang, J., Qu, M. Mei, Q. Pte: Predictive text embedding through large-scale heterogeneous text networks.

[3] Dong, Y., Chawla, N. Swami, A. metapath2vec: Scalable representation learning for heterogeneous networks.

[4] Kuang, D., Yun, S. Park, H. SymNMF: nonnegative low-rank approximation of a similarity matrix for graph clustering. *Journal Of Global Optimization.* **62**, 545–574 (2015)

[5] Deerwester, S., Dumais, S., Furnas, G., Auer, T. Harshman, R. Indexing by latent semantic analysis. *Journal Of The American Society For Information Science.* **41**, 391–407 (1990)

[6] Kim, J., He, Y. Park, H. Algorithms for nonnegative matrix and tensor factorizations: A unified view based on block coordinate descent framework. *Journal Of Global Optimization.* **58**, 285–319 (2014)

[7] Kim, J. Park, H. Fast nonnegative matrix factorization: An active-set-like method and comparisons. *Siam Journal On Scientific Computing.* **33**, 3261–3281 (2011)

[8] Du, R., Drake, B. Park, H. Hybrid clustering based on content and connection structure using joint nonnegative matrix factorization. *Journal Of Global Optimization.* **74**, 861–877 (2019)

[9] Whang, J., Du, R., Jung, S., Lee, G., Drake, B., Liu, Q., Kang, S. Park, H. MEGA: Multi-view semi-supervised clustering of hypergraphs. *Proceedings Of The Vldb Endowment.* **13**, 698–711 (2020)

Fast solution of incompressible flow problems with two-level pressure approximation

Jennifer Pestana[1], *David J. Silvester*[2]

[1] *Department of Mathematics and Statistics, University of Strathclyde, UK*
E-mail: jennifer.pestana@strath.ac.uk
[2] *Department of Mathematics, The University of Manchester, UK*
E-mail: d.silvester@manchester.ac.uk

Abstract

The inf-sup stable Taylor–Hood finite element is widely used when solving Stokes and Navier–Stokes problems. Typically mass is conserved only in a weak sense for Taylor–Hood elements, which can impact the accuracy of solutions. One approach to improving mass conservation properties is to simply augment the Taylor–Hood pressure approximation with piecewise constant functions [1, 2]. The resulting pressure approximation is locally conservative, but the richer pressure approximation space has consequences for the solution of the resulting linear system(s), e.g., a singular coefficient matrix and pressure mass matrix. This talk will describe these challenges, and will discuss ways of recovering efficient iterative solvers for Stokes and Navier–Stokes discretisations.

Acknowledgements: This work was supported by EPSRC grant EP/W033801/1.

References

[1] P. Gresho, R. Lee, S. Chan, and J. Leone, *A new finite element for incompressible or Boussinesq fluids*. In *Third International Conference on Finite Elements in Flow Problems*, pages 204–215. Wiley, New York, 1981.

[2] D. Griffiths, *The effect of pressure approximation on finite element calculations of incompressible flows*. In K.W. Morton and M.J. Baines, editors, *Numerical Methods for Fluid Dynamics*, pages 359–374. Academic Press, San Diego, 1982.

Finite time horizon mixed control of vibrational systems

I. Nakić[1], *M. Pilj Vidaković*[2], *Z. Tomljanović*[3]

[1] *Department of Mathematics, University of Zagreb, Zagreb, Croatia*
E-mail: ivica.nakic@math.hr
[2] *J. J. Strossmayer University of Osijek Department of Mathematics, Osijek, Croatia*
E-mail: mpilj@mathos.hr
[3] *J. J. Strossmayer University of Osijek Department of Mathematics, Osijek, Croatia*
E-mail: ztomljan@mathos.hr

Abstract

We consider a vibrational system control problem over a finite time horizon. The performance measure of the system is taken to be p-mixed H_2 norm which generalizes the standard H_2 norm. We present an algorithm for efficient calculation of this norm in the case when the system is parameter dependent and the number of inputs or outputs of the system is significantly smaller than the order of the system. Our approach is based on a novel procedure which is not based on solving Lyapunov equations and which takes into account the structure of the system. We use a characterization of the H_2 norm given in terms of integrals which we solve using adaptive quadrature rules. This enables us to use recycling strategies as well as parallelization. The efficiency of the new algorithm allows for an analysis of the influence of various system parameters and different finite time horizons on the value of the p-mixed H_2 norm. We illustrate our approach by numerical examples concerning an n-mass oscillator with one damper. Additionally, we show preliminary new results for the case with multiple dampers.

Acknowledgements: Work supported by Croatian Science Foundation, scientific project Vibration Reduction in Mechanical Systems (VIMS, IP-2019-04-6774).

Optimizing the Rayleigh quotient with symmetric constraints and its application to perturbations of structured polynomial eigenvalue problems

Anshul Prajapati[1], *Punit Sharma*[2]

[1] *Department of Mathematics, Indian Institute of Technology Delhi, India*
E-mail: anshul.prajapati@maths.iitd.ac.in
[2] *Department of Mathematics, Indian Institute of Technology Delhi, India*
E-mail: punit.sharma@maths.iitd.ac.in

Abstract

Let $H \in \mathbb{C}^{n,n}$ be Hermitian and $S_0, S_1, \ldots, S_k \in \mathbb{C}^{n,n}$ be symmetric matrices. We consider the problem of maximizing the Rayleigh quotient of H with respect to certain constraints involving symmetric matrices $S_0, S_1, \ldots, S_k$. More precisely, we compute

$$m_{hs_0s_1\ldots s_k}(H, S_0, S_1, \ldots, S_k) := \sup \left\{ \frac{v^*Hv}{v^*v} : \ v \in \mathbb{C}^n \setminus \{0\},\ v^T S_i v = 0 \right. \\ \left. \text{for } i = 0, \ldots, k \right\}, \quad (\mathcal{P})$$

where T and $*$ denote respectively the transpose and the conjugate transpose of a matrix or a vector.

Such problems occur in stability analysis of uncertain systems and in the eigenvalue perturbation theory of matrices and matrix polynomials [1, 2]. The μ-value of $M \in \mathbb{C}^{n,n}$ with respect to perturbations from the structured class $\mathcal{S} \subseteq \mathbb{C}^{n,n}$ is denoted by $\mu_{\mathcal{S}}(M)$ and defined as

$$\mu_{\mathcal{S}}(M) := \left(\inf \{\|\Delta\| : \ \Delta \in \mathcal{S},\ \det(I_n - \Delta M) = 0\}\right)^{-1},$$

where I_n is the identity matrix of size $n \times n$ and $\|\cdot\|$ is the matrix spectral norm. A particular case of problem $(\mathcal{P})$ with only one symmetric constraint (i.e., when $k = 0$) is used to characterize the μ-value of the matrix under skew-symmetric perturbations [2]. Indeed, when $\mathcal{S}$ is the set of all skew-symmetric matrices of size $n \times n$, M. Karow in [2] showed that

$$\mu_{\mathcal{S}}(M) = (m_{hs_0}(H, S_0))^{\frac{1}{2}}, \text{ where } H = M^*M \text{ and } S_0 = M + M^T.$$

Moreover, an explicit computable formula was obtained for $m_{hs_0}(H, S_0)$ in [2, Theorem 6.3] and given by

$$m_{hs_0}(H, S_0) = \inf_{t\in[0,\infty)} \lambda_2 \left(\begin{bmatrix} H & t\overline{S}_0 \\ tS_0 & \overline{H} \end{bmatrix} \right), \tag{1}$$

where $\lambda_2(A)$ stands for the second largest eigenvalue of a Hermitian matrix A. However, the solution to the problem $(\mathcal{P})$ with more than one symmetric constraint is not known.

We derive an upper bound for $(\mathcal{P})$ in terms of minimizing the second largest eigenvalue of a parameter-depending Hermitian matrix. The upper bound is shown to be equal to the exact value of the supremum in $(\mathcal{P})$ if the eigenvalue at the optimal is simple. The results are then applied to derive computable formulas for the structured eigenvalue backward errors of matrix polynomials under consideration. Numerical experiments suggests that our results [5] give better estimation to the supremum in $(\mathcal{P})$ than the one obtained in [4]. This paper [5] is published in Linear Algebra and its Applications.

Acknowledgements: A.P. acknowledges the support of the CSIR Ph.D. grant by Ministry of Science & Technology, Government of India. P.S. acknowledges the support of the DST-Inspire Faculty Award (MI01807-G) by Government of India.

References

[1] J. Doyle. Analysis of feedback systems with structured uncertainties. *IEE Proc. Part D, Control Theory Appl.*, 129: 242–250 (1982).

[2] M. Karow. μ-values and spectral value sets for linear perturbation classes defined by a scalar product. *SIAM J. Matrix Anal. Appl.*, 32: 845–865 (2011).

[3] S. Bora, M. Karow, C. Mehl, P. Sharma. Structured eigenvalue backward errors of matrix pencils and polynomials with palindromic structures. *SIAM J. Matrix Anal. Appl.*, 36: 393–416 (2015).

[4] P. Sharma. Eigenvalue Backward errors of polynomial eigenvalue problems under structure preserving perturbations. *PhD thesis, Department of Mathematics, Indian Institute of Technology Guwahati, India*, (2016).

[5] A. Prajapati, P. Sharma. Optimizing the Rayleigh quotient with symmetric constraints and its application to perturbations of structured polynomial eigenvalue problems. *Linear Algebra Appl.*, 645: 256–277 (2022).

An envelope for the spectrum of a square matrix

Aik. Aretaki[1], *P. Psarrakos*[2], *M. Tsatsomeros*[3]

[1] *Department of Mathematics, University of Thessaly, Greece*
E-mail: kathy@uth.gr
[2] *Department of Mathematics, National Technical University of Athens, Greece*
E-mail: ppsarr@math.ntua.gr
[3] *Department of Mathematics, Washington State University, Pullman, USA*
E-mail: tsat@wsu.edu

Abstract

We introduce and study the envelope of a given square complex matrix, that is, an envelope-type region in the complex plane that contains the eigenvalues of the matrix. This set is the intersection of an infinite number of regions defined by cubic curves. The method of its construction extends the notion of the numerical range, which is known to be an intersection of an infinite number of half-planes. As a consequence, the envelope is contained in the numerical range and represents an improvement in localizing the spectrum of the matrix. The envelope is compact but not necessarily convex or connected, and its connected components have the potential of isolating the eigenvalues of the matrix. We study its geometry, boundary, and number of components, and also examine the envelope of normal matrices and similarities. Moreover, we obtain symmetries of the envelope of a tridiagonal Toeplitz matrix, and explicity characterize envelopes of block-shift matrices, Jordan blocks and two-by-two matrices.

References

[1] M. Adam and M. Tsatsomeros. An eigenvalue inequality and spectrum localization for complex matrices. *Electronic Journal of Linear Algebra*, 15: 239–250 (2006).

[2] Aik. Aretaki, P. Psarrakos and M. Tsatsomeros. The envelope of tridiagonal Toeplitz matrices and block-shift matrices. *Linear Algebra and its Applications*, 532: 60–85 (2017).

[3] P. Psarrakos and M. Tsatsomeros. On the geometry of the envelope of a matrix. *Applied Mathematics and Computation*, 244: 132–141 (2014).

[4] P. Psarrakos and Michael Tsatsomeros. An envelope for the spectrum of a matrix. *Central European Journal of Mathematics*, 10: 292–302 (2012).

The inverse Horn problem

João Filipe Queiró

Department of Mathematics, University of Coimbra, Portugal
E-mail: jfqueiro@mat.uc.pt

Abstract

Alfred Horn's conjecture on eigenvalues of sums of Hermitian matrices was proved more than 20 years ago. In this talk we raise the problem of, given an n-tuple γ in the solution polytope, constructing Hermitian matrices with the required spectra such that their sum has eigenvalues γ.

On Completely Mixed Matrix Games

T. Parthasarathy [1], *G. Ravindran* [2], *Sunil Kumar* [3]

[1] *Chennai Mathematical Institute, India*
E-mail: pacha14@yahoo.com
[2] *SQC & OR, Indian Statistical Institute, India*
E-mail: gravi@hotmail.com
[3] *SQC & OR, Indian Statistical Institute, India*
E-mail: sunny061193@gmail.com

Abstract

A matrix game is known to be a completely mixed game, if all the optimal pairs of strategies of the game are completely mixed. In this paper, we prove that a matrix game A with value zero is completely mixed if and only if the value of the game associated with $A + D_i$ is positive for all i, where D_i is a diagonal matrix whose i^{th} diagonal entry is 1 and else 0. Our result provides a new characterization extending the result of I. Kaplansky (1945). Further, we also provide a few characterizations for a game associated with an odd-ordered skew-symmetric matrix to be completely mixed. We also show that if A is an almost skew-symmetric matrix and the game associated with A has the value positive, then $A+D_i \in Q$ for all i, where D_i is a diagonal matrix whose i^{th} diagonal entry is 1 and else 0. Skew-symmetric matrices and almost skew-symmetric matrices with value positive are P_0 and Q_0 and hence these are processable by Lemke's algorithm.

References

[1] Cottle, R.W., Pang, J.S. and Stone, R.E.: The Linear Complementarity Problem, Academic Press, New York (1992)

[2] Kaplansky, I.: A contribution to von Neumanns theory of games, Ann. Math. 46, 474-479 (1945)

[3] Kaplansky, I.: A contribution to von Neumanns theory of games II, Linear Algebra and its Applications, Elsevier Science Inc. 226-228, 371-373 (1995)

[4] Parthasarathy, T., Ravindran, G. and Sunil Kumar, On Semimonotone Matrices, R_0-Matrices and Q-Matrices, J. of Opt. Theory and App., 195, 131-147, (2022).

On a question of Bhatia, Friedland and Jain

Mandeep Singh

Department of Mathematics, Sant Longowal Institute of Engineering and Technology, Longowal-148106, Punjab, India
E-mail: msrawla@yahoo.com

Abstract

Let $p_1 < p_2 < \cdots < p_n$ be positive numbers, then for any integer m the Loewner matrix associated with the function x^m is given by $L_m = \left[\frac{p_i^m - p_j^m}{p_i - p_j}\right]_{i,j=1}^{n}$. A question was left open in a paper [1] by Bhatia, Friedland and Jain to find formulas for the determinants of the matrices L_m in the case $n = 3$. We aim to answer this question firmly in terms of the Schur polynomials in this paper.

References

[1] R. Bhatia, S. Friedland, T. Jain, *Inertia of Loewner matrices*, Indiana Univ. Math. J. 65 (2016), 1251-1261.

Laplacian spectra of cographs: A twin reduction perspective

Sasmita Barik[1], *Sane Umesh Reddy*[2]

[1] *School of Basic Sciences, IIT Bhubaneswar, Bhubaneswar, India*
E-mail: sasmita@iitbbs.ac.in
[2] *School of Basic Sciences, IIT Bhubaneswar, Bhubaneswar, India*
E-mail: sur11@iitbbs.ac.in

Abstract

A graph is a cograph if and only if it has no induced path on 4 vertices. The twin reduction graph of a cograph G is denoted by R_G. We describe the Laplacian eigenvalues and eigenvectors of a cograph G using R_G and characterize the cographs with even and odd integer eigenvalues, respectively. Further, we provide a complete description of the Laplacian spectrum of H-join of graphs when H is a cograph and obtain bounds for the algebraic connectivity of such graphs.

Acknowledgements: The second author acknowledges the financial support from UGC, Government of India.

References

[1] S. Barik, S. Fallat and S. Kirkland. On Hadamard diagonalizable graphs. *Linear Algebra and its Applications*, 435(8): 1885–1902 (2011).

[2] D. M. Cardoso, M. A. A. Freitas, E. A. Martins and M. Robbiano. Spectra of graphs obtained by a generalization of the join graph operation. *Discrete Mathematics*, 313: 733–741 (2013).

[3] D. G. Corneil, H. Lerchs and L. S. Burlingham. Complement reducible graphs. *Discrete Applied Mathematics*, 3: 163–174 (1981).

[4] J. Lazzarin, O. F. Sosa and F. C. Tura. Laplacian eigenvalues of equivalent cographs. *Linear and Multilinear Algebra*, (2022). https://doi.org/10.1080/03081087.2022.2050168

[5] N. V. R. Mahadev and U. N. Peled. *Threshold Graphs and Related Topics*. Annals of Discrete Mathematics, 56, 1995.

[6] R. Merris. Laplacian graph eigenvectors. *Linear Algebra and its Applications*, 278: 221–236 (1998).

[7] S. S. Mousavi, M. Haeri and M. Mesbahi. Laplacian Dynamics on Cographs: Controllability Analysis Through Joins and Unions. *IEEE Transactions on Automatic Contol*, 66(3): 1383–1390 (2021).

[8] M. Neumann and S. Pati. The Laplacian spectra of graphs with a tree structure. *Linear and Multilinear Algebra*, 57(3): 267–291 (2009).

[9] R. E. A. C. Paley. On Orthogonal Matrices. *Journal of Mathematics and Physics*, 4: 311–320 (1933).

An Algorithm to Compute a Minimal Input-State-Output Representation of a Convolutional Code

Joan-Josep Climent, Diego Napp, Verónica Requena

Departament de Matemàtiques, Universitat d'Alacant,
Campus de Sant Vicent del Raspeig, E-03690 Sant Vicent del Raspeig, Alacant, Spain
e-mails: jcliment@ua.es, diego.napp@ua.es, vrequena@ua.es

There is a close relation between convolutional codes and linear systems. This connection between both has already been established and analysed widely by several authors [4, 5, 6, 7]. From a systems theoretic point of view, a convolutional code is a submodule $\mathcal{C}$ of $\mathbb{F}[z]^n$ (see [8]), where $\mathbb{F}[z]$ is the ring of polynomials in the variable z and coefficients in a finite field $\mathbb{F}$. Since $\mathbb{F}[z]$ is a principle ideal domain, a convolutional code $\mathcal{C}$ has always a well-defined rank k, and there exists $G(z) \in \mathbb{F}[z]^{n\times k}$, of rank k, such that (see [10])

$$\mathcal{C} = \mathrm{im}_{\mathbb{F}[z]}\left(G(z)\right) = \left\{\boldsymbol{v}(z) \in \mathbb{F}[z]^n \mid \boldsymbol{v}(z) = G(z)\boldsymbol{u}(z) \text{ with } \boldsymbol{u}(z) \in \mathbb{F}[z]^k\right\}$$

where $\boldsymbol{u}(z)$ is the **information vector**, $\boldsymbol{v}(z)$ is the corresponding **codeword** and $G(z)$ is the **generator** of $\mathcal{C}$. If $G(z) \in \mathbb{F}[z]^{n\times k}$ is a generator matrix of $\mathcal{C}$ and $U(z) \in \mathbb{F}[z]^{k\times k}$ is unimodular, then $G(z)U(z)$ is also a generator matrix of $\mathcal{C}$. Therefore, the maximum degree of the $k \times k$ minors of all generator matrices of $\mathcal{C}$ is the same, and it is called the degree of $\mathcal{C}$.

On the one hand, a rate k/n convolutional code $\mathcal{C}$ of degree δ can be described by a time invariant linear system (see [9])

$$\left.\begin{aligned}\boldsymbol{x}_{t+1} &= A\boldsymbol{x}_t + B\boldsymbol{u}_t\\ \boldsymbol{y}_t &= C\boldsymbol{x}_t + D\boldsymbol{u}_t\end{aligned}\right\}, \quad \boldsymbol{v}_t = \begin{bmatrix}\boldsymbol{y}_t\\ \boldsymbol{u}_t\end{bmatrix}, \quad t = 0, 1, 2, \ldots, \quad \boldsymbol{x}_0 = \boldsymbol{0}, \tag{1}$$

where $A \in \mathbb{F}^{m\times m}$, $B \in \mathbb{F}^{m\times k}$, $C \in \mathbb{F}^{(n-k)\times m}$ and $D \in \mathbb{F}^{(n-k)\times k}$. We say that (A, B, C, D) is an **input-state-output (ISO) representation** of $G(z)$ of dimension m. Moreover, it is called **minimal** when $m = \delta$.

On the other hand, a rate k/n convolutional code $\mathcal{C}$ of degree δ can be described from the first-order representation (see [5])

$$\mathcal{C} = \left\{\boldsymbol{v}(z) \in \mathbb{F}[z]^n \mid \exists\, \boldsymbol{x}(z) \in \mathbb{F}[z]^\delta : zK\boldsymbol{x}(z) + L\boldsymbol{x}(z) + M\boldsymbol{v}(z) = 0\right\},$$

where $K, L \in \mathbb{F}^{(\delta+n-k)\times\delta}$ and $M \in \mathbb{F}^{(\delta+n-k)\times n}$. The triple (K, L, M) is known as a **first-order representation** of $\mathcal{C}$.

As far as we are aware, obtaining a minimal ISO representation of a convolutional code implies the use of a first-order representation in a long process. In this paper, we present an algorithm to construct a minimal ISO representation

for a convolutional code, using its generator matrix directly without using a first-order representation.

Moreover, the combination of codes can yield a new code with better properties; such combinations have been widely used in coding theory in different forms [1, 2, 3]. We shall focus on the so-called product codes. More concretely, using the algorithm given in this paper, we provide an ISO representation of the product convolutional code based on an ISO representation of each one of the convolutional codes involved in the product.

Acknowledgements: This work was partially supported by the Spanish I+D+i project PID2019-108668GB-I00 of MCIN/AEI/10.13039/501100011033 and the I+D+i project VIGROB-287 of the Universitat d'Alacant.

References

[1] S.D. Cardell, J.-J. Climent. An approach to the performance of SPC product codes under the erasure channel. *Advances in Mathematics of Communications*, 10:11–28 (2016). DOI:10.3934/amc.2016.10.11.

[2] J.-J. Climent, D. Napp, R. Pinto, R. Simões. Series concatenation of 2D convolutional codes by means of input-state-output representations. *International Journal of Control*, 91: 2682–2691 (2018). DOI: 10.1080/00207179.2017.1410573.

[3] N. DeCastro-García, M. García-Planas. Concatenated linear systems over rings and their application to construction of concatenated families of convolutional codes. *Linear Algebra and its Applications*, 542: 624–647 (2018). DOI: 10.1016/j.laa.2017.12.009.

[4] Jr G.D Forney. Convolutional codes I: algebraic structure. *IEEE Transactions on Information Theory*, 16(6): 720–738 (1970). DOI:10.1109/TIT.1970.1054541.

[5] M. Kuijper. *First-order Representations of Linear Systems*. Birkhäuser, Boston, MA (1994).

[6] J.L. Massey, M.K. Sain. Codes, automata, and continuous systems: explicit interconnections. *Transactions on Automatic Control*, 12(6): 644–650 (1994). DOI: 10.1109/TAC.1967.1098736.

[7] J. Rosenthal. Connections between linear systems and convolutional codes. In B. Marcus and J. Rosenthal (eds.), *C*odes, Systems and Graphical Models, volume 123 of *The IMA Volumes in Mathematics and its Applications*, 39–66 (2001). Springer-Verlag, New York. DOI:10.1007/978 − 1 − 4613 − 0165 − 3_2.

[8] J. Rosenthal, J.M. Schumacher, E.V. York. On behaviors and convolutional codes. *IEEE Transactions on Information Theory*, 42(6): 1881–1891 (1996). DOI:10.1109/18.556682.

[9] J. Rosenthal, E.V. York. BCH convolutional codes. *IEEE Transactions on Information Theory*, 45(6): 1833–1844 (1999). DOI:10.1109/18.782104.

[10] E.V. York. *Algebraic Description and Construction of Error Correcting Codes: A Linear Systems Point of View.* PhD thesis, Department of Mathematics, University of Notre Dame, Indiana, USA, 1997.

Computation of the von Neumann entropy of large matrices via trace estimators and rational Krylov methods

Michele Benzi[1], *Michele Rinelli*[2], *Igor Simunec*[3]

[1] *Scuola Normale Superiore, Pisa*
E-mail: michele.benzi@sns.it
[2] *Scuola Normale Superiore, Pisa*
E-mail: michele.rinelli@sns.it
[3] *Scuola Normale Superiore, Pisa*
E-mail: igor.simunec@sns.it

Abstract

We consider the problem of approximating the von Neumann entropy of a large, sparse, symmetric positive semidefinite matrix A, defined as the trace of $-A \log A$. After establishing some useful properties of this matrix function, we consider the use of both polynomial and rational Krylov subspace algorithms within two types of approximations methods, namely, randomized trace estimators and probing techniques based on graph colorings. Numerical experiments on density matrices of different types of networks illustrate the performance of the methods.

References

[1] M. Benzi, M. Rinelli, I. Simunec. Computation of the von Neumann entropy of large matrices via trace estimators and rational Krylov methods. *arXiv preprint arXiv:2212.09642*, (2022).

Restarted pseudo-Lanczos bidiagonalization for the hyperbolic SVD

Fernando Alvarruiz[1], *Carmen Campos*[2], *Jose E. Roman*[1]

[1] *D. Sistemes Informàtics i Computació, Universitat Politècnica de València, València, Spain*
E-mails: fbermejo@dsic.upv.es, jroman@dsic.upv.es
[2] *D. Didáctica de la Matemática, Universitat de València, València, Spain*
E-mail: carmen.campos-gonzalez@uv.es

Abstract

The hyperbolic singular value decomposition (HSVD) was introduced in [1], motivated by some signal processing applications such as the so-called covariance differencing problem. Given $A \in \mathbb{R}^{m\times n}$, the formulation of the HSVD is similar to that of the SVD, except that U is orthogonal with respect to a signature matrix,

$$A = U\Sigma V^*, \qquad U^*\Omega U = \tilde{\Omega}, \tag{1}$$

where $\Omega = \mathrm{diag}(\pm 1)$ is an $m \times m$ signature matrix provided as input, while $\tilde{\Omega}$ is another signature matrix obtained as part of the solution. Sometimes U is said to be a hyperexchange matrix, or also an $(\Omega, \tilde{\Omega})$-orthogonal matrix. With each singular triplet (σ_i, u_i, v_i), there is an associated sign $\tilde{\omega}_i$ (either 1 or -1), the corresponding diagonal element of $\tilde{\Omega}$.

This problem can be solved via an equivalent eigenvalue problem, for instance one of the following cross-product eigenproblems,

$$A^*\Omega A v_i = \sigma_i^2 \tilde{\omega}_i v_i, \tag{2}$$

$$AA^*\Omega u_i = \sigma_i^2 \tilde{\omega}_i u_i. \tag{3}$$

We will also discuss an alternative formulation with an eigenvalue for the cyclic matrix of order $m + n$.

We are interested in the case of large-scale, sparse A. In that case, the above eigenproblems can be approached either by standard Lanczos tridiagonalization or by a symmetric-indefinite Lanczos recurrence (pseudo-Lanczos). Alternatively, we derive a pseudo-Lanczos recurrence for the bidiagonalization of A in order to preserve the HSVD structure in the projected problem.

The methods are implemented in SLEPc, the Scalable Library for Eigenvalue Problem Computations [2]. In the case of bidiagonalization, we show how the pseudo-Lanczos method can be supplemented with a thick restart mechanism that preserves the structure, in a similar way as was done in [3] for generalized symmetric-indefinite eigenvalue problems.

Acknowledgements: Work supported by the Spanish Agencia Estatal de Investigación under grant PID2019-107379RB-I00/AEI/10.13039/501100011033.

References

[1] R. Onn, A. O. Steinhardt, A. Bojanczyk. The hyperbolic singular value decomposition and applications. *IEEE Trans. Signal Proces.*, 39(7): 1575–1588 (1991).

[2] V. Hernandez, J. E. Roman, V. Vidal. SLEPc: A scalable and flexible toolkit for the solution of eigenvalue problems. *ACM Trans. Math. Software*, 31(3): 351–362 (2005).

[3] C. Campos, J. E. Roman. Restarted Q-Arnoldi-type methods exploiting symmetry in quadratic eigenvalue problems. *BIT*, 56(4): 1213–1236 (2016).

A multi-approximation problem via (α, d)-designs

María José Benac [1], *Noelia Belén Rios*[2], *Mariano Ruiz*[3]

[1] *Dto. Académico de Matemática, FCEyT-UNSE and Instituto de Recursos Hídricos - FCEyT- CONICET, Santiago del Estero, Argentina*
E-mail: mjbenac@gmail.com
[2] *Centro de Matematica de La Plata, FCE-UNLP and IAM-CONICET, Buenos Aires, Argentina*
E-mail: nbrios@mate.unlp.edu.ar@
[3] *Centro de Matematica de La Plata, FCE-UNLP and IAM-CONICET, Buenos Aires, Argentina*
E-mail: mruiz@mate.unlp.edu.ar

Abstract

In this talk, we consider a multi-approximation problem on the set of positive semi-definite matrices that comes from finite-dimensional frame theory. Specifically, given a (finite) sequence $\Phi^0 = \{F_i^0\}_{i=1}^m$, with $F_i^0 \in \mathbb{C}^{d_i \times n}$, and a non-increasing sequence of positive weights $\alpha = (\alpha_i)_{i=1}^n$, our purpose is to characterize the best approximants $\Phi = \{F_i\}_{i=1}^m$ of Φ^0 among the set $D(\alpha, d)$ of the so-called (α, d)-*designs*, (where $d = (d_i)_{i=1}^m$). That is, sequences $\Phi = \{F_i\}_{i=1}^m$ such that $F_i \in \mathbb{C}^{d_i \times n}$ and $\sum_{i=1}^m \|f_{i,k}\|^2 = \alpha_k$, where $f_{i,k}$ is the k^{th} column of F_i.

The search of optimizers is done by minimizing the (squared) Joint Frame Operator Distance:

$$\Theta(\Phi) = \sum_{i=1}^m \|F_i^0 (F_i^0)^* - F_i F_i^*\|_2^2.$$

In this talk, we will show a complete characterization of global and local minimizers of Θ in $D(\alpha, d)$, obtained from results in [1]. It allows us to extend the solution of Strawn's conjecture proved in [2] to a multivariate setting.

Acknowledgements: Work (partially) supported by CONICET (PIP 00954CO - 2022), UNLP (11X829), UNSE (23/C190-PIP-2022)

References

[1] M. J. Benac, P. Massey, M. Ruiz, D. Stojanoff, Optimal $(\alpha, \mathbf{d})$-multi-completion of $\mathbf{d}$-designs, *Appl. Comput. Harmon. Anal.*, 62: 331-364 (2023)

[2] P. Massey, N. Rios, D. Stojanoff. Frame completions with prescribed norms: local minimizers and applications. *Adv. Comput. Math.*, 44 no. 1: 51-86 (2018)

Rank distribution of graphs over the field of two elements

Badriah Safarji[1], *Rachel Quinlan*[2]

School of Mathematical and Statistical Sciences, University of Galway, Ireland

Abstract

Let Γ be an undirected graph with vertices $v_1, \ldots, v_n$. A matrix M with entries in a field $\mathbb{F}$ *represents* Γ if the off-diagonal entries of M correspond to edges of Γ in the sense that $M_{ij} \neq 0_{\mathbb{F}}$ if and only if v_i and v_j are adjacent in Γ. There are no conditions on the diagonal entries of M. The *minimum rank problem for graphs* asks for the minimum rank of a matrix representing a given Γ, over a specified field. If the field $\mathbb{F}$ is finite, then the number of matrices representing a given graph over $\mathbb{F}$ is finite, and one may investigate the wider question of the distribution of their ranks.

Every symmetric $n \times n$ matrix represents a graph on the ordered vertex set $(v_1, \ldots, v_n)$. Over any finite field except $\mathbb{F}_2$, the number of $n \times n$ matrices of rank k is an increasing function of k, for $0 \leq k \leq n$. Over $\mathbb{F}_2$, there are fewer matrices of rank n than rank $n-1$. Restricting to symmetric matrices changes this picture slightly, but the numbers of symmetric $n \times n$ $\mathbb{F}_2$-matrices are closely matched, and coincide if n is odd. This suggests that there exist graphs whose $\mathbb{F}_2$-matrix representations of rank $n-1$ outnumber those of rank n. A goal of this project is to characterize graphs with this property. We present some inital steps in this direction.

Polynomial approximations for the matrix logarithm with computation graphs

J. Sastre[1], E. Jarlebring[2], J. Ibáñez[3]

[1] Institute of Telecommunications and Multimedia Applications, Universitat Politècnica de València, Spain
E-mail: jsastrem@upv.es
[2] Dept. of Mathematics, KTH Royal Institute of Technology in Stockholm, Sweden
E-mail: eliasj@kth.se
[3] Institute of Multidisciplinary Mathematics, Universitat Politècnica de València, Spain
E-mail: jjibanez@upv.es

Abstract

One of the most popular method for computing the matrix logarithm is a combination of the inverse scaling and squaring method in conjunction with a Padé approximation [1]. Recently, [2] presented Taylor based algorithm using matrix polynomial evaluation methods which are more efficient than the Paterson–Stockmeyer method [3] in the sense of number of matrix-matrix multiplications. The maximum theoretical efficiency would allow evaluating a polynomial degree 2^k for cost k matrix products (M), $k = 1, 2, \ldots$ [3, Prop. 2], denoted by the optimal degree. However, a nonlinear polynomial system must be solved which may not have a solution, or even if a solution exist, it may be numerically unstable [3, Sec. 3.1]. In [3, 4] the cases for optimal degrees 2^k with cost kM for $k = 3, 4$, are analytically solved. Other evaluation formulas are used in [2] for Taylor based approximations of the matrix logarithm with costs greater than $4M$, however, they are far from the optimal degree. In this work the iterative approach from [5] is used with an objective to obtain an accurate and efficient min–max approximations for costs greater than $4M$. A new algorithm is provided generalizing a scaling and squaring method, maintaining a high efficiency but also opening up to a relative backward error analysis. Simulations on benchmark matrices indicate that the new method is more efficient than state-of-the-art codes with a higher accuracy in comparison to Padé-based approaches.

Acknowledgements: Work supported by the Vicerrectorado de Investigación de la Universitat Politècnica de València (PAID-11-22) and the KTH Royal Institute of Technology in Stockholm.

References

[1] A.H. Al-Mohy, N.J. Higham, Improved Inverse Scaling and Squaring Algorithms for the Matrix Logarithm. *SIAM J. Sci. Comput.*, 34: C153–169 (2012).

[2] J. Ibáñez, J. Sastre, P. Ruiz, J.M. Alonso, E. Defez, An Improved Taylor Algorithm for Computing the Matrix Logarithm. *Mathematics*, 9, 2018 (2021).

[3] J. Sastre, Efficient evaluation of matrix polynomials. *Linear Algebra Appl.*, 539: 229–250 (2018).

[4] J. Sastre, J. Ibáñez, Evaluation of Matrix Polynomials beyond the Paterson-Stockmeyer Method. *Mathematics*, 9, 1600 (2021).

[5] E. Jarlebring, M. Fasi, E. Ringh, Computational graphs for matrix functions. *ACM Trans. on Math. Software*, accepted for publication, (2021).

Quantifying the Topological Stability of a Simplicial Complex

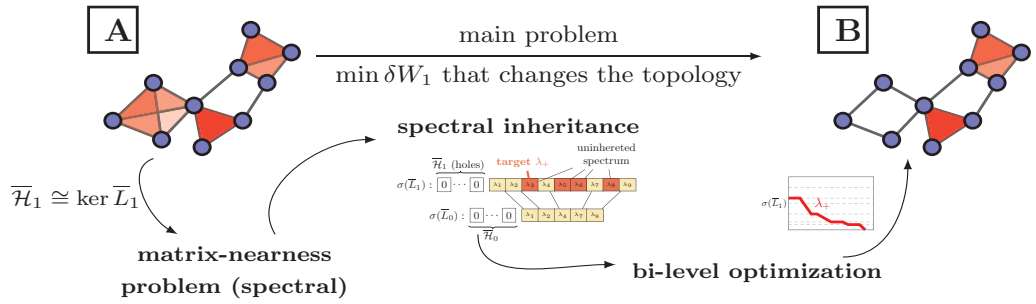
Nicola Guglielmi[*1], *Anton Savostianov*[*2], *Francesco Tudisco*[*3]

[*] *Gran Sasso Science Institute, L'Aquila, Italy*
E-mail: [1]nicola.guglielmi@gssi.it, [2]anton.savostianov@gssi.it,
[3]francesco.tudisco@gssi.it

Abstract

Simplicial complexes $\mathcal{K}$ are generalizations of classical graphs. Their homology groups $\mathcal{H}_k$ are widely used to characterize the structure and the topology of data in chemistry, neuroscience, transportation networks, etc. Exploiting the isomorphism between $\mathcal{H}_k$ and so-called higher-order Laplacian operators L_k, [1], our work investigates the less discussed question of the topological stability of the complex $\mathcal{K}$: how does $\mathcal{H}_k$ change when some edges in $\mathcal{K}$ are perturbed?

By introducing suitable weighted graph operators $\overline{L}_k$, the question is formulated as a matrix-nearness problem, similar to [2], with a spectral objective function that suitably takes into account potential homological pollution due to eigenvalues inherited from previous groups $\mathcal{H}_{k-1}, \mathcal{H}_{k-2}, \ldots$ Given the initial Laplacian operator $\overline{L}_k$, we introduce a continuous flow over the edges of the initial simplex and we develop a bi-level optimization procedure that computes the nearest simplex (or, equivalently, the smallest edge perturbation δW_1) with a different homology by integrating an alternated matrix gradient flow.



The developed numerical method is extensively tested on synthetic and real-life datasets; detailed theoretical background and the full optimization framework are provided in [3].

References

[1] L.-H. Lim. Hodge Laplacians on Graphs *SIAM Review*, 62 Iss. 3 (2020).

[2] E. Andreotti, D. Edelmann, N. Guglielmi, Ch. Lubich. Constrained Graph Partitioning via Matrix Differential Equations. *SIAM Journal on Matrix Analysis and Applications*, 40(1), 2019.

[3] N. Guglielmi, A. Savostianov, F. Tudisco. *Quantifying the structural stability of simplicial homology.* arXiv preprint, 2023.

The Waring problem for matrix algebras

Matej Brešar [1], *Peter Šemrl*[2]

[1] *Department of Mathematics, University of Maribor, Slovenia*
E-mail: matej.bresar@um.si
[2] *Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia*
E-mail: peter.semrl@fmf.uni-lj.si

Abstract

The following problem will be discussed: Given a nonconstant noncommutative polynomial f with coefficients from a field F, is it then true that, for any sufficiently large n, every traceless $n \times n$ matrix T over F is a difference of two elements from $f(M_n(F))$?

Acknowledgements: Work supported by grants J1-2454 and P1-0288 from ARRS Slovenia.

References

[1] M. Brešar. Commutators and images of noncommutative polynomials. *Adv. Math.*, 374: 107346, 21pp. (2020).

[2] M. Brešar, P. Šemrl. The Waring problem for matrix algebras. To appear in *Israel J. Math.*

[3] M. Brešar, P. Šemrl. The Waring problem for matrix algebras, II. Preprint.

A low rank ODE for spectral clustering stability

Nicola Guglielmi[1], *Stefano Sicilia*[1]

[1] *Division of Mathematics, Gran Sasso Science Institute, Italy*
E-mail: nicola.guglielmi@gssi.it, stefano.sicilia@gssi.it

Abstract

Spectral clustering is a well-known technique which identifies k clusters in an undirected graph with weight matrix $W \in \mathbb{R}^{n\times n}$ by exploiting its graph Laplacian

$$L(W) = \mathrm{diag}(W\mathbf{1}) - W, \qquad \mathbf{1} = (1,\dots,1)^T \in \mathbb{R}^n,$$

whose eigenvalues $0 = \lambda_1 \le \lambda_2 \le \cdots \le \lambda_n$ and eigenvectors are related to the k clusters. Since the computation of λ_{k+1} and λ_k affects the reliability of this method, the k-th spectral gap $\lambda_{k+1} - \lambda_k$ is often considered as a stability indicator. This difference can be seen as an unstructured distance between $L(W)$ and an arbitrary symmetric matrix $L_\star$ with vanishing k-th spectral gap.

A more appropriate structured distance to ambiguity such that $L_\star$ represents the Laplacian of a graph has been proposed in [1]. Slightly differently, we consider the objective functional

$$F(\Delta) = \lambda_{k+1}\left(L(W+\Delta)\right) - \lambda_k\left(L(W+\Delta)\right),$$

where Δ is a perturbation such that $W + \Delta$ has non-negative entries and the same pattern of W. We look for an admissible perturbation $\Delta_\star$ of smallest Frobenius norm such that $F(\Delta_\star) = 0$.

In order to solve this optimization problem, we exploit its low rank underlying structure. Similarly to [2], we formulate a rank-4 symmetric matrix ODE whose stationary points are the optimizers sought. The integration of this equation benefits from the low rank structure with a moderate computational effort and memory requirement, as it is shown in some illustrative numerical examples.

Acknowledgements: Work (partially) supported by funds from the Italian MUR within the PRIN-17 Project "Discontinuous dynamical systems: theory, numerics and applications" and by the INdAM Research group GNCS.

References

[1] E. Andreotti, D. Edelmann, N. Guglielmi, C. Lubich. Measuring the stability of spectral clustering. *Linear Algebra and its Applications*, 610: 673–697, 2021

[2] N. Guglielmi, C. Lubich, S. Sicilia. Rank-1 matrix differential equations for structured eigenvalue optimization. *arXiv preprint arXiv:2206.09338*, 2022

Spaces of matrices with a bounded number of eigenvalues

Matjaž Omladič[1], *Klemen Šivic*[2]

[1] *Institute of mathematics, physics and mechanics, Slovenia*
E-mail: matjaz@omladic.net
[2] *Faculty of mathematics and physics, University of Ljubljana, Slovenia*
E-mail: klemen.sivic@fmf.uni-lj.si

Abstract

A seminal result of Gerstenhaber gives the maximal dimension of a vector space of nilpotent matrices. It also exhibits the structure of such a space when the maximal dimension is attained. Some extensions of this result to vector spaces of matrices with a bounded number of eigenvalues have also been studied. In the talk we consider the most general case. For any positive integers n and $k < n$ we give the maximal dimension of a vector space of $n \times n$ matrices with no more than k eigenvalues, which proves the conjecture proposed by Loewy and Radwan. We also describe the structure of such spaces when the maximal dimension is attained.

Acknowledgements: Work partially supported by Slovenian Research Agency, grants P1-0222, N1-0103 and J1-3004.

References

[1] M. Omladič, K. Šivic, *The solution of the Loewy-Radwan conjecture*, arxiv: 2209.09416

Spread Code Constructions from Abelian Non-Cyclic Groups

Joan-Josep Climent, Verónica Requena, Xaro Soler-Escrivà

Departament de Matemàtiques, Universitat d'Alacant,
Campus de Sant Vicent del Raspeig, E-03690 Sant Vicent del Raspeig, Alacant, Spain
e-mails: jcliment@ua.es, vrequena@ua.es, xaro.soler@ua.es

Abstract

Network coding is a part of information theory that describes a method to maximize the rate of a network, which is modeled by a directed acyclic multigraph, with one or multiple sources and multiple receivers. The key point of this method is allowing the intermediate nodes of the network to transmit linear combinations of the inputs they receive [1]. The algebraic approach given in [5], provided a rigorous mathematical setup for error correction when coding in non-coherent networks. In this setting, the transmitted messages (**codewords**) are vector subspaces of a given vector space $\mathbb{F}_q^n$, where $\mathbb{F}_q$ is the finite field of q elements and a **subspace code** is a collection $\mathcal{C}$ of vector subspaces of $\mathbb{F}_q^n$. If all subspaces of $\mathcal{C}$ have the same dimension, $\mathcal{C}$ is called **constant dimension code**. Given two vector subspaces of $\mathbb{F}_q^n$, $\mathcal{U}$ and $\mathcal{V}$, their **subspace distance** is defined as $d_S(\mathcal{U},\mathcal{V}) = \dim(\mathcal{U}) + \dim(\mathcal{V}) - 2\dim(\mathcal{U}\cap\mathcal{V})$. If $\mathcal{C}$ is a constant dimension code, $d(\mathcal{C})$ will be an even number (see more details in [9]).

Consider the general linear group $\mathrm{GL}(n,q)$, and the Grassmannian $\mathcal{G}_q(k,n)$, which is the set of all k-dimensional vector subspaces of $\mathbb{F}_q^n$. The action of $\mathrm{GL}(n,q)$ on $\mathcal{G}_q(k,n)$ provides a relevant way of constructing constant dimension codes as orbits under the action of some specific subgroup of $\mathrm{GL}(n,q)$, called **orbit codes** [10]. Most of the bibliography about these codes focus on the use of cyclic subgroups of $\mathrm{GL}(n,q)$, called **cyclic orbit codes**. Particularly, we can find several works on spread codes with an orbital structure provided by a cyclic group [6, 7]. A **spread code** is a constant dimension code such that all its elements intersect pairwise trivially and their union covers the whole vector space. These codes are clearly a relevant family of constant dimension codes since they reach the maximum distance and, at the same time, the maximum size for that distance [8].

Our objective is focused on the study of orbit codes associated to other types of subgroups of the general linear group. As a first step, in [4] authors approach the study of orbit codes through the action of Abelian non-cyclic subgroups of $\mathrm{GL}(n,q)$, giving an specific construction of maximum distance. Since then, other authors have carried on with this research topic [2, 3]. Nevertheless, as far as we know, the only construction about spread codes obtained through the action of a non-cyclic Abelian group is in [3]. In this paper we analyse in more detail these codes. Specifically, we generalise the results obtained in [3]. For an even

integer n and k a divisor of n, we construct an Abelian non-cyclic orbit code of $\mathbb{F}_q^n$ of dimension k having maximum distance. In this particular construction, we use block matrices composed of companion matrices of a primitive polynomial and identity matrices. Then, we achieve to complete this orbit code with a nice family of k-subspaces of $\mathbb{F}_q^n$ in such a way the resulting code is a k-spread of $\mathbb{F}_q^n$.

Acknowledgements: This work was partially supported by the Spanish I+D+i project PID2019-108668GB-I00 of MCIN/AEI/10.13039/501100011033 and the I+D+i project VIGROB-287 of the Universitat d'Alacant.

References

[1] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung. Network information ow. *IEEE Transactions on Information Theory*, 46(4): 1204-1216 (2000). DOI:10.1109/18.850663.

[2] G. Terra-Bastos, R. Palazzo-Júnior, M. Guerreiro. Abelian non-cyclic orbit codes and multishot subspace codes. *Advances in Mathematics of Communications*, 14(4): 631-650 (2020). DOI: 10.3934/amc.2020035

[3] S. Chen, J. Liang. Construction of spread codes based on Abelian non-cyclic orbit codes. *Linear Algebra and its Applications*, 608: 54-67 (2021).

[4] J.-J. Climent, V. Requena, and X. Soler-Escrivà. A construction of Abelian non-cyclic orbit codes. *Cryptography and Communications*, 11(5): 839-852 (2019). DOI:10.1007/s12095-018-0306-5.

[5] R. Koetter, F.R. Kschischang. Coding for errors and erasures in random network coding. *IEEE Transactions on Information theory*, 54(8): 3579-3591 (2008). DOI:10.1109/TIT.2008.926449.

[6] J. Rosenthal, A.-L Trautmann. A complete characterization of irreducible cyclic orbit codes and their Plücker embedding. *Designs, codes and cryptography*, 66: 275-289 (2013). DOI:10.1007/s10623-012-9691-5.

[7] A.-L. Trautmann, F. Manganiello, M. Braun, J. Rosenthal. Cyclic orbit codes. *IEEE Transactions on Information Theory*, 59(11): 7386-7404 (2013). DOI:10.1109/TIT.2013.2274266.

[8] F. Manganiello, E. Gorla, and J. Rosenthal. Spread codes and spread decoding in network coding. *In Proceedings of the 2008 IEEE International Symposium on Information Theory (ISIT 2008)*, pages 881-885.

[9] A.-L. Horlemann-Trautmann, J. Rosenthal. Constructions of constant dimension codes. In M. Greferath, M. Osvin Pavaevic, N. Silberstein, and M. A. Vázquez-Castro, editors, *Network Coding and Subspace Designs, Signals and Communication Technology*, pages 25-42. Springer International Publishing AG, Cham, Switzerland, 2018. DOI:10.1007/978-3-319-70293-3 2.

[10] A.-L. Trautmann, F. Manganiello, and J. Rosenthal. Orbit codes - A new concept in the area of network coding. *In Proceedings of the 2010 IEEE Information Theory Workshop (ITW 2010)*, Dublin, Ireland, August 2010. IEEE. DOI:10.1109/CIG.2010.5592788.

Matrix version of a three-term recurrence formula with rational coefficients for q-Hermite Sobolev-type orthogonal polynomials

Edmundo J. Huertas[1], *Alberto Lastra*[2], *Anier Soria-Lorente*[3] *and Víctor Soto-Larrosa*[4]

[1] *Dpto. de Física y Matemáticas, Universidad de Alcalá, 28805 Alcalá de Henares, Madrid, Spain*
E-mail: edmundo.huertas@uah.es
[2] *Dpto. de Física y Matemáticas, Universidad de Alcalá, 28805 Alcalá de Henares, Madrid, Spain*
E-mail: alberto.lastra@uah.es
[3] *Dpto. de Tecnología, Universidad de Granma - Ctra. de Bayamo-Manzanillo, Km. 17,500. 85100 - Bayamo, Cuba*
E-mail: sorial@udg.co.cu
[4] *Dpto. de Física y Matemáticas, Universidad de Alcalá, 28805 Alcalá de Henares, Madrid, Spain*
E-mail: v.soto@uah.es

Abstract

The q-Hermite I polynomials are a particular case of the Al-Salam-Carlitz I orthogonal polynomials with parameter $a = -1$, and q stands for their unique parameter, for which we assume that $0 < q < 1$, which means that they belong to the class of orthogonal polynomial solutions of certain second order q-difference equations, known in the literature as the Hahn class. Here we consider a Sobolev-type perturbation of the q-Hermite I inner product, giving rise to the so-called *q-Hermite I-Sobolev type orthogonal polynomials of higher order*. It is well known that these kind of Sobolev-type polynomial families satisfy higher-order recurrence relations which can be expressed as certain high-order banded symmetric semi-infinite matrix. In this work we present a three term version of the aforementioned high-term recurrence formula, but considering rational functions as coefficients, and we also propose the corresponding three diagonal Jacoby-type matrix with entries depending on the variable x.

Acknowledgements: The work of EJH, AL and ASL is supported by Dirección General de Investigación e Innovación, Consejería de Educación e Investigación of the Comunidad de Madrid (Spain), and Universidad de Alcalá under grant CM/JIN/2021-014, Proyectos de I+D para Jóvenes Investigadores de la Universidad de Alcalá 2021. The work of EJH and AL has been also supported by and the Ministerio de Ciencia e Innovación-Agencia Estatal de Investigación MCIN/AEI/10.13039/501100011033 and the European Union "NextGenerationEU'

/PRTR, under grant TED2021-129813A-I00. The work of VSL has been supported by Consejería de Economía, Hacienda y Empleo of the Comunidad de Madrid through "Programa Investigo" funded by the European Union "NextGenerationEU".

References

[1] E. J. Huertas, A. Lastra, Anier Soria-Lorente, Víctor Soto-Larrosa, On Second Order q-Difference Equations Satisfied by Al-Salam–Carlitz I-Sobolev Type Polynomials of Higher Order, *Mathematics*, 8(8): 1300 (2020).

Old Song, New Verse □Easier Spectral Questions via Algebraic Restrictions

Jeffrey Stuart
Pacific Lutheran University
Tacoma, WA 98447 USA
jeffrey.stuart@plu.edu

Abstract

Determining the spectrum of a square matrix $\mathbf{A}$ over a field, one of the fundamental problems in linear algebra, is generally frustratingly hard. The problem itself is simple to formulate, just find the scalars λ such that $\mathbf{Av} = \lambda\mathbf{v}$ for some nonzero vector $\mathbf{v}$. The problem has an obvious algebraic formulation, simply find the roots of the characteristic equation, $\det(\lambda\mathbf{I} - \mathbf{A}) = 0$. As we know, however, accurately constructing the characteristic equation can be computationally expensive, finding its exact roots is generally impossible, and even finding approximate roots can be computationally challenging.

When faced with a surprisingly hard problem, mathematicians often retreat to working on a related, easier problem. For the spectral problem, one manner of retreat is to restrict the class of matrices, which one hopes will lead to an easier problem to solve, even if the results only apply to a smaller set of matrices. One way to restrict the class of matrices is by requiring that the eigenvalues satisfy a stronger condition than the characteristic equation, such as $\lambda^2 - \lambda = 0$, or $\lambda^k - \lambda = 0$ for some positive integer k, or even $\lambda^k - \lambda^\ell = 0$ for some positive integers k and ℓ.

This idea of imposing an algebraic restriction on the spectrum can be redirected away from eigenvalue equations towards algebraic equations involving the matrices themselves. Simple examples are $\mathbf{A}^2 = \mathbf{A}$□or $\mathbf{A}^k = \mathbf{A}$, or even $\mathbf{A}^k = \mathbf{A}^\ell$ where k and ℓ are positive integers. If one is going to look at the consequences of satisfying a matrix polynomial, why stop with just the matrix $\mathbf{A}$? What about $\mathbf{A}^k = \mathbf{A}^T$ or $\mathbf{A}^k = \mathbf{A}^*$? Why stop with just matrices derived from $\mathbf{A}$? One could examine the spectral consequences of requiring $\mathbf{PA} = \mathbf{AP}$ or $\mathbf{PA} = \mathbf{A}^*\mathbf{P}$ for some very special matrix $\mathbf{P}$. Going further with this idea, recent work has explored matrices satisfying $\mathbf{RA}^{s+1} = \mathbf{AR}$ for a class of special matrices $\mathbf{R}$ and where s is a nonnegative integer.

In this talk, I will review some of the spectral results associated with $\mathbf{RA}^{s+1} = \mathbf{AR}$ where $\mathbf{R}^k = \mathbf{I}$ for some positive integer k, and examine extensions to to the generalization $\mathbf{RA}^{s+\ell} = \mathbf{A}^\ell\mathbf{R}$ where $\mathbf{R}^k = \mathbf{I}$ for some positive integers k and ℓ.

Variant of Weyl type Theorems for Toelpitz Operators

Simi Thomas[1], *T. Prasad*[2], *Shery Fernandez*[3]

[1] *Dpt. of Mathematics, Cochin University of Science and Technology, India*
E-mail: simimariumthomas@cusat.ac.in
[2] *Dpt.of Mathematics, University of Calicut, India*
E-mail: prasadt@uoc.ac.in
[3] *Dpt. of Mathematics, Cochin University of Science and Technology, India*
E-mail: sheryfernandez@cusat.ac.in

Abstract

Let $B(\mathcal{H})$ be the algebra of all bounded linear operators on infinite dimensional complex seperable Hilbert space $\mathcal{H}$. An operator $T \in B(\mathcal{H})$ is said to satisfy property (UW_E) if $\sigma_a(T) \setminus \sigma_{uw}(T) = E(T)$, where $\sigma_a(T)$, $\sigma_{uw}(T)$ and $E(T)$ denote the approximate specturm, the upper semi-Weyl spectrum and the set of isolated eigenvalues of T respectively. Here we study property (UW_E) for Toeplitz operators T_ϕ, defined on the Hardy space $H^2(\mathcal{T})$ of the unit circle $\mathcal{T}$ in $\mathbb{C}$ where the symbol $\phi \in L^\infty(\mathcal{T})$ or ϕ is a continuous symbol. In particular, we study stability of property (UW_E) under compact perturbation for functions of Toeplitz operators.

Acknowledgements: The first author is supported by Senior Research Fellowship of University Grants Commission, India.

References

[1] P. Aiena, *On the Spectral mapping theorem for Weyl spectra of Toeplitz operators*, Advances in Operator theory (2020).

[2] P. Aiena, S. Triolo,: *Some remarks on spectral propeties of Toeplitz operators*, Mediterr. j. Math. 16, 135 (2019).

[3] R. G. Douglas,: *Banach algebra techniques in operator theory*, Graduate texts in mathematics, vol. 179, 2nd edn. Springer, New York (1998).

[4] B. Jia and Y. Feng, *Weyl type theorems under compact perturbations*, Mediterr. J. Math. 1-13, (2018)

[5] T. Prasad, Simi Thomas and Shery Fernandez, *Stability of property* (UW_E) *under compact perturbations*, Rend. Circ. Mat. Palermo Series 2, DOI: 10.1007/s000009-020-01506-6 (2022).

Stopping criteria for the coarsest-grid solver in multigrid V-cycle methods

Erin Carson[1], *Kirk M. Soodhalter*[2], *Petr Vacek*[3]

[1] *Faculty of Mathematics and Physics, Charles University, Czech Republic*
E-mail: carson@karlin.mff.cuni.cz
[2] *School of Mathematics, Trinity College Dublin, Ireland*
E-mail: ksoodha@maths.tcd.ie
[3] *Faculty of Mathematics and Physics, Charles University, Czech Republic*
E-mail: vacek@karlin.mff.cuni.cz

Abstract

Multigrid methods are frequently used when solving systems of linear equations, applied either as standalone solvers or as preconditioners for iterative methods. Within each cycle, the approximation is computed using smoothing on fine levels and solving on the coarsest level.

With growth of the size of the problems that are being solved, the size of the problems on the coarsest grid is also growing and their solution can become a computational bottleneck. In practice the problems on the coarsest-grid are often solved approximately, for example by Krylov subspace methods or direct methods based on low rank approximation; see, e.g., [1, 2]. The accuracy of the coarsest-grid solver is typically determined experimentally in order to balance the cost of the solves and the total number of multigrid cycles required for convergence.

In this talk, we present an approach to analyzing the effect of approximate coarsest-grid solves in the multigrid V-cycle method for symmetric positive definite problems. We discuss several stopping criteria derived based on the analysis and suggest a strategy for utilizing them in practice. The results are illustrated through numerical experiments.

References

[1] M. Huber, *Massively parallel and fault-tolerant multigrid solvers on petascale systems*. PhD thesis, Technical University of Munich, Germany, 2019.

[2] A. Buttari et al., Block Low Rank Single Precision Coarse Grid Solvers for Extreme Scale Multigrid Methods. *Numerical Linear Algebra with Applications*, 29, e2407 (2022).

A topological characterization of signed graphs with stable positive semidefinite maximum nullity at most two

Hein van der Holst[1]

[1] *Department of Mathematics, Georgia State University, US*
E-mail: hvanderholst@gsu.edu

Abstract

A signed graph is a pair (G, Σ), where G is an undirected graph (we allow parallel edges but no loops) and $\Sigma \subseteq E(G)$. The edges in Σ are called odd, while the other edges are called even. If (G, Σ) is a signed graph with vertex-set $V = \{1, \ldots, n\}$, $S(G, \Sigma)$ is the set of all real symmetric $n \times n$ matrices $A = [a_{i,j}]$ with $a_{i,j} > 0$ if i and j are adjacent and connected by only odd edges, $a_{i,j} < 0$ if i and j are adjacent and connected by only even edges, $a_{i,j} \in \mathbb{R}$ if i and j are adjacent and connected by both even and odd edges, $a_{i,j} = 0$ if i and j are not adjacent, and $a_{i,i} \in \mathbb{R}$ for all vertices i. The parameter $\nu(G, \Sigma)$ is defined as the largest nullity of any positive semidefinite matrix $A \in S(G, \Sigma)$ satisfying the Strong Arnold Hypothesis. This invariant is closed under taking minors. Arav, Hall, van der Holst, and Li gave a forbidden minor characterization of the class of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 2$. In this talk we present a topological characterization of the class of signed graphs (G, Σ) with $\nu(G, \Sigma) \leq 2$.

Homotopy method for singular multiparameter eigenvalue problems

Zhijun Wang[1], *Bo Dong*[2]

[1] *Department of Computer Science, KU Leuven, Belgium*
E-mail: zhijun.wang@student.kuleuven.be
[2] *School of Mathematical Science, Dalian University of Technology, P.R.China*
E-mail: dongbo@dlut.edu.cn

Abstract

We consider the MEP $A(\mathbf{x},\lambda)=\mathbf{0}$

$$\begin{aligned}(A_{10}+\lambda_1 A_{11}+\lambda_2 A_{12}+\cdots+\lambda_n A_{1n})\mathbf{x}_1&=\mathbf{0},\\ &\cdots\\ (A_{n0}+\lambda_1 A_{n1}+\lambda_2 A_{n2}+\cdots+\lambda_n A_{nn})\mathbf{x}_n&=\mathbf{0},\end{aligned}\tag{1}$$

where $A_{ij}\in\mathbb{C}^{m_i\times m_i}$, $\|\mathbf{x}_i\|_2=1$ for $i,j=1,2,\cdots,n$; $\lambda=(\lambda_1,\cdots,\lambda_n)$ and $\mathbf{x}=\mathbf{x}_1\otimes\cdots\otimes\mathbf{x}_n$ are eigenvalue and eigenvector, respectively.

In [1], the (1) is associated with the following simultaneous eigenvalue problem (SEP)

$$\begin{aligned}\lambda_1\Delta_0\mathbf{x}&=\Delta_1\mathbf{x},\\ &\cdots\\ \lambda_n\Delta_0\mathbf{x}&=\Delta_n\mathbf{x}.\end{aligned}\tag{2}$$

If the MEP (1) is nonsingular, i.e., the associated matrix Δ_0 is nonsingular, the eigenvalues of (1) agree with the eigenvalues of (2). However, several practical problems yield singular MEPs, which is still a challenge to current numerical methods.

In [2], a total degree homotopy method for general MEPs, including singular ones, is proposed. However, the method suffers from generating invalid paths, which becomes a significant obstacle for large sparse problems.

We propose a homotopy continuation method overcoming this obstacle by exploiting the sparse structure of coefficient matrices for general MEPs. For large sparse problems, our method generates significantly less invalid paths than method in [2].

References

[1] F.V. Atkinson, A. B. Mingarelli. *Multiparameter Eigenvalue Problems*. CRC Press, 2010.

[2] B. Dong, B. Yu, Y Yu. A homotopy method for finding all solutions of a multiparameter eigenvalue problem. *SIAM Journal on Matrix Analysis and Applications.*, 37: 550–571 (2016).

Numerical Computation of Quantum Graph Spectra

Chong-Son Dröge[1], *Anna Weller*[2]

[1] *Department of Mathematics and Computer Science, University of Cologne, Germany*
E-mail: droege@math.uni-koeln.de
[2] *Department of Mathematics and Computer Science, University of Cologne, Germany*
E-mail: weller@math.uni-koeln.de

Abstract

In the context of spectral solution methods for partial differential equations on metric graphs, the efficient computation of *quantum graph* spectra is of utmost importance. Under a quantum graph, we here for simplicity understand a metric graph equipped with the negative second order derivative acting on each edge and Neumann-Kirchhoff coupling conditions at the vertices. The spectrum of the quantum graph is then understood as the spectrum of the differential operator acting on the metric graph.
The special situation where all edges in the graph have the same length has been discussed in [1] using a well-known connection of quantum and combinatorial graph spectra. For the general case, we show that we can relate the spectrum to a so-called *Nonlinear Eigenvalue Problem* (NEP): Find eigenvalues $\lambda > 0$ such that a nontrivial $\mathbf{v}$ with

$$\mathbf{H}(\lambda)\mathbf{v} = 0$$

exists. Interestingly, the size of the matrix $\mathbf{H}$ is given by the number of vertices of the graph. The solutions of the NEP can be found as the roots of $\det(\mathbf{H}(\lambda))$ applying for example the Newton-trace method [3] with suitable initial guesses. To find these, we present a workflow to approximate non-equilateral graphs by *extended equilateral graphs* [2]. Moreover, we investigate polynomial approximations of $\mathbf{H}$ and apply standard NEP solvers for polynomial problems.

Acknowledgements: This work was supported by Hypatia.Science, an initiative for the promotion of young female scientists at the Department of Mathematics and Computer Science of the University of Cologne.

References

[1] M. Ainsworth, A. Weller, *A spectral Galerkin method for the solution of reaction-diffusion equations on metric graphs*, Oberwolfach Reports, Workshop Report 36, 2021.

[2] C. Dröge, A. Weller, *Numerical computation of quantum graph spectra as solutions of nonlinear eigenvalue problems,* manuscript in preparation.

[3] S. Güttel, F. Tisseur, *The nonlinear eigenvalue problem*, Acta Numerica, 26, 1-94, 2017.

An inexact matrix-Newton method for solving eigenvector-dependent nonlinear eigenvalue problems

Tom Werner[1]

[1] *Institute for Numerical Analysis, TU Braunschweig, Germany*
E-mail: tom.werner@tu-braunschweig.de

Abstract

The eigenvector-dependent nonlinear eigenvalue problem (NEPv) is a special type of eigenvalue problem where we seek to find k eigenpairs of a Hermitian matrix function $H : \mathbb{C}^{n,k} \to \mathbb{C}^{n,n}$ that depends nonlinearly on the eigenvectors itself. That is, we have to find $V \in \mathbb{C}^{n,k}$ with orthonormal columns and $\Lambda \in \mathbb{C}^{k,k}$ such that $H(V)V = V\Lambda$.
NEPv arise in a variety of applications, most notably in Kohn-Sham density theory and data science applications such as robust linear discriminant analysis [2]. A widely used algorithm to solve NEPv is the self-consistent field (SCF) iteration, which originates from Kohn-Sham density theory.

In this talk, we want to view NEPv as the set of nonlinear matrix equations

$$F(V, \Lambda) = \begin{bmatrix} H(V)V - V\Lambda \\ V^H V - I_k \end{bmatrix} = 0 \tag{1}$$

and present a novel algorithm for solving this problem using Newton's method. Note that Newton's method has been successfully applied to (1) in vectorized form with the drawback of being relatively slow for larger problems due to the quadratic growth of the size of the vectorized problem [1].
In our approach, we apply Newton's method on a matrix level using the Fréchet derivative of F and exploit the structure of the problem by using a global GMRES-approach to solve the Newton-update equation efficiently. This allows us to solve larger problems without a significant slow down compared to the SCF method.
We provide numerical results that show the performance of our algorithm on NEPv originating from different applications. These results indicate that the matrix-Newton approach can compete with SCF in terms of computational time and accuracy.

Acknowledgements: The research that lead to this talk was initiated by a joint work with Dr. Philip Saltenberger.

References

[1] W. Gao, C. Yang, J. Meza. *Solving a Class of Nonlinear Eigenvalue Problems by Newton's Method*. Lawrence Berkeley National Laboratory, Retrieved from https://escholarship.org/uc/item/8j96k0vd. 2009.

[2] Y. Cai, L.-H. Zhang, Z. Bai, R.-C. Li. *On an Eigenvector-Dependent Nonlinear Eigenvalue Problem*. SIAM Journal on Matrix Analysis and Applications, 39(3):1360–1382, 2018.

High-dimensional multi-view clustering methods

Author1[1], *Author2*[2], *Author3*[3]

Alaeddine Zahir[1], *Khalide Jbilou*[1,2], *Ahmed Ratnani*[1]

[1] *Al Khwarizmi department, Mohammed VI Polytechnic University, Morroco*
E-mail: alaeddine.zahir@um6p.ma ahmed.ratnani@um6p.ma
[2] *Approximation and Numerical Analysis, ULCO, France*
E-mail: khalide.jbilou@univ-littoral.fr

Abstract

Multi-view clustering has been widely used in recent years in comparison to single-view clustering, for clear reasons, as it offers more insights into the data, which has brought with it some challenges, such as how to combine these views or features. Most of recent work in this field focuses mainly on tensor representation instead of treating the data as simple matrices. This permits to deal with the high-order correlation between the data which the based matrix approach struggles to capture. Accordingly, we will examine and compare these approaches, particularly in two categories, namely graph-based clustering and subspace-based clustering. We will conduct and report experiments of the main clustering methods over a benchmark datasets.

References

[1] M. E. Kilmer, C. D. Martin, Factorization strategies for third-order tensors, Linear Algebra and its Applications 435 (2011) 641–658.

[2] Feiping Nie and Guohao Cai and Xuelong Li, Multi-View Clustering and Semi-Supervised Classification with Adaptive Neighbours,Thirty-First AAAI Conference on Artificial Intelligence, Vol. 31 No. 1 (2017).

[3] Jianlong Wu and Zhouchen Lin and Hongbin Zha, Essential tensor learning for multi-view spectral clustering, IEEE Transactions on Image Processing, Vol.28, No. 12, (December 2019).

[4] Deyan Xie and Deyan Xie and Quanxue Gao and Siyang Deng and Xiaojun Yang and Xiaojun Yang and Xinbo Gao, Multiple graphs learning with a new weighted tensor nuclear norm, Neural Networks,Vol 133, Pages 57-68, (2021).

Linear maps preserving (p, k) norms of tensor products of matrices

Zejun Huang [1], *Nung-Sing Sze* [2], *Run Zheng* [3]

[1] *College of Mathematics and Statistics, Shenzhen University, China*
E-mail: mathzejun@gmail.com
[2] *Department of Applied Mathematics, The Hong Kong Polytechnic University, Hong Kong*
E-mail: raymond.sze@polyu.edu.hk
[3] *Department of Applied Mathematics, The Hong Kong Polytechnic University, Hong Kong*
E-mail: zheng-run.zheng@connect.polyu.hk

Abstract

Let $m, n \geq 2$ be integers. Denote by M_n the set of $n \times n$ complex matrices. Given a positive integer $k \leq n$ and a real number $p > 2$, the (p, k) norm of a matrix $A \in M_n$ is defined by

$$\|A\|_{(p,k)} = \left[\sum_{i=1}^{k} s_i^p(A)\right]^{\frac{1}{p}},$$

where $s_1(A), \ldots, s_k(A)$ are the largest k singular values of A. We show that a linear map $\phi : M_{mn} \to M_{mn}$ satisfies

$$\|\phi(A \otimes B)\|_{(p,k)} = \|A \otimes B\|_{(p,k)} \quad \text{for all} \quad A \in M_m \text{ and } B \in M_n$$

if and only if there exist unitary matrices $U, V \in M_{mn}$ such that

$$\phi(A \otimes B) = U(\varphi_1(A) \otimes \varphi_2(B))V \quad \text{for all} \quad A \in M_m \text{ and } B \in M_n,$$

where φ_s is the identity map or the transposition map $X \mapsto X^T$ for $s = 1, 2$. The result is also extended to multipartite systems.

Polynomial approximations for the matrix logarithm with computation graphs

E. Jarlebring [1], *J. Sastre*[2], *J. Ibáñez*[3]

1 Dept. of Mathematics, KTH Royal Institute of Technology in Stockholm, Sweden
E-mail: eliasj@kth.se
[2] Institute of Telecommunications and Multimedia Applications, Universitat Politècnica de València, Spain
E-mail: jsastrem@upv.es
[3] Institute of Multidisciplinary Mathematics, Universitat Politècnica de València, Spain
E-mail: jjibanez@upv.es

Abstract

One of the most popular method for computing the matrix logarithm is a combination of the inverse scaling and squaring method in conjunction with a Padé approximation [1]. Recently, [2] presented Taylor based algorithm using matrix polynomial evaluation methods which are more efficient than the Paterson–Stockmeyer method [3] in the sense of number of matrix-matrix multiplications. The maximum theoretical efficiency would allow evaluating a polynomial degree 2^k for cost k matrix products (M), $k = 1, 2, \ldots$ [3, Prop. 2], denoted by the optimal degree [4]. However, a nonlinear polynomial system must be solved which may not have a solution, or even if a solution exist, it may be numerically unstable [3, Sec. 3.1]. In [3, 5] the cases for optimal degrees 2^k with cost kM for $k = 3, 4$, are analytically solved. Other evaluation formulas are used in [2] for Taylor based approximations of the matrix logarithm with costs greater than $4M$, however, they are far from the optimal degree. In this work the iterative approach from [4] is used with an objective to obtain an accurate and efficient min–max approximations for costs greater than $4M$. A new algorithm is provided generalizing a scaling and squaring method, maintaining a high efficiency but also opening up to a relative backward error analysis. Simulations on benchmark matrices indicate that the new method is more efficient than state-of-the-art codes with a higher accuracy in comparison to Padé-based approaches.

Acknowledgements: Work supported by the Vicerrectorado de Investigación de la Universitat Politècnica de València (PAID-11-21 and PAID-11-22) and the KTH Royal Institute of Technology in Stockholm.

References

[1] A.H. Al-Mohy, N.J. Higham, Improved Inverse Scaling and Squaring Algorithms for the Matrix Logarithm. *SIAM J. Sci. Comput.*, 34: C153–169 (2012).

[2] J. Ibáñez, J. Sastre, P. Ruiz, J.M. Alonso, E. Defez, An Improved Taylor Algorithm for Computing the Matrix Logarithm. *Mathematics*, 9, 2018 (2021).

[3] J. Sastre, Efficient evaluation of matrix polynomials. *Linear Algebra Appl.*, 539: 229–250 (2018).

[4] E. Jarlebring, M. Fasi, E. Ringh, Computational graphs for matrix functions. *ACM Trans. on Math. Software*, accepted for publication, (2021).

[5] J. Sastre, J. Ibáñez, Evaluation of Matrix Polynomials beyond the Paterson-Stockmeyer Method. *Mathematics*, 9, 1600 (2021).

On combinatorial matrix majorizations

Geir Dahl[1], *Alexander Guterman*[2], *Pavel Shteyner*[3]

[1] *Department of Mathematics, University of Oslo, Oslo, Norway*
E-mail: geird@math.uio.no
[2] *Department of Mathematics, Bar Ilan University, Ramat Gan, Israel*
E-mail: alexander.guterman@biu.ac.il
[3] *Department of Mathematics, Bar Ilan University, Ramat Gan, Israel*
E-mail: pashteiner@ya.ru

Abstract

Vector majorization is a classical notion useful in many areas of mathematics and its applications. There are many ways to define majorizations for real matrices, generalizing the notion of vector majorization. Different types of matrix majorizations have been applied to different areas of research. For example, directional majorization is useful in economics, while row-stochastic majorization plays an important role in the theory of statistical experiments. We introduce a new concept of majorization, which generalizes several existing notions of matrix majorization, for the families of matrices. The motivation to study this notion comes from mathematical statistics and involves the information content of experiments. We investigate properties of this order both of algebraic, combinatorial, and geometric character. In particular, our results include: the characterization of so-called minimal cover classes; properties of majorizations on the sets of (0,1) and (-1,0,1) matrices; the characterization of matrix maps preserving or converting majorizations.

Symposiums

MSC01. ILAS education

Anthony Cronin, Judi McDonald

Problrm Solving before Instruction
Avi Berman

Magic tricks as a source of examples in Linear Algebra
Fernando Blasco

Comparative Judgement and student engagement with proof writing in linear algebra
Anthony Cronin

A study of quadratic forms in Linear Algebra with GeoGebra
André Lúcio Grande, Rui Duarte

The power and the limits of visualizations
Damjan Kobal

Linear Algebra Education Reform, A Retrospective
Steven Leon

Exciting Eigenvectors: Seeing is Believing
Steve Mackey, Fernando De Teran, Raf Vandebril

Linear Algebra teaching in engineering degrees
Marta Peña

"Give an example of . . ."
Rachel Quinlan

Virtual reality for the teaching of linear geometry
José L. Rodríguez

Minisymposia
Sepideh Stewart, Judi McDonald, Guershon Harel

The Challenges of Teaching Elementary Linear Algebra in a Modern Matrix Based Way
Frank Uhlig

ABSTRACT

Last semester I taught a course on matrix theory, using the Problem Solving before Instruction pedagogy.

In the beginning of each week problems were explained but not solved. They were due, as homework, by the end of the week and in the beginning of the following week the solutions were discussed and new problems were given.

In the talk I will give examples and describe what the students thought of this way of learning.

Magic tricks as a source of examples in Linear Algebra

Fernando Blasco[1]

[1] *Departamento de Matemática Aplicada, Universidad Politćnica de Madrid, ETSIMFMN, Spain*
E-mail: fernando.blasco@upm.es

Abstract Magic tricks are a powerful tool to create interest into mathematical subjects. From a historical point of view, the solution of linear systems of equations has been presented as "magical" in old books. The construction of magic squares is also related to linear algebra concepts, as well as some tricks where the idea of basis plays an important role. In this talk we shall pose some problems related to this topic that can be used as a motivational tool in the classroom. We shall also perform a bunch of linear algebra related magic tricks in order to show how this topics can be brought to the classroom.

References

[1] F. Blasco *Matemagia*. Ariel, Madrid, 2016.

[2] P. Diaconis and R. Graham. *Magical Mathematics: The Mathematical Ideas That Animate Great Magic Tricks*. Princeton University Press, Princeton, 2015.

[3] M. Gardner. *Mathematics, magic and mystery*. Dover, New York, 1956.

[4] C. Mulcahy. *Mathematical Card Magic: Fifty-Two New Effects*. CRC Recreational Mathematics Series, Boca Raton, 2013.

[5] I. Stewart. Cups and Downs. *The College Mathematics Journal*, 43(2): 15–19 (January 2012)

[6] R. Teixeira. Using magic squares to teach linear algebra. *Australian Senior Mathematics Journal*, 32(2): 50–63 (2018).

Comparative Judgement and student engagement with proof writing in linear algebra

Anthony Cronin[1]

[1] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: anthony.cronin@ucd.ie

Abstract

In this talk I will describe a student-centred approach to engaging learners with proof writing and proof comprehension in a second university linear algebra course for mathematics specialists. This novel approach makes use of group-based activities, peer-to-peer feedback, and pairwise comparison of student-generated proofs. By creating structured opportunities to repeatedly engage in robust and meaningful mathematical discourse, we claim our novel approach has scope to add variety and substance to the often narrow array of assessment activities seen in similar classrooms. We present preliminary quantitative evidence for the construct validity of two tasks, alongside qualitative evidence suggesting that these tasks led students to engage in productive mathematical discourse.

Acknowledgements: This is joint work with Dr Ben Davies from the University of Southampton in the UK

A study of quadratic forms in Linear Algebra with GeoGebra

André Lúcio Grande [1]

[1]*Universidade de Aveiro/Portugal*

andreluciogrande@ua.pt

Abstract

This work aims to present several features of the GeoGebra software as a technological tool for teaching quadratic forms in a Linear Algebra and Analytic Geometry course, particularly in the study of Conic Sections and Quadric Surfaces, highlighting their potential and possible contributions. Several studies [1] and [2] present, discuss and highlight the use of technology and its implications in the teaching and learning of Linear Algebra. The methodology used in this work is characterized as being of the qualitative type, focusing on the STEAM approach (Science, Technology, Engineering, Arts, and Mathematics), which favors the discovery by students of the theory obtained through practice in the construction of knowledge through the elaboration of conjectures, testing of hypotheses and validation of results. In this research we will present two activities: the first linked to the study of the ellipse equation and the other related to the paraboloid (?) equation. As a result, we emphasize that in a dynamic, interactive, and intuitive way, GeoGebra allows exploring the objects of study both from an algebraic, geometric, and numerical point of view, greatly helping the study of quadratic forms.

Joint work with Rui Duarte (University of Aveiro).

Acknowledgments: Work supported by University of Aveiro – Portugal.

Referências

[1] Stewart, S. Andrews-Larson, C., & Zandieh, M. *Linear algebra teaching and learning: Themes from recent research and evolving research priorities*. ZDM Mathematics Education, 51(7), 1017-1030, 2019.

[2] Stewart, S., Axler, S., Beezer, R., Boman, E., Catral, M., Harel, G., McDonald, J., Strong, D., & Wawro, M. *The linear algebra curriculum study group (LACSG 2.0) recommendations*. The Notices of American Mathematical Society, 69(5), 813-819, 2022.

The power and the limits of visualizations

Damjan Kobal

Department of Mathematics
Faculty of Mathematiucs and Physics
University of Ljubljana, Slovenia
E-mail: damjan.kobal@fmf.uni-lj.si

Abstract

Visualizations are an important part of mathematics explorations and especially of mathematics teaching and learning. Linear algebra stands at the pivotal position of mathematics learning and right on the crosroads of exploring analogies and generalizations rising from elementary geometric and proportionality principles. Therefore, the intuitive value and power of visualizations are even more important in linear algebra learning. Generally, when we consider visualizations, we think about illustrative geometric presentations by different graphs and scatches. In modern times computer illustrations present a great opportunity for a skilled teacher but also a challenging trap for careless or sloppy approach. With modern technology it is much easier to manipulate student's attention then it is to motivate their understanding. We shall explore some samples of good and motivating visualizations and some of the manipulative implementations with little or negative teaching value.

Linear Algebra Education Reform, A Retrospective

Steven J. Leon

University of Massachusetts Dartmouth
E-mail: leonste@gmail.com

Abstract

In the United States, early Linear Algebra courses emphasized operator theory and were mainly designed for Mathematics majors. Linear Algebra became a standard required course for Mathematics majors in the 1960s. In the following decades, more and more outside disciplines began requiring Linear Algebra and it became increasingly clear that the existing courses were not well suited to the vast majority of students. In this talk, we discuss early efforts to address this problem and to reform linear algebra education. The talk will focus mainly on the efforts of the Linear Algebra Curriculum Study Group, the ILAS Education Committee, and the work done by the ATLAST Project. The speaker will close with a few personal recommendations which are based on over 40 years of experience teaching Linear Algebra.

Exciting Eigenvectors: Seeing is Believing

D. Steven Mackey[1], *Fernando De Terán*[2], *Raf Vandebril*[3]

[1] *Dept. of Mathematics, Western Michigan University, USA*
E-mail: steve.mackey@wmich.edu
[2] *Dept. of Mathematics, Universidad Carlos III de Madrid, Spain*
E-mail: fteran@math.uc3m.es
[3] *Dept. of Computer Science, KU Leuven, Belgium*
E-mail: raf.vandebril@kuleuven.be

Abstract

There is a simple, inexpensive, easy-to-build, and easy-to-operate device (adapted from [1]) that can be used in the classroom to demonstrate to students the physical reality of eigenvectors. In this talk I will show you that device, and tell a bit about how I have used this in various settings, both undergraduate and graduate, since the 1980's. Although I have used it primarily in lecture/demonstration mode, there is considerable scope for adapting this to a more hands-on, direct-engagement-by-students mode. I look to you, the linear algebra community, to develop such adaptations. As time permits, we will also hear about some preliminary classroom experiences with this device from Raf Vandebril and Fernando De Terán.

References

[1] H. V. McIntosh. *Matrix Analysis II: Further Introduction and some Applications to Physical Problems*. 1952.

Linear Algebra teaching in engineering degrees

Marta Peña

Dpt. of Mathematics, Universitat Politècnica de Catalunya, Spain
E-mail: marta.penya@upc.edu

Abstract

There has always been a great concern about the teaching of mathematics in engineering degrees. This concern has increased because students have less interest in these studies, which is mainly due to the low motivation of the students towards mathematics, and which is derived in most cases from the lack of awareness of undergraduate students about the importance of mathematics for their career.

The aim of this work is that students achieve a greater engagement in first academic courses of engineering degrees, through the implementation of real and technological applications in the teaching and learning of Linear Algebra concepts.

"Give an example of . . ."

Rachel Quinlan[1]

[1] *School of Mathematical and Statistical Sciences, University of Galway, Ireland*
E-mail: rachel.quinlan@universityofgalway.ie

Abstract

Students of linear algebra at the University of Galway were presented with a weekly task of the form *Give an example of* - a matrix or linear transformation with some specified property, or a matrix model of their own design, or an object that meets or narrowly misses the criteria of some definition. This talk will share some responses, that reveal insightful or creative thinking on the part of the authors, or reveal possible unintended consequences of certain teaching approaches.

We will argue that (some) tasks of this nature can engage students in authentic and creative mathematical practice, and present opportunities to develop and exercise a sense of personal agency.

Virtual reality for the teaching of linear geometry

José L. Rodríguez[1]

[1] *Department of Mathematics, University of Almería, Spain*
E-mail: jlrodri@ual.es

Abstract

We present some innovative activities carried out in Neotrie VR software [1] with students of the first year of mathematics at the University of Almeria ([2], [3]). These include the relative position of three planes, the use of a graphing calculator to represent parametric equations of planes, the creation of editable points, lines and planes in space, with parallelism and perpendicularity constraints, or the visualization of affine transformations in space.

Acknowledgements: Work partially supported by the Ministry of Science and Innovation grant PID2020-117971GB-C22 and FEDER-Junta de Andalucía grant UAL2020-SEJ-B2086.

References

[1] https://www2.ual.es/neotrie/project/geometria-elemental/

[2] J. L. Rodríguez (2022). Exploring Dynamic Geometry Through Immersive Virtual Reality and Distance Teaching. In: Richard, P.R., Vélez, M.P., Van Vaerenbergh, S. (eds) Mathematics Education in the Age of Artificial Intelligence. Mathematics Education in the Digital Era, vol 17. Springer, Cham.

[3] J. L. Rodríguez (2023). *Exploring dynamic geometry through immersive virtual reality and distance teaching*. Proceedings of the Asian Technology Conference in Mathematics 2022. pp. 218-230.

The structure and nature of linear algebra

Sepideh Stewart[1], *Judi McDonald*[2], *Guershon Harel*[3]

[1] *Department of Mathematics, University of Oklahoma, USA*
E-mail: sepidehstewart@ou.edu
[2] *Department of Mathematics and Statistics, Washington State University, USA*
E-mail: jmcdonald1@wsu.edu
[3] *Department of Mathematics, University of California San Diego, USA*
E-mail: gharel@ucsd.edu

Abstract

In this talk, we will examine the structure of linear algebra proofs and their nature. We will identify some proof techniques relying on the absence of a property. We will also explore the existing structure that connects and condenses powerful ideas. In particular, we show the visual power of matrices and their role in building, capturing, and condensing abstract ideas. We will conclude with some instructional insights from our experiences teaching proof-based second courses in linear algebra.

The Challenges of Teaching Elementary Linear Algebra in a Modern Matrix Based Way

Frank Uhlig
Department of Mathematics and Statistics,
Auburn University, Auburn, AL 36849-5310, USA
uhligfd@auburn.edu

Abstract

We assess the situation of our elementary Linear Algebra classes in the US holistically and through personal history recollections. Possible remedies for our elementary Linear Algebra's teaching problems are discussed and a change from abstract algebraic taught classes to a concrete matrix based first course is considered. The challenges of such modernization attempts for this course are laid out in light of our increased after-Covid use of e-books and e-primers.

We specifically address the useless and needless, but ubiquitous use of determinants, characteristic polynomials and polynomial root finding methods that are propagated in our elementary text books and are used in the majority of our elementary Linear Algebra classes for the matrix eigenvalue problem but that have no practical use whatsoever and offer no solution for finding matrix eigenvalues.

This paper challenges all mathematicians as we have misinformed and miseducated our students badly for decades in elementary Linear Algebra now and urges a switch to a new, fully matrix theoretical approach that covers all classical subjects in a practical and computable way.

MSC02. New faces of spectral graph theory

Aida Abiad, Gabriel Coutinho, Krystal Guo

Spectra of normal Cayley graphs
Soffia Arnadottir

NEPS of Complex Unit Gain Graphs
Francesco Belardo, Maurizio Brunetti, Suliman Khan

Clique complexes of strongly regular graphs and their eigenvalues
Sebastian Cioaba, Krystal Guo, Chunxu Ji, Mutasim Mim

Probing the Structure of Graph Nullspaces with Zero Loci
Joshua Cooper, Grant Fickes

On Sidorenko's conjecture for determinants and Gaussian Markov random fields
Péter Csikvári, Balázs Szegedy

On the spectra and algebraic connectivity of token graphs of a cycle
Cristina Dalfó, Monica Andrea Reyes, Miguel Angel Fiol, Arnau Messegué

On classes of diminimal trees
Carlos Hoppen

Spectra of Trees
Thomás Jung Spier, Emanuel Juliano, Gabriel Coutinho

Algebraic connectivity of maximal outerplanar graphs
Claudia Justel, Rafael de Paula, Carla Oliveira

Constructing cospectral hypergraphs
Antonina Khramova, Aida Abiad

On the spectra of weighted digraphs
Miriam Pisonero

Perfect state transfer in quantum walks on orientable maps
Vincent Schmeits, Krystal Guo

Cospectrality results on generalized Johnson and Grassmann graphs
Robin Simoens, Aida Abiad, Jozefien D'haeseleer

Semidefinite and eigenvalue bounds for bicliques and biidependent sets
Luis Felipe Vargas, Monique Laurent, Sven Polak

Spectra of normal Cayley graphs

Arnbjörg Soffía Árnadóttir[1]

[1] *DTU Compute, Technical University of Denmark, Denmark*
E-mail: sofar@dtu.dk

Abstract

A normal Cayley graph for a group G is a Cayley graph whose connection set is a union of conjugacy classes of G. Such a graph lies in an association scheme and its spectrum can be calculated using the irreducible characters of G. In this talk, we will explore these connections between groups, association schemes and character theory.

Acknowledgements: Work supported by Independent Research Fund Denmark, 8021-00249B AlgoGraph

NEPS of Complex Unit Gain Graphs

Francesco Belardo[1], *Maurizio Brunetti*[1], *Suliman Khan*[2]

[1] *Department of Mathematics and Applications, University of Naples Federico II, Italy*
E-mails: fbelardo@unina.it, mbrunett@unina.it
[2] *Department of Mathematics and Physics, University of Campania Luigi Vanvitelli*
E-mail: suliman.khan@unicampania.it

Abstract

A complex unit gain graph, or $\mathbb{T}$-gain graph, is a gain graph whose gains belong to the multiplicative group of complex units. Generalizing a classical construction for simple graphs due to Cvektović [1], and its extension to signed graphs due to Germina et al. [2], we give a suitably defined non-complete extended p-sums (NEPS, for short) of $\mathbb{T}$-gain graphs. Structural properties of NEPS like balance and some spectral properties and invariants of their adjacency and Laplacian matrices are considered, including the energy and the symmetry of the adjacency spectrum.

Acknowledgements: The authors are are grateful to GNSAGA of InDAM for the support provided.

References

[1] D. Cvetković, Graphs and their spectra (Grafovi i njihovi spektri) (Thesis), *Univ. Beograd Publ. Elektrotehn. Fak.*, Ser. Mat. Fiz. 354–356: 1–50 (1971).

[2] K.A. Germina, Shahul Hameed K, T. Zaslavsky, On products and line graphs of signed graphs, their eigenvalues and energy, Linear Algebra Appl. 435: 2432–2450 (2011).

[3] F. Belardo, M. Brunetti, S. Khan, NEPS of Complex Unit Gain Graphs, in preparation.

Clique complexes of strongly regular graphs and their eigenvalues

Sebastian M. Cioabă[1], *Krystal Guo*[2], *Chunxu Ji*[3], *and Mutasim Mim*[1]

[1] *Department of Mathematical Sciences, University of Delaware, Newark, DE 19716-2553, U.S.A.*
E-mail: {cioaba@udel,mutasimm}.edu
[2] *University of Amsterdam, The Netherlands*
E-mail: k.guo@uva.nl
[3] *Department of Mathematics, California State University San Marcos, San Marcos, CA 92096, U.S.A.*

Abstract

It is well known that non-isomorphic strongly regular graphs with the same parameters must be cospectral (have the same eigenvalues). We investigate whether the spectra of higher order Laplacians associated with these graphs can distinguish them. In this paper, we study the clique complexes of strongly regular graphs, and determine the spectra of the triangle complexes of Hamming graphs, Triangular graphs and several other strongly regular graphs. In many cases, the spectrum of the triangle complex distinguishes between strongly regular graphs with the same parameters, but we find some examples where that is not the case.

Acknowledgements: This research has been partially supported by NSF grants DMS-1600768 and CIF-1815922 and a and a JSPS Invitational Fellowship for Research in Japan S19016.

Probing the Structure of Graph Nullspaces with Zero Loci

Joshua Cooper, Grant Fickes

Department of Mathematics, University of South Carolina, Columbia, SC USA

cooper@math.sc.edu

The adjacency nullity of graphs and hypergraphs is something of a mystery, though some results are known for narrow classes of graphs such as trees. There is, however, rich structure in their nullspaces (and, for hypergraphs, their nullvarieties), visible by partitioning nullvectors according to their zero loci: vertex sets which are indices of their zero coordinates. This set system is the lattice of flats of a "kernel matroid", a subsystem of which are the "stalled" sets closed under skew zero forcing (SZF), a graph percolation/infection model known to have connections with rank and nullity. These set systems have interesting descriptions in terms of matchings, vertex covers, and edges' influence on rank – especially for trees. For a wide variety of graphs, the lattice of SZF-closed sets is also a matroid, a fact which can be used to obtain a polynomial-time algorithm for computing the skew zero forcing number. This contrasts with the general case, where we show that the corresponding decision problem is NP-hard. We also define skew zero forcing for hypergraphs, and show that, for linear hypertrees, the poset of SZF-closed sets is dual to the lattice of ideals of the hypergraph's nullvariety; while, for complete hypergraphs, the SZF-closed sets and the zero loci of nullvectors are more loosely related.

On Sidorenko's conjecture for determinants and Gaussian Markov random fields

Péter Csikvári[1], *Balázs Szegedy*[2]

[1] *Alfréd Rényi Institute of Mathematics, Hungary E-mail:* peter.csikvari@gmail.com [2] *Alfréd Rényi Institute of Mathematics, Hungary E-mail:* szegedyb@gmail.com

Abstract

We study a class of determinant inequalities that are closely related to Sidorenko's famous conjecture (also conjectured by Erdős and Simonovits in a different form). Our main result can also be interpreted as an entropy inequality for Gaussian Markov random fields (GMRF). We call a GMRF on a finite graph G homogeneous if the marginal distributions on the edges are all identical. We show that if G is bipartite, then the differential entropy of any homogeneous GMRF on G is at least $|E(G)|$ times the edge entropy plus $|V(G)|?2|E(G)|$ times the point entropy. We also show that in the case of non-negative correlation on edges, the result holds for an arbitrary graph G. The connection between Sidorenkos conjecture and GMRF's is established via a large deviation principle on high dimensional spheres combined with graph limit theory. It is also observed that the system we study exhibits a phase transition on large girth regular graphs. Connection with Ihara zeta function and the number of spanning trees is also discussed.

References

[1] P. Csikvári and B. Szegedy. *On Sidorenko's conjecture for determinants and Gaussian Markov random fields*, Random Structures & Algorithms (2022), https://doi.org/10.1002/rsa.21112

On the spectra and algebraic connectivity of token graphs of a cycle

M. A. Reyes[1], *C. Dalfó*[1], *M. A. Fiol*[2], *and A. Messegué*[1]

[1] *Dept. de Matemàtica, Universitat de Lleida, Lleida/Igualada*
E-mail: {monicaandrea.reyes,cristina.dalfo,visitant.arnau.messegue}@udl.cat
[2] *Dept. de Matemàtiques, Universitat Politècnica de Catalunya, Barcelona*
E-mail: miguel.angel.fiol@upc.edu

Abstract

The k-token graph $F_k(G)$ of a graph G is the graph whose vertices are the k-subsets of vertices from G, two of which being adjacent whenever their symmetric difference is a pair of adjacent vertices in G. Recently, it was conjectured that the algebraic connectivity (or second Laplacian eigenvalue) of $F_k(G)$ equals the algebraic connectivity of G. In this paper, we first give results that relate the algebraic connectivities of a token graph and the same graph after removing a vertex. When applied to 2-tokens graphs, these results allow us to prove the conjecture for three infinite families: the odd graphs O_r for all r, the multipartite complete graphs $K_{n_1,n_2,\ldots,n_r}$ for all $n_1, n_2, \ldots, n_r$, and the cycle graphs C_n. In the case of cycles, we present a new method that allows us to compute the whole spectrum of $F_2(C_n)$. As a consequence, we prove that the conjecture also holds for uniclyclic graphs.

Acknowledgements: This research has been supported by AGAUR under project 2021SGR00434 and MICINN under project PID2020-115442RB-I00.

References

[1] K. Audenaert, C. Godsil, G. Royle, and T. Rudolph, Symmetric squares of graphs, *J. Combin. Theory B* **97** (2007) 74–90.

[2] C. Dalfó, F. Duque, R. Fabila-Monroy, M. A. Fiol, C. Huemer, A. L. Trujillo-Negrete, and F. J. Zaragoza Martínez, On the Laplacian spectra of token graphs, *Linear Algebra Appl.* **625** (2021) 322–348.

[3] C. Dalfó, and M. A. Fiol, On the algebraic connectivity of token graphs, submitted (2022).

[4] C. Dalfó, M. A. Fiol, and A. Messegué, Some bounds on the algebraic connectivity of token graphs, submitted (2022).

[5] C. Dalfó, M. A. Fiol, M. Miller, J. Ryan, and J. Širáň, An algebraic approach to lifts of digraphs, *Discrete Appl. Math.* **269** (2019) 68–76.

[6] C. Dalfó, M. A. Fiol, S. Pavlíková, and J. Širán, A note on the spectra and eigenspaces of the universal adjacency matrices of arbitrary lifts of graphs, *Linear Multilinear Algebra* (2022), DOI: 10.1080/03081087.2022.2042174.

[7] C. Dalfó, M. A. Fiol, and J. Širáň, The spectra of lifted digraphs, *J. Algebraic Combin.* **50** (2019) 419–426.

[8] R. Fabila-Monroy, D. Flores-Peñaloza, C. Huemer, F. Hurtado, J. Urrutia, and D. R. Wood, Token graphs, *Graphs Combin.* **28** (2012), no. 3, 365–380.

[9] M. Fiedler, Algebraic connectivity of graphs, *Czech. Math. Journal* **23** (1973), no. 2, 298–305.

[10] R. Grone, R. Merris, and V. S. Sunder, The Laplacian spectrum of a graph, *SIAM J. Matrix Anal. Appl.* **11** (1990) 218–238.

[11] S. Ibarra and L. M. Rivera, The automorphism groups of some token graphs, `arXiv:1907.06008v3[math.CO]`

[12] W.-C. Yueh, Eigenvalues of several tridiagonal matrices, *Appl. Math. E-Notes* **5** (2005) 66–74.

On classes of diminimal trees

Carlos Hoppen[1]

[1] *Instituto de Matemática e Estatística, Universidade Federal do Rio Grande do Sul, Brazil*
E-mail: choppen@ufrgs.br

Abstract

As usual, a tree is an acyclic and connected graph, and its diameter is the maximum number of edges on a path connecting two of its vertices. A tree T with vertex set $[n] = \{1, \dots, n\}$ and diameter $d \in \{0, \dots, n-1\}$ is said to be *diameter-minimal* (or *diminimal*) if there exists a real symmetric matrix $M = (m_{ij}) \in \mathbb{R}^{n \times n}$ such that:

(i) For all $i \neq j$, $m_{ij} \neq 0$ if and only if $\{i, j\}$ is an edge of T.

(ii) The spectrum of M contains exactly $d + 1$ distinct eigenvalues.

A tree T satisfying (i) is said to be the *underlying graph* of M, and we let $\mathcal{S}(T)$ denote the set of symmetric matrices with underlying tree T.

The definition of diminimal tree was motivated by a result of Leal-Duarte and Johnson [6], which states that, for any symmetric matrix M whose underlying graph is a tree T of diameter d, the number $\mu(M)$ of distinct eigenvalues of M is at least $d + 1$. This implies that

$$q(T) := \min\{\mu(M) \colon M \in \mathcal{S}(T)\} \geq d + 1. \tag{1}$$

The authors of [6] asked whether the inequality in (1) was actually an equation. As it turns out, in general, the answer is no, and there exist trees T_d of any given diameter $d \geq 6$ for which $q(T_d) > d + 1$. Examples of this behavior may be found in [2, 4, 5], for instance. Note that the trees for which (1) holds with equality are precisely the diminimal trees defined above. A matrix $M \in \mathcal{S}(T)$ such that $q(T) = \mu(M)$ is called a *minimal matrix* associated with T.

In this talk, I shall survey recent results about diminimal trees. The emphasis will be on recent results in collaboration with Allem, Braga, Oliveira, Sibemberg and Trevisan [1], which describe infinite families of diminimal trees. More precisely, we start with a decomposition by Johnson and Saiago [4], by which each tree T of diameter d is associated with one of a finite set $\mathcal{C}_d$ of 'irreducible' trees, known as *seeds*, with the property that T can be generated from the corresponding seed by a sequence of diameter-preserving operations known as *branch duplications*. For every $d \geq 6$, we then identify two seeds of diameter d (if d is even) and three seeds of diameter d (if d is odd) with the property that any tree generated from them by a sequence of branch duplications is diminimal. This result is proved by induction. Interestingly, the induction hypothesis

is much stronger. For instance, minimal matrices associated with different trees in the family may be chosen with the same set of eigenvalues. Moreover, it is possible to perturb some values in this set in a way that there exist minimal matrices with the new set of perturbed eigenvalues. One of the main technical tools is an eigenvalue location algorithm by Jacobs and Trevisan [3].

Acknowledgements: Work partially supported by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (Proj. MathAmsud 88881.694479/2022-01), Fundação de Amparo à Pesquisa do Estado do Rio Grande do Sul (Proj. 19/2551-0001727-8) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (Proj. 315132/2021-3).

References

[1] L.E. Allem, R.O. Braga, C. Hoppen, E.R. Oliveira, L.S. Sibemberg and V. Trevisan. *Diminimal families of arbitrary diameter*. arXiv:2302.00835, 29pp. (2023).

[2] F. Barioli and S. Fallat. On two conjectures regarding an inverse eigenvalue problem for acyclic symmetric matrices. *The Electronic Journal of Linear Algebra*, 11: 41–50, (2004).

[3] D.P. Jacobs and V. Trevisan. Locating the eigenvalues of trees. *Linear Algebra and its Applications*, 434(1):81–88, (2011).

[4] C.R. Johnson and C.M. Saiago. *Eigenvalues, Multiplicities and Graphs*. Cambridge University Press, Cambridge, UK, 2018.

[5] I.-J. Kim and B.L. Shader. Classification of trees each of whose associated acyclic matrices with distinct diagonal entries has distinct eigenvalues. *Bull. Korean Math. Soc.*, 28:95–99 (2008).

[6] A. Leal-Duarte and C.R. Johnson. On the minimum number of distinct eigenvalues for a symmetric matrix whose graph is a given tree. *Mathematical Inequalities and Applications*, 5: 175–180 (2002).

Spectra of Trees

Thomás Jung Spier

Dpt. of Computer Science, Universidade Federal de Minas Gerais, Brazil
E-mail: thomasjung@dcc.ufmg.br

Abstract

In this talk, we explore some recent results about the spectrum of trees. We show applications for the study of strong cospectrality [1, 4, 5, 6], perfect state transfer [2, 5, 6] and integral trees [3, 5, 6].

This is joint work with Emanuel Juliano and Gabriel Coutinho.

Acknowledgements: Author acknowledges the funding from FAPEMIG that supported this research.

References

[1] C. D. Godsil, J. Smith. Strongly Cospectral Vertices. *arXiv:1709.07975*, (2017).

[2] C. D. Godsil. When can perfect state transfer occur? *Electronic Journal of Linear Algebra*, 23:877890, (2012).

[3] F. Harary, A. J. Schwenk. Which graphs have integral spectra? *Graphs and combinatorics*, pages 45–51, (1974).

[4] E. Juliano, G. Coutinho, T. J. Spier. Strong Cospectrality in Trees. Accepted to *Algebraic Combinatorics* (2023).

[5] E. Juliano, G. Coutinho, T. J. Spier. The spectrum of symmetric decorated paths. Preprint submitted (2023).

[6] E. Juliano, G. Coutinho, T. J. Spier. Perfect State Transfer in Trees. Preprint under preparation (2023).

Algebraic connectivity of maximal outerplanar graphs

Claudia Marcela Justel[1], *Rafael Leal de Paula*[2], *Carla Silva Oliveira*[3]

[1] *Instituto Militar de Engenharia (IME), Brazil*
E-mail: cjustel@ime.eb.br
[2] *Instituto Militar de Engenharia (IME), Brazil*
E-mail: paula.rafael@ime.eb.br
[3] *Escola Nacional de Ciências Estatísticas (ENCE/IBGE), Brazil*
E-mail: carla.oliveira@ibge.gov.br

Abstract

Inside the class of chordal graphs, there are two subclasses called k-trees and k-paths that have maximal cliques and minimal vertex separators with constant sizes ($k+1$ and k, respectively). A k-tree can be defined recursively as follows: a complete graph on k vertices is a k-tree, and a k-tree on $n+1$ vertices can be determined by adding to a k-tree on n vertices a new vertex adjacent to k mutually adjacent vertices [2]. A k-path is a k-tree with only 2 simplicial vertices [9]. By definition, we conclude that 2-trees and 2-paths do not contain subgraph homeomorphic to K_5 and $K_{3,3}$, so they are planar graphs [6]. Moreover, 2-paths not contain subgraph homeomorphic to K_4 and $K_{2,3}$ and its number of edges is $2n-3$, consequently they are maximal outerplanar graphs [7],[4].

Spectral properties of planar and outerplanar graphs have been studied as we can see in [1],[8],[10],[11],[12]. Cvetković et al., [5], and Boots et al., [3], conjectured which planar and outerplanar graphs have the maximum spectral radius. Tait, M. and Tobin, J., [10], proved these conjectures for n large enough. In 2021, Lin e Ning, [8], show the conjecture for outerplanar is true for all $n \geq 2$ except for $n = 6$. Yu et al., [11], present upper bounds for the signless Laplacian spectral radius of planar graphs, outerplanar graphs and Halin graphs in terms of order and maximum degree. Moreover, presents extremal graphs for the signless Laplacian spectral radius for a special type of outerplanar graph, without inner triangles, which are 2-path graphs. Yu et al., [12], analyzed some extremal results for the spectral radius of $A_\alpha-$ matrix (or $\alpha - index$). Besides, proved that the only outerplanar graph with maximum $\alpha - index$ is $K_1 \vee P_{n-1}$. Barrière et al., [1], analyzed the asymptotic behavior for the maximum value of the second smallest eigenvalue of the Laplacian matrix, that is, algebraic connectivity, of planar bipartite and outerplanar graphs with a fixed number of vertices.

In this paper, we present some results for the maximum value of algebraic connectivity of maximal outerplanar and 2-path graphs when the number of vertices is fixed.

Acknowledgements: The research of C. S. Oliveira is supported by CNPq Grant 304548/2020-0.

References

[1] Barrière, L., Huemer, C., Mitsche, D., Orden, D. On the Fiedler value of large planar graphs. *Linear Algebra and its Applications*, 439:7, 2070–2084 (2013).

[2] Beineke, L.W., Pippert, R.E. Properties and characterizations of k-trees. *Mathematika*, 18, 141-151 (1971).

[3] Boots, B. N. and Royle, G. F. A Conjecture on the maximum value of the principal eigenvalue of planar graph. *Geogr. Anal.*, 23(3), 276-282, (1991).

[4] Chartrand, G., Harary, F. Planar permutation graphs. *Ann. Inst. Henri Poincaré* 3, 433-438 (1967).

[5] Cvetković, D. and Rowlinson, P. The largest eigenvalue of a graph: survey. *Linear and Multilinear Algebra*, 20(1-2), 3-33, (1990).

[6] Diestel, R. *Graph Theory*. Springer Verlag, New York (2000).

[7] Harary, F. *Graph Theory*, Addison-Wesley, Reading,. MA (1972).

[8] Lin, H., Ning, B. A complete solution to the Cvetković - Rowlinson conjecture.*Journal of Graph Theory* 97(3):441–450 (2021).

[9] Markenzon, L., Justel, C., Paciornik, N. Subclasses of k-trees: characterization and recognition. *Discrete Appl. Math.* 154, 818-825 (2006).

[10] Tait, M., Tobin, J. Three conjectures in extremal spectral graph theory. *J. Combin.Theory, Ser. B* 126, 137–161 (2017)

[11] Yu, G., Feng, L., Ilic, A., and Stevanovic, D. The signless laplacian spectral radius of bounded degree graphs on surfaces. *Applicable Analysis and Discrete Mathematics* 9, 332–346 (2015a).

[12] Yu, Z., Kang, L., Liu, L., and Shan, E. (2019). The extremal α-index of outerplanar and planar graphs.*Applied Mathematics and Computation* 343(C):90–99 (2019).

Constructing cospectral hypergraphs

Aida Abiad[123], *Antonina P. Khramova*[1]

[1] *Department of Mathematics and Computer Science, Eindhoven University of Technology, The Netherlands*
E-mail: a.abiad.monge@tue.nl, a.khramova@tue.nl
[2] *Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium*
[3] *Department of Mathematics and Data Science, Vrije Universiteit Brussel, Belgium*

Abstract

Spectral hypergraph theory mainly concerns using hypergraph spectra to obtain structural information about the given hypergraph. This field has attracted a lot of attention over the last years. The spectrum of a hypergraph can be defined in different ways. In this talk, we will focus on the spectrum of two well-known hypergraph representations: adjacency tensors and integer matrices with entries defined by the number of edges that two vertices share. Two hypergraphs are cospectral if they share the same spectrum with respect to a certain representation. By studying cospectral hypergraphs, we aim to understand which hypergraph properties cannot be deduced from their spectra. In this talk, we will show new methods for constructing cospectral uniform hypergraphs.

On the spectra of weighted digraphs

Miriam Pisonero

Dpt. Matemática Aplicada, Universidad de Valladolid/IMUVA, España
E-mail: mpisonero@uva.es

Abstract

The Nonnegative Inverse Eigenvalue Problem (NIEP) is the problem of characterizing all possible spectra of entrywise nonnegative matrices, or equivalently, all possible spectra of weighted digraphs. Note that a nonnegative matrix can be seen as the adjacency matrix of a weighted digraph. In this talk we make a brief overwiew of what is known about the problem and its variations. Finally, we discuss when zero is an eiegenvalue of the weighted digraphs whose adjacency matrix is weakly diagonally dominant, showing the connection of this with the parity of the greatest common divisor of the cycle lengths of the digraph.

Acknowledgements: Work (partially) supported by PID2021-122501NB-I00.

References

[1] C. R. Johnson, C. Marijuán, M. Pisonero. Diagonal dominance and invertibility of matrices. *Special Matrices* 11: 20220181 (2023).

[2] A. I. Julio, C. Marijuán, M. Pisonero, R. L. Soto. Universal Realizability in Low Dimension. *Linear Algebra and Appl.* 619: 107–1366 (2021).

Perfect state transfer in quantum walks on orientable maps

Krystal Guo[1], *Vincent Schmeits*[2]

[1] *Korteweg-de Vries Institute for Mathematics, University of Amsterdam, The Netherlands. QuSoft (Research center for Quantum software & technology), The Netherlands.*
E-mail: k.guo@uva.nl
[2] *Korteweg-de Vries Institute for Mathematics, University of Amsterdam, The Netherlands*
E-mail: v.f.schmeits@uva.nl

Abstract

Introduced by Zhan [2], a vertex-face walk is a type of discrete-time quantum walk on the arcs of an orientable map (i.e. a cellularly embedded graph on an orientable surface). The unitary transition matrix for the walk is the product of two reflections corresponding to the incidence relations between the arcs and the vertices and faces of the map. If the initial state of the walk is a uniform superposition of the arcs incident to some vertex u, and if after some number of steps in the walk, the state is a superposition of arcs incident to some other vertex v, we say that there is perfect state transfer (PST) from u to v. The walk is periodic at u if $v = u$. We give families of examples of maps that exhibit perfect state transfer and periodicity. We also show that, under some projection, the evolution of any two-reflection discrete-time quantum walk satisfies a Chebyshev recurrence.

References

[1] K. Guo, V. Schmeits. Perfect state transfer in quantum walks on orientable maps. arXiv:2211.12841 [math.CO], (2022) https://doi.org/10.48550/arXiv.2211.12841

[2] H. Zhan, Quantum walks on embeddings. *Journal of Algebraic Combinatorics*, 53(4): 1187–1213 (2020).

Cospectrality results on generalized Johnson and Grassmann graphs

Aida Abiad[1], *Jozefien D'haeseleer*[2], *Robin Simoens*[3]

[1] *Dpt. of Mathematics and Computer Science, Eindhoven University of Technology, The Netherlands*
Dpt. of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium
Dpt. of Mathematics and Data Science, Vrije Universiteit Brussel, Belgium
E-mail: a.abiad.monge@tue.nl
[2] *Dpt. of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium*
E-mail: Jozefien.Dhaeseleer@UGent.be
[3] *Dpt. of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium*
E-mail: Robin.Simoens@UGent.be

Abstract

Can we characterize a graph by its spectrum? In 2003, van Dam and Haemers conjectured that the answer is positive for almost all graphs [1]. This conjecture, which plays a special role in the graph isomorphism problem, has only been solved for some specific families of graphs. In this talk, we use a switching method to prove that the answer is negative for some graph classes in the Johnson and Grassman schemes.

References

[1] E.R. van Dam and W.H. Haemers. Which graphs are determined by their spectrum?. *Linear Algebra Appl.* **373** (2003), 241–272.

Semidefinite and eigenvalue bounds for bicliques and biidependent sets

Monique Laurent[1], *Sven Polak*[2], *Luis Felipe Vargas*[3]

[1] *Centrum Wiskunde & Informatica and Tilburg University, The Netherlands*
E-mail: monique.laurent@cwi.nl
[2] *Centrum Wiskunde & Informatica, The Netherlands*
E-mail: s.c.polak@cwi.nl
[3] *Centrum Wiskunde & Informatica, The Netherlands*
E-mail: luis.vargas@cwi.nl

Abstract

A biindependent pair in a bipartite graph $G = (V_1 \cup V_2, E)$ is a pair (A, B), where $A \subseteq V_1, B \subseteq V_2$ and the union $A \cup B$ is independent in G. The parameters $g(G)$ and $h(G)$ are defined, respectively, as the maximum product $|A| \cdot |B|$ and the maximum ratio $\frac{|A| \cdot |B|}{|A|+|B|}$ taken over biindependent sets in G. These parameters are NP-hard to compute and have applications for bounding the maximum product-free sets in groups and for bounding the nonnegative rank of a matrix. In this talk we define semidefinite programming bounds for $g(G)$ and $h(G)$ and show that they they can be seen as quadratic variations of the Lovász ϑ-number $\vartheta(G)$. In addition, we formulate a closed-form eigenvalue bounds, which coincide with the semidefinite bounds for edge-transitive graphs.

Acknowledgements: This work is partially supported by the European Union's Framework Programme for Research and Innovation Horizon 2020 under the Marie Skłodowska-Curie Actions Grant Agreement No. 813211 (POEMA).

References

[1] M. Laurent, S.C. Polak, L.F. Vargas, *Semidefinite bounds for bicliques and biindependent pairs*. Preprint, arXiv:2302.08886.

MSC03. Nonnegative matrices: spectral properties

Carlos Marijuán, Julio Moro

Connecting the Hermite-Biehler Theorem to the Nonnegative Inverse Eigenvalue Problem
Richard ELLARD, Helena SMIGOC

Nonnegative Jacobi matrix realizations in low dimension
Andres M. ENCINAS BACHILLER, M. José JIMÉNEZ, Carlos MARIJUÁN, Margarida MITJANA, Miriam PISONERO

Powers of Sparsest Matrices Realising the Karpelevic Arcs
Priyanka JOSHI, Helena SMIGOV, Stephen KIRKLAND

On Hankel matrices and the symmetric nonnegative inverse eigenvalue problem
Ana I. JULIO, Roberto C. DÍAZ, Timoteo HERRERA

More on polynomials preserving nonnegative matrices
Raphael LOEWY

Universal realizability on the border
Carlos MARIJUÁN

A combinatorial characterization of C-realizable lists in the nonnegative inverse eigenvalue problem
Julio MORO, Carlos MARIJUÁN

Šmigoc's glue for universal realizability on the left half-plane
Ricardo L SOTO, Jaime H. ALFARO

Connecting the Hermite-Biehler Theorem to the Nonnegative Inverse Eigenvalue Problem

Richard Ellard[1], *Helena Šmigoc*[2]

[1] *School of Mathematics and Statistics, Technological University Dublin, Ireland*
E-mail: richard.ellard@tudublin.ie
[2] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: helena.smigoc@ucd.ie

Abstract

The famous Hermite-Biehler Theorem [1, 2] states that a real polynomial $f(x) = p(x^2) + xq(x^2)$ is Hurwitz stable (all of the roots of f lie in the open left half-plane) if and only if the leading coefficients of p and q have the same sign and all the roots of $p(-x^2)$ and $xq(-x^2)$ are real and interlace. More generally, the number and relative positions of the nonnegative roots of $p(-x)$ and $q(-x)$ determine the number of roots of f which lie in the left (or right) half-plane.

The *Nonnegative Inverse Eigenvalue Problem* (NIEP) asks for a characterisation of those lists of complex numbers which are *realisable* as the spectrum of some (entrywise) nonnegative matrix. An important special case arises when the Perron eigenvalue is the only root of the characteristic polynomial f in the right half-plane, and, in this special case, a complete characterisation was given by Laffey and Šmigoc [3] which employed a rather long and technical argument.

By examining the relationship between the roots of f and those of p and q from a simple algorithmic perspective, we give a new—and perhaps more elegant—proof of the Laffey-Šmigoc characterisation which provides a deeper insight into the result.

Acknowledgements: Work (partially) supported by Science Foundation Ireland under grant 11/RFP.1/MTH/3157.

References

[1] C. Hermite. Sur les nombre des racines d'une équation algébrique comprise entre des limites données. *J. Reine Angew. Math*, 52: 39–51 (1856).

[2] M. Biehler. Sur une classe d'équations algébriques dont toutes les racines sont réelles. *J. Reine Angew. Math*, 87: 350–352 (1879).

[3] T. J. Laffey, H. Šmigoc. Nonnegative realization of spectra having negative real parts. *Linear Algebra Appl.*, 416 (1): 148–159 (2006).

Nonnegative Jacobi matrix realizations in low dimension

Encinas, A.M.[1], *Jiménez, M.J.*[2], *Marijuán, C.*[3], *Mitjana, M.*[4], *Pisonero, M.*[5]

[1] *Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: andres.marcos.encinas@upc.edu
[2] *Dpt. 3, Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: maria.jose.jimenez@upc.edu
[3] *Dpt. 3, Dpt. Matemátiques, Universidad de Valladolid, Spain*
E-mail: cmarijuan@uva.es
[4] *Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: margarida.mitjana@upc.edu
[5] *Dpt. 3, Dpt. Matemátiques, Universidad de Valladolid, Spain*
E-mail: mpisonero@uva.es

Abstract

We denote by $\mathsf{J}(\mathsf{a},\mathsf{b})$ the real symmetric Jacobi matrix with main diagonal $\mathsf{a} = (a_1, \ldots, a_n)$ and secondary diagonals $\mathsf{b} = (b_1, \ldots, b_{n-1})$, where $\mathsf{b} > 0$. Our aims are to characterize the spectra of nonnegative irreducible symmetric Jacobi matrices of size $n \leq 6$ and also to obtain all realizations by Jacobi matrices of this type. Our work is strongly based on the characterizations given by S. Friedland and A.A. Melkman in their celebrated 1979 paper.

Acknowledgements: This work has been partly supported by the Spanish Research Council (Ministerio de Ciencia e Innovación) under project PID2021-122501NB-I00 and by the Universitat Politècnica de Catalunya under funds AGRUP-UPC.

Powers of Sparsest Matrices Realising the Karpelevic Arcs

Priyanka Joshi[1], *Stephen Kirkland*[2], *Helena Šmigoc*[1]

[1] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: priyanka.joshi@ucdconnect.ie, helena.smigoc@ucd.ie
[2] *Department of Mathematics, University of Manitoba, Canada*
E-mail: Stephen.Kirkland@umanitoba.ca

Abstract

Karpelevič [1] described Θ_n, the region in the complex plane consisting of all eigenvalues of all stochastic matrices of order n. The boundary of Θ_n is a disjoint union of arcs, known as the Karpelevič arcs. Johnson and Paparella [2] considered the question of constructing stochastic matrices realising the boundary of the Karpelevič region. Kirkland and Šmigoc [3] characterized the sparsest realising matrices for the Karpelevič arcs of order n.

We study the powers of the sparsest realising matrices characterised in [3]. In particular, we find the necessary and sufficient conditions that a sparsest realising matrix associated with the Karpelevič arc of order n has to satisfy in order to be a power of another stochastic matrix. We present our results in terms of the digraphs associated with these sparsest realising matrices.

Acknowledgements: Work supported by Science Foundation Ireland (SFI) under Grant Number SFI 18/CRT/6049.

References

[1] F. Karpelevič *On the characteristic roots of matrices with nonnegative elements*. in Eleven Papers Translated from the Russian, Amer. Math. Soc. Transl. (2) 140:79-100, (1988).

[2] C. Johnson and P. Paparella. *A matricial view of the Karpelevič theorem.* Linear Algebra Appl., 520 (2017), 1–15.

[3] S. Kirkland and H. Šmigoc. *Stochastic matrices realising the boundary of the Karpelevič region.* Linear Algebra Appl., 635 (2022), 116–138.

More on polynomials preserving nonnegative matrices

Raphael Loewy[1]

[1] *Dept. of Mathematics, Technion-Israel Institute of Technology, Israel*
E-mail: loewy@technion.ac.il

Abstract

Let n be an arbitrary positive integer. We consider polynomials preserving the set of $n \times n$ nonnegative (elementwise) matrices. Let

$$\mathcal{P}_n = \{p \in \mathbb{C}[x] : p(A) \geq 0 \text{ for all } A \geq 0, A \in \mathbb{R}^{n,n}\}.$$

Loewy and London [4] defined $\mathcal{P}_n$, motivated by its connection to the well known Nonnegative Inverse Eigenvalue Problem (NIEP). Indeed, if a list $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ of complex numbers is the spectrum of an $n \times n$ nonnegative matrix A, then the list $p(\Lambda) := (p(\lambda_1), p(\lambda_2), \ldots, p(\lambda_n))$ is the spectrum of the nonnegative matrix $p(A)$, provided $p \in \mathcal{P}_n$. But the investigation of $\mathcal{P}_n$ is of independent interest.

The following are clear:(i) If $p \in \mathcal{P}_n$ then all its coefficients are real. (ii) If all coefficients of p are nonnegative then $p \in \mathcal{P}_n$. (iii) $\mathcal{P}_{n+1} \subseteq \mathcal{P}_n$. Clark and Paparella [1, 2] conjectured that $\mathcal{P}_{n+1} \subset \mathcal{P}_n$, and proved it for $n = 1, 2$. Loewy [3] proved this conjecture by showing that there exists $a = a(n) > 0$ such that $1 + x + x^2 + \cdots + x^{n-1} - ax^n + x^{n+1} + \cdots + x^{2n}$ is in $\mathcal{P}_n$ but not in $\mathcal{P}_{n+1}$. It turns out in the proof that $a(n) \to 0$ *as* $n \to \infty$, and it is desirable to find $a > 0$ that is independent of n. In this talk we show such a exists.

To restrict ourselves to finite dimensional spaces, let m be a positive integer and define $\mathcal{P}_{n,m} = \{p \in \mathcal{P}_n : degree(p) \leq m\}$. Then, $\mathcal{P}_{n,m}$ can be considered as a closed, convex cone in $\mathbb{R}^{m+1}$. It is known that a polynomial $p \in \mathcal{P}_{n,2n}$ can have only one negative coefficient, namely that of x^n. We discuss the possible number of negative coefficients of $p \in \mathcal{P}_{n,m}$ for $m > 2n$.

Finally, it is of interest to know whether $\mathcal{P}_{n,m}$ is a polyhedral cone. We show that $\mathcal{P}_{2,m}$ is nonpolyhedral for every $m \geq 4$.

References

[3] R. Loewy. Proof of a conjecture on polynomials preserving nonnegative matrices. *arXiv:2022.00110*.

[4] R. Loewy and D. London, A note on an inverse problem for nonnegative matrices. *Linear and Multilinear Algebra*, 6:83-90 (1978/79).

Universal Realizability on the border

Carlos Marijuán

Dpt. Matemática Aplicada, University of Valladolid/IMUVA, Spain
E-mail: cmarijuan@uva.es

Abstract

The *Nonnegative Inverse Eigenvalue Problem* (NIEP) consists of the characterization of the lists of complex numbers that are spectra of nonnegative matrices. We say that a list $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ is *realizable* if it is the spectrum of a nonnegative matrix. We say that the realizable list Λ is *universally realizable* if, for every possible Jordan canonical form allowed by Λ, there is a nonnegative matrix with spectrum Λ. The *Universal Realizability Problem* (URP) consists of the characterization of the lists that are universally realizable.

In terms of n, the NIEP is completely solved only for $n \leq 4$, and for $n = 5$ with trace zero. It is clear that for $n \leq 3$ the concepts of universally realizable and realizable are equivalent. The URP is also solved for $n \leq 4$ and for $n = 5$ with trace zero in the real case, and the solutions are different to the NIEP. In this talk we study the universal realizability of nonreal spectra of size 5 with trace zero on the border of realizability. We use techniques from Graph Theory and Linear Algebra.

Acknowledgements: W ork supported by grant PGC2018-096446-B-C21 funded by MCIN/AEI/ 10.13039/501100011033, "ERDF A way of making Europe"

References

[1] J. Torre-Mayo, M.R. Abril-Raymundo, E. Alarcia-Estévez, C. Marijuán, M. Pisonero. The nonnegative inverse eigenvalue problem from the coefficients of the characteristic polynomial, EBL digraphs. *Linear Algebra Appl.*, 426: 729-773 (2007).

[2] A. I. Julio, C. Marijuán, M. Pisonero, R. L. Soto. On universal realizability of spectra, *Linear Algebra Appl.* 563 353–372 (2019).

[3] A. I. Julio, C. Marijuán, M. Pisonero, R. L. Soto. Universal realizability in low dimension, *Linear Algebra Appl.* 619: 107–136 (2021) .

[4] C. Marijuán, R. L. Soto. How diagonalizably realizable implies universally realizable. *Submited* (2023).

A combinatorial characterization of C-realizable lists in the nonnegative inverse eigenvalue problem

Julio Moro[1], *Carlos Marijuán*[2]

[1] *Departamento de Matemáticas, Universidad Carlos III de Madrid, Leganés (Spain)*
E-mail: jmoro@math.uc3m.es
[2] *Departamento de Matemáticas, Universidad de Valladolid/IMUVA, Valladolid (Spain)*
E-mail: cmarijuan@uva.es

Abstract

The Real (respectively, Symmetric) Nonnegative Inverse Eigenvalue Problem (RNIEP - respectively, SNIEP) deals with characterizing the possible real spectra of entrywise nonnegative (respectively, symmetric nonnegative) matrices. Any list of real numbers which is the spectrum of an entrywise nonnegative (respectively, symmetric nonnegative) matrix is said to be realizable (respectively, symmetrically realizable).

Among all realizable lists a subclass has been identified as those 'realizable by compensation' (in short, C-realizable), which was shown in [1] to include most of the subclasses associated with sufficient realizability conditions known so far in the RNIEP. Later on, it was proved in [2] that any C-realizable list is in particular symmetrically realizable.

In this talk we present a combinatorial characterization of C-realizable lists, first for the special case of zero-sum lists [3], and then for arbitrary ones. One of the consequences of this characterization is that the set of zero-sum C-realizable lists is the union of polyhedral cones whose faces are described by equations involving only linear combinations with coefficients 1 and -1 of the entries in the list. Lists with positive sum are C-realizable if and only if there exists a shifted version with zero sum satisfying the aforementioned conditions.

Acknowledgements: Work (partially) supported by Grant PGC2018-096446-B-C21 funded by MCIN/AEI/10.13039/501100011033, and Grants MTM2017-84098-P and MTM2017-90682-REDT of the spanish Ministerio de Economía y Competitividad.

References

[1] A. Borobia, J. Moro and R. Soto A unified view on compensation criteria in the real nonnegative inverse eigenvalue problem, *Linear Algebra Appl.*, 428: 2574—2584 (2008).

[2] R. Ellard and H. Šmigoc, Connecting sufficient conditions for the Symmetric Nonnegative Inverse Eigenvalue Problem, *Linear Algebra Appl.*, 498: 521–552.

[3] C. Marijuán and J. Moro, A characterization of trace-zero sets realizable by compensation in the SNIEP, *Linear Algebra Appl.*, 615: 42-–76 (2021), DOI: 10.1016/j.laa.2020.12.021

Šmigoc's glue for universal realizability on the left half-plane

Ricardo L. Soto[1], *Jaime H. Alfaro*[2]

[1] *Dpt. Matemáticas, Universidad Católica del Norte, Chile*
E-mail: rsoto@ucn.cl
[2] *Dpt. Matemáticas, Universidad Católica del Norte, Chile*
E-mail: jaime.alfaro@ucn.cl

Abstract

A list $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ of complex numbers is said to be *realizable* if it is the spectrum of a nonnegative matrix. Λ is said to be *universally realizable* ($\mathcal{UR}$) if it is realizable for each possible Jordan canonical form allowed by Λ. In this paper, using companion matrices and applying a procedure by Šmigoc, is provided a sufficient condition for the *universal realizability* of left half-plane spectra, that is, $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ with $\lambda_1 > 0$, $Re\lambda_i \leq 0$, $i = 2, \ldots, n$. It is also shown how the effect of adding a negative real number to a not $\mathcal{UR}$ left half-plane list of complex numbers, makes the new list $\mathcal{UR}$, and a family of left half-plane lists that are $\mathcal{UR}$ is characterized.

Acknowledgements: Work (partially) supported by Universidad Católica del Norte-VRIDT 036-2020, NÚCLEO 6 UCN VRIDT-083-2020, Chile.

References

[1] M. Collao, M. Salas, R. L. Soto, *Spectra universally realizable by doubly stochastic matrices*, Special Matrices 6 (2018) 301-309.

[2] R. C. Díaz, R. L. Soto, *Nonnegative inverse elementary divisors problem in the left half-plane*, Linear Multilinear Algebra 64 (2016) 258-268.

[3] C. R. Johnson, A. I. Julio, R. L. Soto, *Nonnegative realizability with Jordan structure*, Linear Algebra Appl. 587 (2020) 302-313.

[4] T. J. Laffey, H. Šmigoc, *Nonnegative realization of spectra having negative real parts*, Linear Algebra Appl. 416 (2006) 148-159.

[5] H. Minc, *Inverse elementary divisor problem for nonnegative matrices*, Proc. of the Amer. Math. Society 83 (1981) 665-669.

[6] H. Šmigoc, *The inverse eigenvalue problem for nonnegative matrices*, Linear Algebra Appl. 393 (2004) 365-374.

[7] R. L. Soto, R. C. Díaz, H. Nina, M. Salas, *Nonnegative matrices with prescribed spectrum and elementary divisors*, Linear Algebra Appl. 439 (2013) 3591-3604.

MSC04. Green's function on networks and its applications

Ángeles Carmona,
Andrés M. Encinas,
María José Jiménez,
Margarida Mitjana

Green's functions every where
Ángeles Carmona Mejías, Andrés M. Encinas, María José Jiménez

Capacity on graphs: submodularity and simplex geometry
Karel Devriendt

Generalized diffusion equation on graphs/networks
Fernando Diaz-Diaz, Ernesto Estrada

A Riesz Decomposition Theorem for Schrödinger Operators on Graphs
Florian Fischer

Combinatorial inversion in edge-weighted unicyclic graphs
Daniel A. Jaume, Cristian Panelo

Group Inverse and equilibrium measure on Random Walks
Àlvar Martín Llopis, Ángeles Carmona, Maria José Jiménez, Andrés Encinas

Recovering piecewise constant conductances on networks with boundary
Álvaro Samperio, Ángeles Carmona, Maria José Jiménez, Andrés Encinas
Spectral Gap Problems of Periodic Jacobi Operators
Kumar V B Kiran

Green's functions every where

Carmona, Á.[1], *Encinas, A.M.*[2], *Jiménez, M.J.*[3]

[1] *Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: angeles.carmona@upc.edu
[2] *Dpt. 3, Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: andres.marcos.encinas@upc.edu
[3] *Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: maria.jose.jimenez@upc.edu

Abstract

Green's function has been a powerful tool for solving differential equations or partial differential equations with boundary conditions since the work of George Green. Green's function is the resolvent kernel for problems raised in terms of the Laplace operator. In the discrete setting; i.e., when considering networks, kernels can be seem as matrices and hence the Green's function is nothing more than the group inverse (singular case) or the inverse (nonsingular case) of the Laplacian matrix. Hence, it is crucial to know both properties and expressions for Green's functions in order to understand the properties fulfilled by solution of the raised problems. For instance, in the setting of networks Green's functions appear in relation with discrete vector calculus, random walks, machine learning, pagerank problems, and so on. We will review some of the main properties and goodness of Green's functions.

Acknowledgements: This work has been partly supported by the Spanish Research Council (Ministerio de Ciencia e Innovación) under project PID2021-122501NB-I00 and by the Universitat Politècnica de Catalunya under funds AGRUP-UPC.

Capacity on graphs: submodularity and simplex geometry

Karel Devriendt[1]

[1] *Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany*
E-mail: karel.devriendt@mis.mpg.de

Abstract

Capacity is an important concept in potential theory and electrical engineering. It is a non-additive generalization of a measure and captures both the "size" of a set and its "shape".
The definition of capacity can be generalized to a graph $G = (V, E)$ based on the Dirichlet form $\mathcal{E}(f) := \sum_E [f(u) - f(v)]^2$ where the sum runs over all adjacent vertices $\{u, v\} \in E$ in the graph and $f : V \to \mathbb{R}$ is a function on the vertices. The capacity $\text{cap}(A, B)$ between two disjoint non-empty sets $A, B \subset V$ is defined as

$$\text{cap}(A, B) := \min \{\mathcal{E}(f) \ : \ f(A) = 0, f(B) = 1\} .$$

Song et al. [1] studied the reciprocal of $\text{cap}(A, B)$ as a generalization of the effective resistance and showed, for instance, how to calculate the capacity using Kron reduction or based on the Moore–Penrose pseudoinverse Laplacian of a modified graph — this follows because $\mathcal{E}(f)$ is the quadratic form determined by the graph Laplacian.

A first new result discussed in this talk is the following:

Theorem 1 (Capacity is submodular)

$$\text{cap}(T, A \cup B) \leq \text{cap}(T, A) + \text{cap}(T, B) - \text{cap}(T, A \cap B) \text{ for all } A, B \subseteq V \backslash T. \quad (1)$$

For a fixed set T, the function $\text{cap}(T, \cdot)$ is a function on subsets of $V \backslash T$, and inequality (1) says that this set function is submodular. Submodularity is a strong property with many applications, for instance related to optimization and multi-criteria decision making [2, 3].

A second new result relates capacity to the following result of Fiedler [4]: the vertices of a graph G can be embedded $\varphi : V \to \mathbb{R}^{|V|-1}$ such that $\text{cap}(a, b) = 1/\|\varphi(a) - \varphi(b)\|^2$ for all vertices $a, b \in V$, and the resulting points $\varphi(V)$ are the vertices of a simplex.

Theorem 2 (Capacity is inverse distance) *The capacity between two sets A, B is the inverse squared distance between the faces with vertices $\varphi(A), \varphi(B)$:*

$$\text{cap}(A, B) = \frac{1}{d^2(\varphi(A), \varphi(B))},$$

where $d(\varphi(A), \varphi(B))$ is the Euclidean distance between the affine subspaces generated by the points $\varphi(A), \varphi(B)$.

References

[1] Y. Song, D. J. Hill, T. Liu. On extension of effective resistance with application to graph Laplacian definiteness and power network stability. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 66(11): 4415 - 4428 (2019)

[2] L. Lovász. Submodular functions and convexity. In: *A. Bachem, B. Korte, M. Grötschel (eds) Mathematical Programming The State of the Art*, Springer, Berlin–Heidelberg, 1983.

[3] M. Grabisch *Set Functions, Games and Capacities in Decision Making*. Springer, Cham, 2016.

[4] M. Fiedler *Matrices and Graphs in Geometry*. Cambridge University Press, Cambridge, 2011.

Generalized diffusion equation on graphs/networks

Fernando Diaz-Diaz[1], *Ernesto Estrada*[2]

[1] *Institute of Cross-Disciplinary Physics and Complex Systems, IFISC (UIB-CSIC), 07122 Palma de Mallorca, Spain*
E-mails: fernandodiaz@ifisc.uib-csic.es
estrada@ifisc.uib-csic.es

Abstract

Diffusion is an ubiquitous phenomenon present in many physical, biological and social systems. However, in the last decades, researchers have discovered many examples of so-called *anomalous diffusion*, arising because of the presence of long range interactions or memory effects. Anomalous diffusion is characterized by the presence of power laws in the time evolution of full-width at half-maximum and and the maximum probability ($P_{max} \propto t^{-\gamma}$), such that $\gamma < 0.5$ can be identified with subdiffusive systems and $\gamma > 0.5$ with superdiffusive ones.

Several tools have been derived to analyze anomalous diffusion, many of them based on continuous time random walks (CTRW) and fractional diffusion equations [1]. However, most of these approaches neglected the networked structure of many of these systems. To overcome this limitation of previous models, we present here, based on the results published in [2], a generalized diffusion equation for networks, using Caputo time-fractional derivatives to model memory effects and d-path Laplacian operators [3] to model long-range interactions. We analytically proved that the solution of this equation is able to recreate diffusive, subdiffusive and superdiffusive scenarios, and found the parameter regimes where the different types of anomalous diffusion appear. We also performed computational simulations that confirm the presence of super-, sub- and diffusive regimes. Finally, we tested the practical applicability of our generalized diffusion equation by modelling the movement of proteins through a DNA chain.

References

[1] R. Metzler and J. Klafter, *The random walk's guide to anomalous diffusion: a fractional dynamics approach*, Physics Reports, vol. 339, pp. 1–77, Dec. 2000.

[2] F. Diaz-Diaz and E. Estrada, *Time and space generalized diffusion equation on graph/networks*, Chaos, Solitons and Fractals, vol. 156, p. 111791, Mar. 2022.

[3] E. Estrada, E. Hameed, N. Hatano, and M. Langer, *Path Laplacian operators and superdiffusive processes on graphs*. I. One-dimensional case," Linear Algebra and its Applications, vol. 523, pp. 307–334, June 2017. arXiv: 1604.00555.

A Riesz Decomposition Theorem for Schrödinger Operators on Graphs

Florian Fischer[1]

[1] *Institute of Mathematics, University of Potsdam, Germany*
E-mail: florifis@uni-potsdam.de

Abstract

In the classical potential theory on the Euclidean space and in the potential theory of transient Markov chains a unique decomposition of superharmonic functions into a harmonic and a potential part is well-known. In this talk the basic concepts and ideas to gain such a decomposition for non-negative Schrödinger operators on weighted infinite graphs will be shown. The talk is based on joint work with Matthias Keller, see [1].

Acknowledgements: F. F. thanks the Heinrich-Böll-Stiftung for the support.

References

[1] Florian Fischer and Matthias Keller. Riesz decompositions for Schrödinger operators on graphs. *J. Math. Anal. Appl.*, 495(1):22, 2021.

Combinatorial inversion in edge-weighted unicyclic graphs

Daniel A. Jaume[1], *Cristian Panelo*[2]

[1] *Universidad Nacional de San Luis. Argentina*
E-mail: djaume@unsl.edu.ar
[2] *Universidad Nacional de San Luis. Argentina*
E-mail: crpanelo@unsl.edu.ar

Abstract

Combinatorial expressions of the inverse of the adjacency matrix non-singular unicyclic graph were given in [1]. In this work, we extend those results to edge-weighted unicyclic graphs.

Based in the characterization of non-singular unicyclic graphs was given by Guo, Yan and Yeh, see [2], we give three families of edge-weighted unicyclic graphs, called Class WI, Class WIIa and Class WIIb, and find a combinatorial formula for the inverse of adjacency matrix in each family. The inverse of an edge-weighted non-singular unicyclic graph U is the sum of the matrix $WInv_1(U)$ and, if necessary, a correction matrix, $WInv_2(U)$ or $WInv_3(U)$, depending on the class to which the underlying unicyclic graph of U belongs. We show that an edge-weighted unicyclic graph of Class WIIb can be singular.

We prove that the minimum rank of the family of zero-diagonal matrices of order n, whose underlying graph is a non-singular unicyclic graph, is n if its underlying graph is of Class I or IIa, and is $n-2$ if its underlying graph is of Class IIb. We prove that the singularity of a matrix whose underlying graph is of Class IIb is forest stable and cyclic unstable.

Acknowledgements: This work was partially supported by the Universidad Nacional de San Luis, Argentina, grant PROICO 03-0918, MATH AmSud, grant 21-MATH-05, Agencia I+D+i, Argentina, grants PICT 2020-Serie A-00549 and PICT-2021-CAT-II-00105.

References

[1] Jaume, Daniel A and Panelo, Cristian and Toledo, Maikon Machado and Molina, Gonzalo. *On the structure of the inverse of non-singular unicyclic graphs*. Discrete Applied Mathematics, Vol. 321, pag 316 – 332, 2022.

[2] Guo, Ji-Ming and Yan, Weigen and Yeh, Yeong-Nan. *On the nullity and the matching number of unicyclic graphs*. Linear Algebra and its applications, Vol. 431, pag 1293 – 1301, 2009.

Group Inverse and equilibrium measure on Random Walks

Carmona, Á.[1], *Jiménez, M.J.*[2], *Encinas, A.M.*[3], *Martín, À.*[4]

[1] *Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: angeles.carmona@upc.edu
[2] *Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: maria.jose.jimenez@upc.edu
[3] *Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: andres.marcos.encinas@upc.edu
[4] *Dpt. Matemàtiques, Universitat Politècnica de Catalunya, Spain*
E-mail: alvar.martin@upc.edu

Abstract

In this manuscript we show the central role of the group inverse of the Laplacian in the study of random walks on networks. Moreover, we take advantage of the relation of group inverse and equilibrium measures and we obtain expressions for the mean first passage time and for Kemeny's constant in terms of equilibrium measures. For networks with symmetries we can obtain the analytic expression of the above parameters such as distance bi-regular graphs or barbell networks.

Acknowledgements: This work has been partly supported by the Spanish Research Council (Ministerio de Ciencia e Innovación) under project PID2021-122501NB-I00 and by the Universitat Politècnica de Catalunya under funds AGRUP-UPC.

References

[1] Á. Carmona, A.M. Encinas, M.J. Jiménez, À. Martín. Mean First Passage Time and Kemeny's constant using generalized inverses of the combinatorial Laplacian. *Submitted*, (2022).

[2] E. Bendito, et al. A formula for the Kirchhoff index. *International journal of quantum chemistry* 108.6 (2008): 1200–1206.

Recovering piecewise constant conductances on networks with boundary

A. Carmona[1], *A. M. Encinas*[2], *M. J. Jiménez*[3], *Á. Samperio*[4]

[1] *Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, Spain*
E-mail: angeles.carmona@upc.edu
[2] *Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, Spain*
E-mail: andres.marcos.encinas@upc.edu
[3] *Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, Spain*
E-mail: maria.jose.jimenez@upc.edu
[4] *IMUVA (Instituto de Investigación en Matemáticas), Universidad de Valladolid, Spain*
E-mail: alvaro.samperio@uva.es

Abstract

The problem of recovering the conductances of a well-connected spider network with boundary from its Dirichlet-to-Robin map is ill-posed for large networks, so despite there is an exact algorithm to solve it [1], the resulting network is very different from the original one. This problem is the discrete analogous to Calderon's Inverse Problem, in which knowing a-priori that the conductivity is piecewise constant with a bounded number of unknown values makes the problem Lipschitz stable [2].

We propose to introduce the hypothesis analogous for the discrete problem that the conductances are constant in each element of a partition of the set of edges with a small number of elements and we formulate the problem as a polynomial optimization one, in which we minimize the difference between the Dirichlet-to-Robin map of the recovered network and the given one plus a term which penalizes the deviation from this hypothesis. We show examples in which we are able to accurately recover the conductances solving this problem.

Acknowledgements: [123]The authors were supported by the Research Project PID2021-122501NB-I00. [4]The author was supported by a FPI grant of the Research Project PGC2018-096446-B-C21 (with the help of FEDER Program).

References

[1] C. Araúz, A. Carmona, A.M. Encinas. Overdetermined partial boundary value problems on finite networks. *Journal of Mathematical Analysis and Applications*, 423: 191–207 (2015).

[2] G. Alessandrini, S. Vessella. Lipschitz stability for the inverse conductivity problem. *Advances in Applied Mathematics.*, 35: 207–241 (2005).

Spectral Gap Problems of Periodic Jacobi Operators

V B Kiran Kumar

Department of Mathematics, Cochin University of Science And Technology, Kochi, India
E-mail: vbk@cusat.ac.in

Abstract

Periodic Jacobi operators arise naturally in many quantum mechanics problems and form an important class of operators, both in pure theoretical interests and in many applied mathematics problems. We can formulate the well-known inverse spectral theory problems and spectral gap issues of such operators in terms of the spectral theory of block Toeplitz operators. We consider the classical result proved by Göran Borg in 1946[1], and its discrete versions. This theorem states that the periodic potential of the one-dimensional Schrödinger operator is constant almost everywhere if and only if its spectrum is connected. The discrete version and generalizations of the result were also known since 1975 (see [2] for, eg.). Such results are referred to as Borg-type theorems. In this talk, I will present a short survey on the recent developments in this area, including the pseudospectral analogues.

Acknowledgements: Work (partially) supported by Kerala State Council for Science, Technology and Environment via the KSCSTE YSA Project.

References

[1] Borg, Göran, *Eine Umkehrung der Sturm-Liouvilleschen Eigenwertaufgabe. Bestimmung der Differentialgleichung durch die Eigenwerte.*(German) Acta Math. 78 (1946), 1–96.

[2] H. Flaschka, *Discrete and periodic illustrations of some aspects of the inverse method*, Lecture Notes in Physics 38 (1975), 441–466.

[3] L. Golinskii, V. B. Kiran Kumar, M.N.N. Namboodiri, S. Serra-Capizzano, *A note on a discrete version of Borg's Theorem via Toeplitz-Laurent operators with matrix-valued symbols*, Bulletino U.M.I(9) VI(2013), 205-218.

[4] V. B. Kiran Kumar, M.N.N. Namboodiri, S. Serra-Capizzano, *Perturbation of operators and approximation of spectrum*, Proc. Indian Acad. Sci. (Math. Sci.) Vol. 124(2), 2014, 205–224.

[5] V. B. Kiran Kumar, Krishna Kumar, G. *A Note on Discrete Borg-type theorems*, Linear Multilinear Algebra 66(7), 2017,1418–1433

[6] V. B. Kiran Kumar, Krishna Kumar, G. *On Stability in the Borg's theorem for Non-normal Periodic Jacobi Operators*(submitted)

MSC05. Bounded rank perturbations in matrix theory and related problems

Itziar Baragaña, Froilán M. Dopico, Alicia Roca

Generic Hermitian matrix pencils with bounded rank
Fernando De Terán, Andrii Dmytryshyn, Froilán M. Dopico

Generic skew-symmetric matrix polynomials with bounded rank and fixed even grade
Andrii Dmytryshyn, Fernando De Terán, Froilán M. Dopico

Minimal rank perturbations of matrix pencils
Marija Dodig, Marko Stosic

Minimal rank factorizations of low rank polynomial matrices
Froilán Dopico, Andrii Dmytryshyn, Paul Van Dooren

Rank-one eigenvalues assignment for Hermitian matrix pencils
Hannes Gernandt, Carsten Trunk

Small rank perturbations of H-expansive matrices
Dawie Janse van Rensburg, Gilbert Groenewald, Andre Ran

Weyr characteristics perturbation results for matrix pencils
Francisco Martínez Pería, Hannes Gernandt, Friedrich Philipp, Carsten Trunk

Solving singular generalized eigenvalue problems: perturbation, projection and structure preservation
Christian Mehl, Michiel Hochstenbach, Bor Plestenjak

Stabilization of port-Hamiltonian systems by low rank output feedback
Volker Mehrmann

Eigenvalues of rank one perturbations of singular M-matrices
Andre Ran

Rank-one perturbation of linear relations via matrix pencils
Alicia Roca, Itziar Baragaña

Combinatorics in matrix pencils completion and rank perturbation problems
Marko Stosic, Marija Dodig

Kernel and range representation of matrix pencils
Carsten Trunk

An interlacing result for Hermitian matrices in Minkowski space
Madelein van Straaten, Dawie Janse van Rensburg, André Ran

Jordan-like decompositions of linear relations
Henrik Winkler

Rank one perturbations of matrices with applications in graph theory
Michał Wojtylak, Andre Ran, Marek Skrzypczyk

Generic Hermitian matrix pencils with bounded rank

Fernando De Terán[1], *Andrii Dmytryshyn*[2], *Froilán M. Dopico*[3]

[1] *Departamento de Matemáticas, Universidad Carlos III de Madrid, Spain*
E-mail: fteran@math.uc3m.es
[2] *School of Science and Technology, Örebro University, Sweden*
E-mail: andrii.dmytryshyn@oru.se
[3] *Departamento de Matemáticas, Universidad Carlos III de Madrid, Spain*
E-mail: dopico@math.uc3m.es

Abstract

In this talk, we will describe the generic complete eigenstructures of complex Hermitian $n \times n$ matrix pencils with rank at most r (with $r \leq n$). To do this, we prove that the set of such pencils is the union of a finite number of bundle closures, where each bundle is the set of complex Hermitian $n \times n$ pencils with the same complete eigenstructure (up to the specific values of the finite eigenvalues). We also obtain the explicit number of such bundles and their codimension. The cases $r = n$, corresponding to general Hermitian pencils, and $r < n$ exhibit surprising differences, since for $r < n$ the generic complete eigenstructures can contain only real eigenvalues, while for $r = n$ they can contain real and non-real eigenvalues. Moreover, we will see that the sign characteristic of the real eigenvalues plays a relevant role for determining the generic eigenstructures of Hermitian pencils.

Acknowledgements: Work (partially) supported by the Agencia Estatal de Investigación of Spain through grant PID2019-106362GB-I00/AEI/10.13039/501100011033 and by the Madrid Government (Comunidad de Madrid-Spain) under the Multiannual Agreement with UC3M in the line of Excellence of University Professors (EPUC3M23), and in the context of the V PRICIT (Regional Programme of Research and Technological Innovation), and by the Swedish Research Council (VR) grant 2021-05393.

Generic skew-symmetric matrix polynomials with bounded rank and fixed even grade

Fernando De Terán[1], *Andrii Dmytryshyn*[2], *Froilán M. Dopico*[1]

[1] *Departamento de Matemáticas, Universidad Carlos III de Madrid, Avenida de la Universidad 30, 28911, Leganés, Spain.*
E-mail: fteran@math.uc3m.es, dopico@math.uc3m.es
[2] *School of Science and Technology, Örebro University, 701 82, Örebro, Sweden.*
E-mail: andrii.dmytryshyn@oru.se

Abstract

Generic matrix polynomials with symmetries and bounded rank are known only in the case when their grade is odd [1, 2]. In this presentation, we adress the even-grade case. To be exact, we show that the set of $m \times m$ complex skew-symmetric matrix polynomials of even grade d, i.e., of degree at most d, and (normal) rank at most $2r$ is the closure of the single set of matrix polynomials with the certain, explicitly described, complete eigenstructure. This complete eigenstructure corresponds to the most generic $m \times m$ complex skew-symmetric matrix polynomials of even grade d and rank at most $2r$.

Acknowledgements: The work of A. Dmytryshyn was supported by the Swedish Research Council (VR) grant 2021-05393. The work of F. De Terán and F. M. Dopico has been partially funded by the Agencia Estatal de Investigación of Spain through grants PID2019-106362GB-I00 MCIN/ AEI/ 10.13039/ 501100011033/ and by the Madrid Government (Comunidad de Madrid-Spain) under the Multiannual Agreement with UC3M in the line of Excellence of University Professors (EPUC3M23), and in the context of the V PRICIT (Regional Programme of Research and Technological Innovation).

References

[1] F. De Terán, A. Dmytryshyn, and F. M. Dopico. Generic symmetric matrix polynomials with bounded rank and fixed odd grade. *SIAM Journal on Matrix Analysis and Applications*, 41(3): 1033–1058 (2020).

[2] A. Dmytryshyn and F. M. Dopico. Generic skew-symmetric matrix polynomials with fixed rank and fixed odd grade. *Linear Algebra and its Applications*, 536:1–18, (2018).

Minimal rank perturbations of matrix pencils

Marija Dodig [1,3], *Marko Stošić* [2,3]

[1] *Departamento de Matemática, Faculdade de Ciências da Universidade de Lisboa, Lisbon, Portugal*
E-mail: msdodig@fc.ul.pt
[2] *CAMGSD, Departamento de Matemática, Instituto Superior Técnico, Lisbon, Portugal*
[3] *Mathematical Institute SANU, Serbia*

Abstract

In this talk we will present the connection between bounded rank perturbations of matrix pencils, and the general matrix pencil completion problem. We shall discuss situations in which double general matrix pencil completion problem can be reached. In particular, the idea of minimality in both problems will be explained. Recent result on bounded rank perturbations of matrix pencils without non-trivial homogeneous invariant factors will be given as an example.

Acknowledgements: Work partially supported by Portuguese Fundação para a Ciência e a Tecnologia (FCT), project no. EXPL/MAT-PUR/0584/2021.

Minimal rank factorizations of low rank polynomial matrices

Andrii Dmytryshyn[1], *Froilán Dopico*[2], *Paul Van Dooren*[3]

[1] *School of Science and Technology, Örebro University, Örebro, Sweden*
E-mail: andrii.dmytryshyn@oru.se
[2] *Departamento de Matemáticas, Universidad Carlos III de Madrid, Leganés, Spain*
E-mail: dopico@math.uc3m.es
[3] *Department of Mathematical Engineering, Université catholique de Louvain, Louvain-la-Neuve, Belgium*
E-mail: paul.vandooren@uclouvain.be

Abstract

The description of the sets of matrix pencils with bounded rank r was presented in [1] and [3] from two different perspectives. In [1], these sets are described as the union of the closures of a small number of sets with certain Kronecker canonical forms. In contrast, in [3], the same sets are described in a completely explicit way as the union of certain sets of pencils expressed as the product of two pencils with rank r and with specified grades for the columns of the first factor and the rows of the second factor. Such a explicit product description was fundamental for characterizing the generic change of the partial multiplicities of regular matrix pencils under low-rank perturbations [2]. In the case of the sets of polynomial matrices of bounded rank and bounded degree, a description similar to that in [1] in terms of a few generic eigenstructures was presented in [4], but a description in terms of a product of two polynomial matrices with rank r and with specified grades for their columns and rows, respectively, is not yet available in the literature. We present such a description in this talk.

Acknowledgements: Work partially supported by the Agencia Estatal de Investigación of Spain through grant PID2019-106362GB-I00 of MCIN/AEI/ 10.13039/501100011033/ and by the Swedish Research Council (VR) under grant 2021-05393.

References

[1] F. De Terán and F.M. Dopico, *A note on generic Kronecker orbits of matrix pencils with fixed rank*, SIAM Journal on Matrix Analysis and Applications, 30 (2008), pp. 491-496.

[2] F. De Terán and F. M. Dopico. *Generic change of the partial multiplicities of regular matrix pencils under low-rank perturbations*, SIAM J. Matrix Anal. Appl., 37 (2016) 823–835.

[3] F. De Terán, F.M. Dopico, and J.M. Landsberg, *An explicit description of the irreducible components of the set of matrix pencils with bounded normal rank*, Linear Algebra and its Applications, 520 (2017), pp. 80-103.

[4] A. Dmytryshyn and F.M. Dopico, *Generic complete eigestructures for sets of matrix polynomials with bounded rank and degree*, Linear Algebra and its Applications, 535 (2017), pp. 213-230.

Rank-one eigenvalue assignment for Hermitian matrix pencils

Hannes Gernandt [1] *and Carsten Trunk*[2]

[1] *Brandenburgische Unversität Cottbus-Senftenberg, Germany*
E-mail: hannes.gernandt@ieg.fraunhofer.de
[2] *Technische Universität Ilmenau, Germany*
E-mail: carsten.trunk@tu-ilmenau.de

Abstract

Eigenstructure assignment using low-rank perturbations problems is useful in many control engineering applications and has recieved a lot of attention in recent years. In this talk, we focus on eigenvalue assignment of regular Hermitian matrix pencils of the form

$$\lambda E - A + (\alpha\lambda - \beta)uu^H, \quad E = E^H \in \mathbb{C}^{n\times n},\ A = A^H \in \mathbb{C}^{n\times n},\ \alpha, \beta \in \mathbb{R},\ u \in \mathbb{C}^n.$$

In contrast to the unstructured case, i.e. when E and A are not necessarily Hermitian it is known that arbitrary eigenvalue configurations are possible after a single unstructured rank-one perurtbation [1].

Using the Thompson canonical form and by analyzing the characteristic signs, we are able to determine the possible eigenvalue configurations after Hermitian rank-one perturbations.

References

[1] H. Gernandt, C. Trunk. Eigenvalue placement for regular matrix pencils under rank-one perturbations. *SIAM J. Matrix Anal. Appl.*, 38: 134–154 (2017).

Small rank perturbations of H-expansive matrices

GJ Groenewald [1], *DB Janse van Rensburg*[1], *ACM Ran*[2]

[1] *Mathematics and Applied Mathematics, North-West University, South Africa*
E-mail: Gilbert.Groenewald@nwu.ac.za
E-mail: Dawie.Jansevanrensburg@nwu.ac.za
[2] *Faculteit Exacte Wetenschappen, Vrije Universiteit, The Netherlands*
E-mail: a.c.m.ran@vu.nl

Abstract

In this talk we will discuss small rank perturbations of H-expansive and H-unitary matrices paying particular attention to the location of eigenvalues of these matrices with respect to the unit circle.

Acknowledgements: Work (partially) supported by PAA

References

[1] . Mehl, V. Mehrmann, A.C.M. Ran, L. Rodman: Eigenvalue perturbation theory of symplectic, orthogonal, and unitary matrices under generic structured rank one perturbations. *BIT Numerical Mathematics* 54 (2014), 219–255.

[2] .H. Fourie, G.J. Groenewald, D.B. Janse van Rensburg, and A.C.M. Ran: Rank one perturbations of H-positive real matrices. *Linear Algebra and its Applications* 439 (2013), 653–674.

[3] .C.M. Ran, M. Wojtylak: Global properties of eigenvalues of parametric rank one perturbations for unstructured and structured matrices. *Complex Analysis and Operator Theory*, (2021) 15:44

Weyr characteristics perturbation results for matrix pencils

Hannes Gernandt[1], *Francisco Martínez Pería*[2], *Friedrich Philipp*[3], *Carsten Trunk*[4]

[1] *Technische Universität Berlin, Germany*
E-mail: gernandt@math.tu-berlin.de
[2] *Instituto Argentino de Matemática, CONICET, Argentina, and Centro de Matemática de La Plata, Universidad Nacional de La Plata, Argentina*
E-mail: francisco@mate.unlp.edu.ar
[3] *Technische Universität Ilmenau, Germany*
E-mail: friedrich.philipp@tu-ilmenau.de
[4] *Technische Universität Ilmenau, Germany*
E-mail: carsten.trunk@tu-ilmenau.de

Abstract

There is a lot of recent work on low-rank perturbations of matrix pencils, some of them consider generic perturbations and some others consider non-generic perturbations. Most of the results for non-generic perturbations are mainly based on the estimation of invariant factors and the concept of majorization of finite sequences.

In this talk we follow a different approach, treating rank-one perturbations only. Our goal is to obtain perturbation results for matrix pencils via the Weyr characteristic of their kernel or range representations (which are some related linear relations).

Recent results for perturbations of linear relations describe the change of the Weyr characteristic under perturbations [1, 2], and allow to give bounds for the change of the Weyr characteristic of the perturbed matrix pencil. The choice whether to use kernel or range representation depends on the appearance of the rank-one pencil.

This is talk is based in a joint work with Hannes Gernandt, Friedrich Philipp and Carsten Trunk.

References

[1] L. Leben, F. Martínez Pería, F. Philipp, C. Trunk, H. Winkler. Finite rank perturbations of linear relations and singular matrix pencils. *Complex Anal. Oper. Theory*, 15: 37 (2021).

[2] H. Gernandt, F. Martínez Pería, F. Philipp, C. Trunk. On characteristic invariants of matrix pencils and linear relations. arXiv:2203.08296

Solving singular generalized eigenvalue problems: perturbation, projection and structure preservation

Michiel Hochstenbach[1], *Christian Mehl*[2], *Bor Plestenjak*[3]

[1] *Department of Mathematics and Computer Science, TU Eindhoven, The Netherlands*
E-mail: m.e.hochstenbach@tue.nl
[2] *Institute of Mathematics, TU Berlin, Germany*
E-mail: mehl@math.tu-berlin.de
[3] *Department of Mathematics, University of Ljubljana, Slovenia*
E-mail: Bor.Plestenjak@fmf.uni-lj.si

Abstract

While the regular generalized eigenvalue problem is well understood and many algorithms for its solution have been developed, the singular generalized eigenvalue problem has been a challenge for many decades. Only recently, several new algorithms for its solution have been suggested.

One particular approach to tackle the problem is a method based on applying *rank-completing perturbations* that was developed in [1]. These perturbations have a rank equal to the size of the given singular pencil minus its normal rank so that they "complete" the rank of the singular pencil to full rank. Generically, such a perturbation results in a regular matrix pencil whose set of eigenvalues contains the eigenvalues of the original singular matrix pencil (called true eigenvalues) as well as additional random eigenvalues. The true eigenvalues can be separated from the random eigenvalues by using special orthogonality relation of the corresponding left and right eigenvectors.

In the talk, we show how the method from [1] can be developed further, to obtain a more efficient method that uses projection to a random regular subpencil of the original singular pencil rather than a perturbation. In this way, the method can also be applied to rectangular pencils. Another focus will be on structure preservation, i.e., on investigating the case of Hermitian pencils (i.e., both coefficient matrices are Hermitian) or pencils with a related symmetry structure. This case is challenging, because left and right eigenvalues may coincide which will lead to difficulties in separating the true eigenvalues from the random eigenvalues.

References

[1] M. Hochstenbach, C. Mehl, B. Plestenjak. Solving singular generalized eigenvalue problems by a rank-completing perturbation. *SIAM J. Matrix Anal. Appl.*, 40:1022–1046 (2019).

Stabilization of port-Hamiltonian systems by low rank output feedback

Volker Mehrmann[1]

[1] *Inst. f. Math. MA4-5, TU Berlin, Germany*
E-mail: mehrmann@math.tu-berlin.de

Abstract

Port-Hamiltonian (pH) systems are an ubiquitous modeling class for many physical systems. The pH class has many important properties, in particular, stability of the system is guaranteed via the associated Hamiltonian which represents a Lyapunov function. In the linear case the only part of the system that may not be asymptotically stable is associated with purely imaginary eigenvalues. Although stability is guaranteed, to achieve asymptotic stability it is then necessary to carry out a stabilization via output feedback. For pH systems then the effects of the involved dissipation term (which leads to the part of the system which is asymptotically stable) and the part of the feedback (which has to move purely imaginary eigenvalues from the imaginary axis) are intertwined, so that often the necessary feedback is of very low rank. In this talk we will characterize the necessary feedback structure via a canonical form under unitary transformations and show how a minimal norm, minimal rank feedback can be constructed.

References

[1] F. Achleitner, A. Arnold, and V. Mehrmann. Hypocoercivity and hypocontractivity concepts for linear dynamical systems. *Electronic Journal of Linear Algebra* Vol. 38, 883–911, 2023.

[2] N. Gillis, V. Mehrmann, and P. Sharma, Computing nearest stable matrix pairs. *Numerical Linear Algebra with Applications*, Vol. 25, e2153, 2018.

[3] C. Mehl, V. Mehrmann, and M. Wojtylak, Linear algebra properties of dissipative Hamiltonian descriptor systems *SIAM Journal Matrix Analysis and Applications*, Vol. 39, 1489–1519, 2018.

[4] V. Mehrmann and B. Unger, Control of port-Hamiltonian differential-algebraic systems and applications, http://arxiv.org/abs/2201.06590, 2022. *Acta Numerica*, to appear, 2023.

Eigenvalues of rank one perturbations of singular M-matrices

André Ran[1]

[1] *Department of Mathematics, Vrije Universiteit Amsterdam, The Netherlands, and Focus Area Pure and Applied Analytics, North-West University, Potchefstroom, South Africa*

E-mail: a.c.m.ran@vu.nl

Abstract

Let H be an (entrywise) nonnegative matrix, then $A = \rho(H)I - H$ is a singular M-matrix. With the exception of zero, all eigenvalues of A are in the open right half plane. Let v and w be nonnegative vectors, and consider for $t > 0$ the matrix $B(t) = A + tvw^T$.

In the talk we will discuss the following results. All real eigenvalues of $B(t)$ are nonnegative, and there is a $t_0 > 0$ such that for $0 < t < t_0$ the matrix $B(t)$ has all its eigenvalues in the open right half plane. This does not extend to all $t > 0$ in general, except for the two-dimensional case. In the three dimensional case, under an additional condition there is a t_1 such that $B(t)$ has all its eigenvalues in the open right half plane for $t > t_1$, but this does not extend to any higher dimension.

Acknowledgements: The work is based on research supported in part by the National Research Foundation of South Africa (Grant Number 145688).

References

[1] J. Bierkens, A.C.M. Ran. A singular M-matrix perturbed by a nonnegative rank one matrix has positive principal minors; is it D-stable? *Linear Algebra Appl.* 457 (2014) 191–208

[2] B. Anehila, A.C.M. Ran. A note on a conjecture concerning rank one perturbations of singular M-matrices, *Quaestiones Math.* 45:10 (2022), 1529-1537

Rank-one perturbation of linear relations via matrix pencils

Itziar Baragaña[1], *Alicia Roca*[2]

[1] *Dpto. de Ciencia de la Computación e IA, Universidad del País Vasco/Euskal Herriko Unibertsitatea, UPV/EHU, Spain*
E-mail: itziar.baragana@ehu.eus
[2] *Dpto. de Matemática Aplicada, Universitat Politècnica de València/IMM, Spain*
E-mail: aroca@mat.upv.es

Abstract

Associated to a matrix pencil two linear relations can be defined: the kernel and the range representations. In [3] the Weyr characteristic of a linear relation is introduced, the relationship between the Weyr characteristic of a pencil and that of their two representations is investigated, and the results are used to estimate the invariant characteristics of matrix pencils under rank one perturbations.

We go back on the rank-one perturbation problem of linear relations and, using the above relationship, we characterize in terms of the Weyr characteristics of two linear relations S and T, when T is a rank-one perturbation of S. We show that this problem can be stated as a pencil completion problem, and we provide a solution using the results of [1] and [2].

Acknowledgements: Work supported by grant PID2021-124827NB-I00 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe", by the "European Union".

References

[1] I. Baragaña, A. Roca. Rank-one perturbations of matrix pencils. *Linear Algebra Appl.*, 606: 170-191 (2020).

[2] M. Dodig, M. Stošić. The general matrix pencil completion problem: a minimal case, *SIAM Journal on Matrix Analysis and Applications*, 40 (1): 347-369 (2019).

[3] H. Gernandt, F. Martínez Pería, F. Philipp and C. Trunk. On characteristic invariants of matrix pencils and linear relations, *arXiv e-prints (2022)*, arXiv:2203.08296.

Combinatorics in matrix pencils completion and rank perturbation problems

Marija Dodig [1,3], *Marko Stošić* [2,3]

[1] *Departamento de Matemática, Faculdade de Ciências da Universidade de Lisboa, Lisbon, Portugal*
[2] *CAMGSD, Departamento de Matemática, Instituto Superior Técnico, Lisbon, Portugal*
E-mail: mstosic@isr.ist.utl.pt
[3] *Mathematical Institute SANU, Serbia*

Abstract

In bounded rank perturbation problems of matrix pencils, and related matrix pencils completion problems, frequently one encounters combinatorial problems involving partitions of integers and polynomial chains – a problems of independent combinatorial interest. We will give an overview of some of the most interesting and important combinatorial results, as well as some open problems, with a particular emphasis on the ones that recently had applications in matrix pencils perturbation problems.

Acknowledgements: Work partially supported by Portuguese Fundação para a Ciência e a Tecnologia (FCT), project no. EXPL/MAT-PUR/0584/2021.

Kernel and range representation of matrix pencils

Carsten Trunk[1]

[1] *Institut für Mathematik, TU Ilmenau, Germany*
E-mail: carsten.trunk@tu-ilmenau.de

Abstract

Let $A - \lambda E$ be a matrix pencil with square or rectangular matrices A, E. A representation of the form

$$\{(Ez, Az) \ : \ z \in \mathbb{C}^d\}$$

is called *image representation*. Similarly, a representation of the form

$$\{(x, y) \ : \ Ax = Ey\} = \ker [A, -E]$$

is called *kernel representation*.

Obviously, the kernel and the range representation are subspaces of a larger vector space. We are interested in the connection (spectrum, eigenvalues etc.) between the pencil and its kernel and range representation. These concepts are well-known in the literature, see, e.g., [1, 2], but were not used (so far).

Acknowledgements: This is based on joint papers with Thomas Berger (Paderborn, Germany), Hannes Gernandt (Berlin), Francisco Martínez Pería (La Plata, Argentina), Friedrich Philipp (Ilmenau) and Henrik Winkler (Ilmenau).

References

[1] P. Benner and R. Byers. Evaluating products of matrix pencils and collapsing matrix products. *Numer. Linear Algebra Appl.*, 8: 357–380, (2001).

[2] P. Benner and R. Byers. An arithmetic for matrix pencils: theory and new algorithms. *Numer. Math.*, 103: 539–573, (2006).

An interlacing result for Hermitian matrices in Minkowski space

D.B. Janse van Rensburg[1], *A.C.M. Ran*[2], *M. van Straaten*[1]

[1] *Mathematics and Applied Mathematics, North-West University, South Africa*
E-mail: dawie.jansevanrensburg@nwu.ac.za, madelein.vanstraaten@nwu.ac.za
[2] *Department of Mathematics, VU Amsterdam, the Netherlands*
E-mail: a.c.m.ran@vu.nl

Abstract

The well known interlacing problem is studied, but here we consider the result for Hermitian matrices in the Minkowski space, an indefinite inner product space with one negative square. More specific, we consider the $n \times n$ matrix $A = \begin{bmatrix} J & u \\ -u^* & a \end{bmatrix}$ with $a \in \mathbb{R}$, $J = J^*$ and $u \in \mathbb{C}^{n-1}$. Then A is H-selfadjoint with respect to the matrix $H = I_{n-1} \oplus (-1)$. The canonical form for the pair (A, H) plays an important role and the sign characteristic coupled to the pair is also discussed. Inspired by some of the results in the paper [1].

Acknowledgements: Work partially supported by DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences.

References

[1] W.-R. Xu, N. Bebiano, G.-L. Chen. A reduction algorithm for reconstructing periodic Jacobi matrices in Minkowski spaces. *Applied Math. and Comp.*, 419: 126853 (2022).

Jordan-like decompositions of linear relations

Henrik Winkler[1]

[1] *Institut f. Mathematik, TU Ilmenau, Germany*
E-mail: henrik.winkler@tu-ilmenau.de

Abstract

Let $\mathfrak{H}$ be a finite-dimensional linear space. Any subspace of $\mathfrak{H} \times \mathfrak{H}$ can be considered as a linear relation. Each linear relation has a direct sum decomposition with respect to the following three types of linear relations:

1. *multishifts*, i.e., injective operators without eigenvalues;
2. *Jordan relations*, i.e., relations with a finite number of eigenvalues (including possibly ∞), which are made up of Jordan chains;
3. *completely singular relations*, i.e., multivalued relations which are made up of so-called singular chains; their eigenvalues fill up the set of complex numbers including ∞.

In particular, the structure of symmetric linear relations in indefinite-inner product spaces $\mathfrak{H}$ is considered.

Acknowledgements: This talk is based on a joint work with Thomas Berger (Paderborn), Timo Reis (Ilmenau), Henk de Snoo (Groningen), and Carsten Trunk (Ilmenau).

Rank one perturbations of matrices with applications in graph theory

Michał Wojtylak[1]

[1] *Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, Poland*
E-mail: michal.wojtylak@uj.edu.pl

Abstract

The problem of rank one perturbations of a matrix seems to be well studied. The behaviour of eigenvalues of $A + \tau uv^*$ is described globally and locally. We will review some results from [1] and show how they fit in the harmonic analysis on graphs setting. In particular we will talk about quadratically embaddable graphs. Joint work with Andre Ran and Marek Skrzypczyk will be presented.

References

[1] A.C.M. Ran, M. Wojtylak, Global Properties of Eigenvalues of Parametric Rank One Perturbations for Unstructured and Structured Matrices, *CAOT* 2021 (15:21).

MSC06. Matrix and operator means

Sejong Kim, Sushil Singla,
Tin-Yau Tam

Majorization and properties on Spectral geometric mean
Luyining Gan

The Endpoint Geodesic Problem on Symmetric Spaces with Applications
Knut Hüper, Fatima Silva Leite

Superfast iterative refinement of Low Rank Approximation of a Matrix
Victor Y. Pan

Non-homogeneous gradient equations for sum of squares of Wasserstein metric
Jinmi Hwang, Sejong Kim, Vatsalkumar N. Mer

Linearity of Cartan and Wasserstein geodesics
Sejong Kim, Hayoung Choi, Yongdo Lim

Matrix/Operator Mean Lagniappe
Jimmie Lawson, Yongdo Lim

Regression on the manifold of fixed rank positive semidefinite matrices
Hosoo Lee

Geometric means on some matrix manifolds
Luís Machado

Algebraic properties of operations on positive definite cones in operator algebras corresponding to various versions of Heron means
Lajos Molnár

Approximation results for generalized operator means
Miklós Pálfia, Zoltán Léka, Zsigmond Tarcsay

Operator means of positive definite compact operators and their properties
Sushil Singla

Matrix Means on Grassmann Manifolds
Tin-Yau Tam, Xiang Xiang Wang

Majorization and properties on Spectral geometric mean

Luyining Gan[1]

[1] *Department of Mathematics and Statistics, University of Nevada Reno, USA*
E-mail: lgan@unr.edu

Abstract

In this talk, we will introduce the relation between the metric geometric mean, spectral geometric mean and Wasserstein mean of the positive definite matrices in terms of (weak) log-majorization relation. In addition, we will also introduce some new properties of the weighted spectral geometric mean, like geodesic property and tolerance relation.

Acknowledgements: The work of L. Gan partially supported by AMS-Simons Travel Grant 2022–2024.

The Endpoint Geodesic Problem on Symmetric Spaces with Applications

Knut Hüper[1], *Fátima Silva Leite*[2]

[1] *Institute of Mathematics, Julius-Maximilians-Universität Würzburg, Germany*
E-mail: hueper@mathematik.uni-wuerzburg.de
[2] *Institute of Systems and Robotics - Coimbra, and*
Department of Mathematics, University of Coimbra, Portugal
E-mail: fleite@mat.uc.pt

Abstract

An important task for interpolation problems and statistics on symmetric spaces is the efficient computation of (geometric) means of data and in particular the computation of midpoints of smooth curves connecting two data points. While closed form solutions for the so called endpoint geodesic problem on general symmetric spaces are well known, often explicit exponentiation of matrices and/or SVD computations are still required. In most cases, these computations are rather expensive. We present much simpler closed form expressions for the particular case of Grassmannians, where only constant, linear and quadratic functions in the data points and scalar trigonometric functions are involved. We also comment on the general idea putting other important symmetric spaces, compact and noncompact ones, into perspective.

Acknowledgements: The first author has been supported by German BMBF-Projekt 05M20WWA: Verbundprojekt 05M2020 - DyCA. The second author has been supported by Fundação para a Ciência e Tecnologia (FCT) under the project UIDP/00048/2020.

Superfast iterative refinement of Low Rank Approximation of a Matrix

Victor Y. Pan[1]

[1] *Ph.D. Programs in Computer Science and Mathematics The Graduate Center of the City University of New York New York, NY 10036 USA and Department of Computer Science, Lehman College of the City University of New York, Bronx, NY 10468 USA*

E-mail: victor.pan@lehman.cuny.edu

Abstract

We call a matrix algorithm superfast (aka running at sublinear cost) if it involves much fewer flops and memory cells than the matrix has entries. Using such algorithms is highly desired or even imperative in computations with Big Data, which involve immense matrices and are quite typically reduced to computation of low rank approximation (LRA) of an input matrix. Any superfast LRA algorithm falls miserably on some inputs, even if using randomization is allowed, but we prove that some superfast LRA algorithms output reasonable or even nearly optimal solutions for a large input class. Moreover, we propose, analyze, and test a novel superfast algorithm for iterative refinement of any crude but sufficiently close low rank approximation of a matrix. The results of our numerical tests are in good accordance with our formal study.

Acknowledgements: Our work has been partially supported by NSF Grants CCF–1116736, CCF–1563942 and CCF–1733834 and PSC CUNY Award 69813 00 48.

References

[1] Qi Luan, Victor Y. Pan, John Svadlenka, The Dual JL Transforms and Superfast Matrix Algorithms, arXiv:1906.04929, last revised 4 Dec 2022.

Non-homogeneous gradient equations for sum of squares of Wasserstein metric

Jinmi Hwang[1], *Sejong Kim*[2], *Vatsalkumar N. Mer*[3]

[1] *Department of Mathematics, Sungkyunkwan University, Republic of Korea*
E-mail: jinmi0401@skku.edu
[2] *Department of Mathematics, Chungbuk National University, Republic of Korea*
E-mail: skim@chungbuk.ac.kr
[3] *Department of Mathematics, Chungbuk National University, Republic of Korea*
E-mail: vnm232657@gmail.com

Abstract

We investigate Euclidean and Riemannian gradient equations on the open convex cone $\mathbb{P}_m$ of all $m \times m$ positive definite Hermitian matrices

$$\nabla \left[\sum_{j=1}^{m} w_j d_W^2(X, A_j) \right] = F(X)$$

where d_W denotes the Wasserstein metric, and F is a differentiable map on $\mathbb{P}_m$. The special case where $F(X) = 0$ is the equation vanishing the gradient of the weighted sum of squares of the Wasserstein metrics. Its unique solution is known as the Wasserstein mean of $A_1, \ldots, A_n$. We show the existence and uniqueness of the solution for these non-homogeneous gradient equations, and furthermore, establish the boundedness of solutions.

Acknowledgements: The work of J. Hwang was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education (No. NRF-2022R1I1A1A01068411). The work of S. Kim was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. NRF-2022R1A2C4001306).

References

[1] R. Bhatia, T. Jain and Y. Lim, On the Bures-Wasserstein distance between positive definite matrices, *Expo. Math.*, 37 , no. 2, 165–191 (2019).

[2] Y. Lim, F. Hiai and J. Lawson. Nonhomogeneous Karcher eqautions with vector fields on positive definite matrices. *Eur. J. Math.*, 7 , 1291–1328 (2021).

Linearity of Cartan and Wasserstein geodesics

Hayoung Choi [1], *Sejong Kim* [2], *Yongdo Lim* [3]

[1] *Department of Mathematics, Kyungpook National University, South Korea*
E-mail: hayoung.choi@knu.ac.kr
[2] *Department of Mathematics, Chungbuk National University, South Korea*
E-mail: skim@chungbuk.ac.kr
[3] *Department of Mathematics, Sungkyunkwan University, South Korea*
E-mail: ylim@skku.edu

Abstract

The open convex cone of positive definite Hermitian matrices has two important Riemannian geometries [1, 2], where the weighted geometric (Cartan) means and Wasserstein means appear as the corresponding geodesics. In this talk we discuss their linearity problem, which means when the Cartan and Wasserstein geodesics lie in the space of spanned by two given positive definite matrices. We give a complete characterization for the Cartan geodesic and partially for the Wasserstein geodesic.

Acknowledgements: The work of Sejong Kim is supported by the National Research Foundation of Korea grant funded by the Korea government (MSIT) (No. NRF-2022R1A2C4001306).

References

[1] R. Bhatia, *Positive Definite Matrices*, Princeton Series in Applied Mathematics, Princeton University Press, 2007.

[2] R. Bhatia, T. Jain, and Y. Lim, *On the Bures-Wasserstein distance between positive definite matrices*, Expo. Math. **37** (2019), no. 2, 165-191.

Matrix/Operator Mean Lagniappe

Jimmie Lawson

Department of Mathematics, Louisiana State University, USA
E-mail: lawson@math.lsu.edu

Abstract

In my professional home state of Louisiana (USA), the word "lagniappe" is frequently used for a bonus, something extra that is often unanticipated. Mathematical research sometimes provides lagniappe, new and unexpected results that go beyond the original field of inquiry. In recent research in the area of Matrix and Operator Geometric Means, I have had two significant experiences of such lagniappe, which will be the focus of my presentation. The first is, as far as I can tell, a previously unknown converse to the foundational inverse function theorem of analysis. In its simplest form the converse states that if a C^r-function f has a Lipschitz inverse f^{-1} on a neighborhood of $f(x)$, then $f'(x)$ is invertible. The second arose in the intensive study of the Cartan (alternatively Karcher or least squares) mean in the setting of positive definite 2×2-matrices of determinant 1, denoted $\mathbf{SPD}_2$. It was recognized that this space can serve as a model for three-dimensional hyperbolic geometry, which gives rise to new computational tools for the geometry and geometric tools for the matrix theory. We provide some illustrations.

Acknowledgements: Work partially supported by the Louisiana State University Boyd Support Fund.

References

[1] J.D. Lawson. An inverse function theorem converse. *J. Math. Anal. Appl.*, 486: 6 pp. (2020)

[2] J.D. Lawson, Y. Lim. From hyperbolic geometry to 2×2 Hermitian matrices and back. *Journal of Geometry*, 112: 15 pp. (2021).

Regression on the manifold of fixed rank positive semidefinite matrices

Hosoo Lee

Elementary Education Research Institute, Jeju National University, Korea
E-mail: hosoo@jejunu.ac.kr

Abstract

We consider the problem of finding a geodesic curve that best fits a given set of time-labelled points on the manifold of fixed rank positive semidefinite matrices. Building upon the polar decomposition of vectors in the plane, we develop a quotient geometry for the manifold of fixed rank positive semidefinite matrices. The natural metric decomposes as the sum of the Riemannian metric on the cone of positive definite matrices and the standard metric of the Grassman manifold. The quality of a geodesic curve is measured by a term that penalizes its lack of fit to the data. The corresponding objective function is determined by the natural metric.

Regression on the manifold of fixed rank positive semidefinite matrices

Hosoo Lee

Elementary Education Research Institute, Jeju National University, Korea
E-mail: hosoo@jejunu.ac.kr

Abstract

We consider the problem of finding a geodesic curve that best fits a given set of time-labelled points on the manifold of fixed rank positive semidefinite matrices. Building upon the polar decomposition of vectors in the plane, we develop a quotient geometry for the manifold of fixed rank positive semidefinite matrices. The natural metric decomposes as the sum of the Riemannian metric on the cone of positive definite matrices and the standard metric of the Grassman manifold. The quality of a geodesic curve is measured by a term that penalizes its lack of fit to the data. The corresponding objective function is determined by the natural metric.

Geometric means on some matrix manifolds

Luís Machado

Institute of Systems and Robotics - University of Coimbra, Rua Silvio Lima - Polo II, 3030-290 Coimbra, Portugal
Department of Mathematics - University of Trás-os-Montes e Alto Douro (UTAD), Quinta de Prados, 5000-801 Vila Real, Portugal
E-mail: lmiguel@utad.pt

Abstract

Over the past few years, many of the tasks coming from computer vision and image analysis require averaging manifold-valued data. Examples include, but are not limited to, filtering, training, clustering, visualization, segmentation, classification, recognition, grouping and motion analysis [3]. Matrix Lie groups and quotient spaces play a crucial role in these applications, since they are the natural representatives of symmetries. Due to the lack of a linear structure in those spaces, the standard averaging schemes cannot be applied and therefore generalizations of the concept of mean were required. The mean in the context of Riemannian geometry, introduced originally by E. Cartan in the 1920s, has had a great impact on the progress of various fields of science and technology. Typically, we are given a finite set of points and look for a point that minimizes the sum of the squared geodesic distances to each one of the given points.

In this work, the Riemannian mean is obtained through a limiting process of a variational problem whose extremals are piecewise geodesics that best fit a given set of data points [1]. We will then make an incursion through matrix manifolds that play important roles in vision applications, such as, the orthogonal group, the real Grassmannian [2] and the set of symmetric and positive definite matrices.

Acknowledgements: Work supported by Fundação para a Ciência e Tecnologia (FCT) under the project UIDP/00048/2020.

References

[1] L. Machado, K. Hüper, F. Silva Leite. Riemannian means as solutions of variational problems. *LMS. J. Comput. Math.*, 9: 86–103 (2006).

[2] E. Batzies, K. Hüper, L. Machado, F. Silva Leite. Geometric means and geodesic regression on Grassmannians. *Linear Algebra Appl.*, 466: 83–101 (2015).

[3] P. Turaga, A. Srivastava. *Riemannian computing in computer vision.* Springer Cham, 2016.

Algebraic properties of operations on positive definite cones in operator algebras corresponding to various versions of Heron means

Lajos Molnár

ELKH-SZTE Analysis and Applications Research Group, Bolyai Institute, University of Szeged, H-6720 Szeged, Aradi vértanúk tere 1., Hungary, and Department of Analysis, Institute of Mathematics, Budapest University of Technology and Economics, Műegyetem rkp. 3., H-1111 Budapest, Hungary
E-mail: molnarl@math.u-szeged.hu

Abstract

There are several variants of Heron type means on the positive definite cone $\mathcal{A}^{++}$ in a (unital) C^*-algebra $\mathcal{A}$. Among them, we mention the conventional Heron mean defined as

$$\left(\frac{A^{1/2}+B^{1/2}}{2}\right)^2,$$

the Kubo-Ando type Heron mean defined as

$$A^{1/2}\left(\frac{I+(A^{-1/2}BA^{-1/2})^{1/2}}{2}\right)^2 A^{1/2} = \frac{A+B+2A\sharp B}{4},$$

(where $\sharp$ denotes the Kubo-Ando geometric mean) and the recently introduced Wasserstein mean

$$\frac{A+B+A(A^{-1}\sharp B)+(A^{-1}\sharp B)A}{4}.$$

Apparently, for commuting elements A, B in $\mathcal{A}^{++}$, those three means all coincide. In the talk, we discuss the converse question: whether the equality of any two of those three means for a particular pair of elements implies the commutativity of those elements.

From the conventional Heron mean we easily get a well-behaving operation on positive definite cones. Indeed, if we multiply it by 4, we obtain a semigroup operation that also satisfies the so-called bisymmetry equation:

$$(A\circ B)\circ(C\circ D) = (A\circ C)\circ(B\circ D).$$

In this talk, we study questions to what extent do analogous algebraic properties hold for the Kubo-Ando type Heron mean and for the Wasserstein mean?

Besides pointing out certain weaker forms of associativity appearing in the theories of non-associative algebras (especially in loop theory), a typical result

among the ones that we are going to present is the following. Denote by $\diamond$ the operation obtained from the Kubo-Ando type Heron mean (by multiplying it by 4). Let $\mathcal{A}$ be a C^*-algebra and let $A, B \in \mathcal{A}^{++}$. Then the equality

$$(X \diamond A) \diamond B = X \diamond (A \diamond B)$$

holds for all $X \in \mathcal{A}^{++}$ if and only if $B^{-1/2}AB^{-1/2}$ is a central element (it commutes with all elements of $\mathcal{A}$).

From this, we immediately obtain the following. Assuming that there exists an element $A \in \mathcal{A}^{++}$ such that

$$(X \diamond A) \diamond Y = X \diamond (A \diamond Y)$$

holds for all $X, Y \in \mathcal{A}^{++}$, the algebra $\mathcal{A}$ is necessarily commutative.

Acknowledgements: This work has been supported by the project TKP2021-NVA-09 provided by the Ministry of Innovation and Technology of Hungary from the National Research, Development and Innovation Fund, financed under the TKP2021-NVA funding scheme. The research has also been supported by the National Research, Development and Innovation Office of Hungary, NKFIH, Grant No. K134944.

Approximation results for generalized operator means

Zoltán Léka[1], *Miklós Pálfia*[2], *Zsigmond Tarcsay*[3]

[1] *Dpt. of Mathematics, Óbuda University, Hungary*
E-mail: leka.zoltan@nik.uni-obuda.hu
[2] *Dpt. of Mathematics, Corvinus University of Budapest and University of Szeged, Hungary*
E-mail: miklos.palfia@uni-corvinus.hu
[3] *Dpt. of Mathematics, Corvinus University of Budapest and ELTE*
E-mail: zsigmond.tarcsay@uni-corvinus.hu

Abstract

In this talk we will investigate zeros of nonlinear operators in a Thompson metric space. Inspired by the work of Gaubert and Qu from 2014, we study exponentially contracting continuous and discrete time flows generated by these nonlinear operators. We establish the operator norm convergence of deterministic and stochastic resolvent and proximal type algorithms, in particular versions coming from a Trotter-Kato type formula. This generalizes recent strong law of large numbers and so called 'nodice' results proved for the Karcher mean of positive operators by Lim and Pálfia. Applications include generalization of these results from the Karcher mean to other, so called generalized Karcher means introduced in 2016. The talk is based on an ongoing joint work with Léka Zoltán and Zsigmond Tarcsay.

Acknowledgements: Work partially supported by the Hungarian National Research, Development and Innovation Office NKFIH, Grant No. FK128972 and by project TKP2021-NVA-09. Project no. TKP2021-NVA-09 has been implemented with the support provided by the Ministry of Innovation and Technology of Hungary from the National Research, Development and Innovation Fund, financed under the TKP2021-NVA funding scheme.

References

[1] S. Gaubert, Z. Qu. The contraction rate in Thompson's part metric of order-preserving flows on a cone – Application to generalized Riccati equations. *J. Differential Equations*, 256: 2902—2948 (2014).

[2] Y. Lim, M. Pálfia. Strong law of large numbers for the L^1-Karcher mean. *Journal of Functional Analysis*, 279:7: 108672 (2020).

Operator means of positive definite compact operators and their properties

Sushil Singla

Department of Mathematics, Indian Statistical Institute (ISI) Delhi Center, India
E-mail: ss774@snu.edu.in

Abstract

Matrix geometric mean has been interests to many mathematicians in the last few decades and its connection with Riemannian structure of positive matrices has been widely explored, see [2, 3]. This connection was further explored for positive Hilbert-Schidmt operators in [5]. Matrix inequalities has been a subject of interest for many mathematicians and a complementary Golden-Thompson trace Inequality was proved by Hiai and Petz in [4]. In their proof, they used the following limit related to geometrix mean of inequalities:

$$\lim_{r\to 0}(A^r \#_t B^r)^{1/r} = e^{(1-t)logA+tlogB}.$$

It is also related to symmetric form of Lie-Trotter formula given as:

$$\lim_{r\to 0^+}(A^{(1-t)r/2}B^{tr}A^{(1-t)r/2}) = e^{(1-t)logA+tlogB}.$$

These limit has also been studied when $r \to \infty$, under the name Reciprocal Lie-Trotter formula, see [1]. Only the existence of limit is known and no closed form formula is known for such a limit. We will present various properties of matrix exponentials and corresponding inequalities for operator mean of positive compact operators.

References

[1] K. M. R. Audenaert, F. Hiai. Reciprocal Lie-Trotter formula. *Linear Multilinear Algebra* 64: 1220–1235 (2016).

[2] R. Bhatia *Positive Definite Matrices*. Princeton Series in Applied Mathematics, Princeton University Press, Princeton, NJ, 2007.

[3] R. Bhatia, J. Holbrook. Riemannian geometry and matrix geometric means. *Linear Algebra Appl.* 413: 594–618 (2006).

[4] F. Hiai, D. Petz. The Golden-Thompson trace inequality is complemented. *Linear Algebra Appl.* 181: 153–185, (1993).

[5] J. Lawson, Y. Lim. The least squares mean of positive Hilbert-Schmidt operators. *J. Math. Anal. Appl.* 403: 365–375 (2013).

Matrix Means on Grassmann Manifolds

Tin-Yau Tam[1] , *Xiang Xiang Wang*[2]

[1] *Department of Mathematics and Statistics, University of Nevada, Reno, USA*
E-mail: ttam@unr.edu
[2] *Department of Mathematics and Statistics, University of Nevada, Reno, USA*
E-mail: xiangxiangw@unr.edu

Abstract

The Grassmann manifolds $\mathrm{Gr}_{n,k}(\mathbb{F})$ appear in the form of the set of k-dimensional subspaces of $\mathbb{F}^n$, the space of rank k orthogonal projectors $P : \mathbb{F}^n \to \mathbb{F}^n$, where $\mathbb{F}$ is the real field or the complex field. They can be identified with the symmetric space $\mathrm{O}(n)/(\mathrm{O}(k)\times\mathrm{O}(n-k))$ for the real case and $\mathrm{U}(n)/(\mathrm{U}(k)\times\mathrm{U}(n-k))$ for the complex case. Motivated by the geometric mean of two positive definite matrices [1], the mid-point of two elements in $\mathrm{Gr}_{n,k}(\mathbb{F})$ is defined as their geometric mean [2]. Geometric inequalities will be presented in the context of elliptical geometry. The dual geometry (hyperbolic) and its implication, for example, Hua's inequality [3], will be discussed.

References

[1] R. Bhatia. *Positive Definite Matrices*. Princeton University Press, Princeton, New Jersey, 2007.

[2] E. Batzies, K. Hüper, L. Machado, F. Silva Leite. Geometric mean and geodesic regression on Grassmannians. *Linear Algebra Appl.* 466: 83–101 (2015),

[3] L.-G. Hua. On an inequality of Harnack's type. *Sci. Sinica.* 14: 791 (1965).

MSC07. The interplay between linear-multilinear algebra and rational approximation

Claude Brezinski,
Michela Redivo-Zaglia,
Ahmed Salam

Efficient computation of the Wright function
Lidia Aceto, Fabio Durastante

Rational extrapolation methods, Anderson acceleration, and solution of systems of equations
Claude Brezinski

Numerical approximation of the symbol of an operator with local spectral mean values evaluations
Jean-Paul Chehab

A tensor bidiagonalization method for higher-order singular value decomposition with applications
Anas El Hachimi

Efficient Inversion of Matrix ϕ-Functions of Low Order
Luca Gemignani

Structured-barycentric forms and the AAA framework for modeling second-order dynamics from data
Ion Victor Gosea, Serkan Gugercin, Steffen W. R. Werner

A Rational Preconditioner for Multi-dimensional Riesz Fractional Diffusion Equations
Mariarosa Mazza, Lidia Aceto

The Short-term Rational Lanczos Method and Applications
Stefano Pozza, Davide Palitta, Valeria Simoncini

Extrapolation methods for choosing a regularization parameter
Giuseppe Rodriguez, Claude Brezinski, Caterina Fenu, Michela Redivo Zaglia

Applications of trace estimation techniques
Yousef Saad

On generalized inverse of a vector, with applications to vector epsilon algorithm
Ahmed Salam

Computing the generalized rational minimax approximation
Nir Sharon

Error bounds for the approximation of matrix functions with rational Krylov methods
Igor Simunec

Perfect shifts for Hessenberg-Hessenberg pencils
Marc Van Barel, Nicola Mastronardi, Raf Vandebril, Paul Van Dooren

Efficient computation of the Wright function

Lidia Aceto[1], Fabio Durastante[2]

[1] Dpt. of Science and Technological Innovation, University of Eastern Piedmont, Italy
E-mail: lidia.aceto@uniupo.it
[2] Dpt. of Mathematics, University of Pisa, Italy
E-mail: fabio.durastante@unipi.it

Abstract

The Wright function is a generalization of the exponential function defined by

$$W_{\lambda,\mu}(z) := \sum_{n=0}^{\infty} \frac{z^n}{n!\,\Gamma(\lambda n + \mu)}, \quad \lambda > -1,\ \mu \in \mathbb{C}.$$

Although several representations of the Wright function have been introduced and many of its analytical properties have already been well-studied (see, e.g., [2, 3, 4, 5]), its numerical evaluation is still an active research area.

In this talk we present a new algorithm for the efficient computation of a particular expression of the Wright function of interest in applications, namely

$$f_{\lambda,\mu}(t;x) := t^{\mu-1} W_{\lambda,\mu}(-|x|t^{\lambda}), \qquad t \geq 0, x \in \mathbb{R}, \quad \lambda \in (-1,0), \mu \in \mathbb{C}.$$

This algorithm is based on the numerical inversion of the Laplace transform

$$f_{\lambda,\mu}(t;x) = \frac{1}{2\pi i} \int_{\mathcal{C}} e^{st} F_{\lambda,\mu}(s;x) ds, \qquad F_{\lambda,\mu}(s;x) = s^{-\mu} e^{-|x| s^{-\lambda}}.$$

To determine the contour $\mathcal{C}$ which is a suitable deformation of the Bromwich line and the number of quadrature nodes, we use the error analysis for obtaining a result with a certified bound. The proposed procedure is innovative with respect to previous attempts in the literature because it avoids dealing with oscillatory integrals and is fairly easy to implement.

We present some numerical experiments that validate both the theoretical estimates of the error and the applicability of the proposed method for representing the solutions of fractional differential problems, [1].

Acknowledgements: Work partially supported by GNCS-INdAM.

References

[2] Y. Luchko. *The Wright function and its applications.* In: Basic Theory, Volume 1, De Gruyter, 2019, pp. 241–268.

[3] F. Mainardi, A. Consiglio. The Wright functions of the second kind in Mathematical Physics. *Mathematics*, 8: Article-Number 884 (2020).

[4] F. Mainardi, A. Mura, G. Pagnini. The M-Wright function in time-fractional diffusion processes: a tutorial survey. *arXiv preprint*, arXiv:1004.2950 (2010).

[5] B. Stanković. On the function of E. M. Wright. *Publ. Inst. Math. (Beograd) (N.S.)*, 10(24): 113–124 (1970).

Rational extrapolation methods, Anderson acceleration, and solution of systems of equations

C. Brezinski

[1] *Laboratoire Paul Painlevé, University of Lille, France*
E-mail: claude.brezinski@univ-lille.fr

Abstract

Joint work with Stefano Cipolla, Michela Redivo-Zaglia and Yousef Saad.

In computational sciences it is often necessary to obtain the limit of a sequence of elements of a vector space that converges slowly to its limit or even diverges.

In some situations, we may be able to obtain a new sequence that converges faster by modifying the method that produces it. However, in many instances, the process by which the sequence is produced is hidden into a black box.

Thus, a solution is to transform this sequence into a new sequence which, under some assumptions, converges faster.

Among these general techniques Shanks' transformation is arguably the best all-purpose method for accelerating the convergence of sequences.

The aim of this talk is to present a general framework for Shanks' transformation(s) of sequences of elements in a vector space.

This framework includes the Minimal Polynomial Extrapolation (MPE), the Reduced Rank Extrapolation (RRE), the Modified Minimal Polynomial Extrapolation (MMPE), the Topological Shanks transformation (TEA and STEA algorithm), and in some sense also Anderson Acceleration (AA).

Their application to the solution of systems of linear and nonlinear equations will be discussed.

References

[1] C. Brezinski, M. Redivo-Zaglia, Y. Saad, Shanks sequence transformations and Anderson acceleration, SIAM Rev., 60 (2018) 646-669.

[2] C. Brezinski, M. Redivo-Zaglia, *Extrapolation and Rational Approximation. The Works of the Main Contributors*, Springer Nature, Cham, Switzerland, 2020, viii + 406 pages.

[3] C. Brezinski, S. Cipolla, M. Redivo-Zaglia, Y. Saad, Shanks and Anderson-type acceleration techniques for systems of nonlinear equations, IMA J. Numer. Anal., 42 (2022) 3058-3093.

Numerical approximation of the symbol of an operator with local spectral mean values evaluations

Jean-Paul Chehab[1]

[1] *Laboratoire LAMFA (UMR CNRS 7352), Université de Picardie Jules Verne, 33 rue Saint Leu, 80039 Amiens Cédex, France*

E-mail: Jean-Paul.Chehab@u-picardie.fr

Abstract

We aim here at approximating the symbol of a linear operator $\mathcal{A}$ by using numerical data and fitting procedures; the symbol is assumed to be a real and positive function ϕ of the wavelength λ, and in practice, a function of the eigenvalues λ_h of a matrix A that corresponds to the discretization of $\mathcal{A}$.

We propose to adapt the matrix trace estimator developed in [2] to successive distinct spectral bands in order to build a piecewise constant function as an approximation of ϕ. To decompose the spectral interval into band of frequencies, we propose several approaches, from the formal spectral to the multigrid one [1, 4]. Rational approximations to the symbol can then be proposed.

We apply the numerical procedure for capturing an additional linear damping term in hydrodynamics models [3, 5].

References

[1] H. Abboud, C. Al Kosseifi and J.-P. Chehab, A stabilized bi-grid method for Allen-Cahn equation in finite elements, Computational and Applied Mathematics, **38** (2019) 35.

[2] Zhaojun Bai, Mark Fahey, Gene Golub, Some large-scale matrix computation problems, Journal of Computational and Applied Mathematics 74 (1996) 71-89

[3] J.L. Bona, H. Kalisch, Singularity formation in the generalizd Benjamin-One equations, DCDS-B, 11(1) (2004): 27-45

[4] J.-P. Chehab, Stabilization and Numerical Filtering for the Modeling and the Simulation of time dependent PDEs, DCDS-S, 14(8) (2021): 2693–2728

[5] E. Ott and R. N. Sudan, Damping of solitary waves, Physics of Fluids, **13** (1970), 1432–1435.

A tensor bidiagonalization method for higher-order singular value decomposition with applications

A. El Hachimi[1], K. Jbilou[2], A. Ratnani[3], L. Reichel[4]

[1] Mohammed VI Polytechnic University, Green City, Morocco. Université du Littoral Cote d'Opale, LMPA, 50 rue F. Buisson, 62228 Calais-Cedex, France.
E-mail: anas.elhachimi1997@gmail.com
[2] Université du Littoral Cote d'Opale, LMPA, 50 rue F. Buisson, 62228 Calais-Cedex, France. Mohammed VI Polytechnic University, Green City, Morocco.
E-mail: khalide.jbilou@univ-littoral.fr
[3] Mohammed VI Polytechnic University, Green City, Morocco.
E-mail: ahmed.RATNANI@um6p.ma
[4] Department of Mathematical Sciences, Kent State University, Kent, OH 44242, USA.
E-mail: reichel@math.kent.edu

Abstract

The need to know a few singular triplets associated with the largest singular values of third-order tensors arises in data compression and extraction. This paper describes a new method for their computation using the t-product. Methods for determining a couple of singular triplets associated with the smallest singular values also are presented. The proposed methods generalize available restarted Lanczos bidiagonalization methods for computing a few of the largest or smallest singular triplets of a matrix. The methods of this paper use Ritz and harmonic Ritz lateral slices to determine accurate approximations of the largest and smallest singular triplets, respectively. Computed examples show applications to data compression and face recognition.

References

[1] K. Braman, N. Hao, R. C. Hoover, M. E. Kilmer. Third-order tensors as operators on matrices: A theoretical and computational framework with applications in imaging. *SIAM Journal on Matrix Analysis and Applications.*, 34: 148–172 (2013).

[2] M. Hached, K. Jbilou, C. Koukouvinos, M. Mitrouli. A multidimensional principal component analysis via the C-product Golub-Kahan-SVD for classification and face recognition. *Mathematics.*, 9: 1249 (2021).

[3] M. E. Kimler, C. D. Martin. Factorization strategies for third-order tensors, *Lin. Alg. Appl.*, 435: 641—658 (2011).

[4] J. Baglama, L. Reichel. Augmented implicitly restarted Lanczos bidiagonalization methods. *SIAM Journal on Scientific Computing.*, 27: 19–42 (2005).

Efficient Inversion of Matrix ϕ-Functions of Low Order

Luca Gemignani

Department of Computer Science, University of Pisa, Italy
E-mail: luca.gemignani@unipi.it

Abstract

The talk is concerned with efficient numerical methods for solving a linear system $\phi(A)\boldsymbol{x} = \boldsymbol{b}$, where $\phi(z)$ is a ϕ-function and $A \in \mathbb{R}^{N\times N}$. More specifically, we are interested in the computation of $\phi(A)^{-1}\boldsymbol{b}$ for the case where $\phi(z) = \phi_1(z) = \dfrac{e^z-1}{z}$, $\phi(z) = \phi_2(z) = \dfrac{e^z-1-z}{z^2}$. A fast numerical algorithm for computing $\psi_1(A)$ and $\psi_1(A)\boldsymbol{b}$ with $\psi_1(z) = 1/\phi_1(z)$, $\phi_1(z) = \dfrac{e^z-1}{z}$, has been presented in [1, 2]. The algorithm exploits a partial fraction decomposition of the meromorphic function $\psi_1(z)$ and it is particularly suited for the application to structured matrices for which fast linear solvers exist. The same approach cannot be extended to other functions $\psi_\ell(z) = 1/\phi_\ell(z)$ with $\ell > 1$ due to the lack of explicit closed–form expressions of their poles. In this talk we discuss some iterative schemes based on the algorithm introduced in [1, 2] for computing both $\phi_2(A)^{-1}$ and $\phi_2(A)^{-1}\boldsymbol{b}$ [3]. These schemes relies on Newton's iteration for matrix inversion and Krylov-type linear solvers. Adaptations of these schemes for structured matrices are considered. In particular the cases of banded and more generally quasiseparable matrices are investigated. Numerical results are presented to show the effectiveness of our proposed algorithms.

References

[1] P. Boito, Y. Eidelman, and L. Gemignani. Efficient solution of parameter-dependent quasiseparable systems and computation of meromorphic matrix functions. *Numer. Linear Algebra Appl.*, 25(6):e2141, 13, 2018.

[2] P. Boito, Y. Eidelman, and L. Gemignani. Computing the reciprocal of a ϕ-function by rational approximation. *Adv. Comput. Math.*, 48(1), 2022.

[3] L. Gemignani. Efficient Inversion of Matrix ϕ-Functions of Low Order. Tech. Rep. arXiv:2302.07565.

Structured-barycentric forms and the AAA framework for modeling second-order dynamics from data

Ion Victor Gosea[1], *Serkan Gugercin* [2], *Steffen W. R. Werner* [3]

[1] *CSC Group, Max Planck Institute, Magdeburg, Germany*
E-mail: gosea@mpi-magdeburg.mpg.de
[2] *Department of Mathematics and Division of Computational Modeling and Data Analytics, Academy of Data Science, Virginia Tech, Blacksburg, VA, USA*
E-mail: gugercin@vt.edu
[3] *Courant Institute of Mathematical Sciences, New York University, New York, USA*
E-mail: steffen.werner@nyu.edu

Abstract

Barycentric forms [1] appear naturally in data-driven approximation of linear dynamical systems, by means of interpolation, and by means of least squares fit in [2-5]. The barycentric form of rational functions allows computationally efficient constructions of rational approximants. In this contribution, one first goal will be to extend these classical barycentric forms to the class of second-order systems, which appear naturally in the modeling of mechanical structures and electro-mechanical systems.

The AAA (Adaptive Antoulas Anderson) algorithm [6] is a rational approximation tool that is used to fit a rational function to a set of data measurements. It is a fast and robust method that was successfully extended to a series of applications in recent years, ranging from approximation theory to system and control theory. The AAA algorithm can also be interpreted as a data-driven tool for reduced-order modeling of dynamical systems from frequency domain measurements [3-4]. This is precisely the application that we have in mind for the current contribution.

Although the AAA algorithm has already been applied for fitting linear systems with first-order dynamics (unstructured case), we present here an extension of AAA to fitting systems with second-order dynamics (structured case). Toward this goal, the development of structured barycentric forms associated with the transfer function of second-order systems is needed. These allow the construction of reduced-order models from given frequency domain data, by combining interpolation and least-squares fit. As in the original AAA algorithm [6], the interpolation points are chosen based on a greedy selection criterion. Finally, various numerical test cases including, for example, the behavior of underwater drones and micro-mechanical gyroscopes are used to verify the developed theory.

References

[1] J.-P. Berrut and L. N. Trefethen, Barycentric Lagrange interpolation. SIAM Rev., 46(3):501–517, 2004

[2] A. C. Antoulas and B. D. O. Anderson, On the scalar rational interpolation problem. IMA J. Math. Control Inf., 3(2–3):61–8, 1986.

[3] A.C. Rodriguez, L. Balicki, S. Gugercin, The p-AAA algorithm for data driven modeling of parametric dynamical systems. arXiv preprint, arXiv:2003.06536, 2022.

[4] I. V. Gosea and S. Gugercin, Data-driven modeling of linear dynamical systems with quadratic output in the AAA framework. J. Sci. Comput., Springer, 91 (16), 2022.

[5] B. Gustavsen, A. Semlyen, Rational approximation of frequency domain responses by vector fitting. IEEE Trans. Power Deliv. 14(3), 1052–1061, 1999.

[6] Y. Nakatsukasa, O. Sete, L. N. Trefethen, The AAA Algorithm for Rational Approximation, SIAM J. Sci. Comput., Vol. 40, No. 3, A1494–A1522, 2018.

A Rational Preconditioner for Multi-dimensional Riesz Fractional Diffusion Equations

Lidia Aceto[1], *Mariarosa Mazza*[2]

[1] *University of Eastern Piedmont, Italy*
E-mail: lidia.aceto@uniupo.it
[2] *University of Insubria, Italy*
E-mail: mariarosa.mazza@uninsubria.it

Abstract

In this talk we propose a rational preconditioner for an efficient numerical solution of linear systems arising from the discretization of multi-dimensional Riesz fractional diffusion equations. In particular, the discrete problem is obtained by employing finite difference or finite element methods to approximate the fractional derivatives of order α with $\alpha \in (1, 2]$. The proposed preconditioner is then defined as a rational approximation of the Riesz operator expressed as the integral of the standard heat diffusion semigroup. We show that, being the sum of k inverses of shifted Laplacian matrices, the resulting preconditioner belongs to the generalized locally Toeplitz class, a wide algebra of matrix sequences that can be linked to a function representing the asymptotic eigenvalue distribution as the matrix size diverges. As a consequence, we are able to provide the asymptotic description of the spectrum of the preconditioned matrices and we show that, despite the lack of clustering just as for the Laplacian, our preconditioner for α close to 1 and $k \neq 1$ reasonably small, provides better results than the Laplacian itself, while sharing the same computational complexity.

Acknowledgements: Work (partially) supported by GNCS-INdAM project CUP E53C22001930001.

The Short-term Rational Lanczos Method and Applications

Davide Palitta[1], *Stefano Pozza*[2], *Valeria Simoncini*[3]

[1] *Department of Mathematics, University of Bologna, Italy*
E-mail: davide.palitta@unibo.it
[2] *Faculty of Mathematics and Physics, Charles University, Czech Republic*
E-mail: pozza@karlin.mff.cuni.cz
[3] *Department of Mathematics, University of Bologna, Italy*
E-mail: valeria.simoncini@unibo.it

Abstract

Rational Krylov subspaces have become a reference tool in dimension reduction procedures for several application problems. When data matrices are symmetric, a short-term recurrence can be used to generate an associated orthonormal basis. In the past this procedure was abandoned because it requires twice the number of linear system solves per iteration than with the classical long-term method. We propose an implementation that allows one to obtain key rational subspace matrices without explicitly storing the whole orthonormal basis, with a moderate computational overhead associated with sparse system solves. Several applications are discussed to illustrate the advantages of the proposed procedure.

Acknowledgements: Work supported by Charles University Research programs No. PRIMUS/21/SCI/009 and UNCE/SCI/023, and by the Magica project ANR-20-CE29-0007 funded by the French National Research Agency.

References

[1] D. Palitta, S. Pozza, V. Simoncini. The Short-Term Rational Lanczos Method and Applications. *SIAM J. Sci. Comput.*, 44: A2843–A2870 (2022).

Extrapolation methods for choosing a regularization parameter

Claude Brezinski[1], *Caterina Fenu*[2], *Michela Redivo Zaglia*[3], *Giuseppe Rodriguez*[2]

[1] *Laboratoire Paul Painlevé, University of Lille, France*
E-mail: claude.brezinski@univ-lille.fr
[2] *Department of Mathematics and Computer Science, University of Cagliari, Italy*
E-mail: kate.fenu@unica.it, rodriguez@unica.it
[3] *Department of Mathematics, University of Padua, Italy*
E-mail: Michela.RedivoZaglia@unipd.it

Abstract

Various extrapolation methods have been proposed for the regularized solution of severely ill-conditioned linear systems; see, e.g., [1, 2, 3].

In this talk, after reviewing some of these methods, an extrapolation procedure for choosing the truncation parameter in TSVD/TGSVD will be presented. Its effectiveness will be tested by numerical experiments and compared to other existing methods.

References

[1] A. Bouhamidi, K. Jbilou, L. Reichel, H. Sadok, and Z. Wang. Vector extrapolation applied to truncated singular value decomposition and truncated iteration. *J. Eng.Math.*, 93(1):99–112, (2015).

[2] C. Brezinski, M. Redivo-Zaglia, G. Rodriguez, and S. Seatzu. Extrapolation techniques for ill-conditioned linear systems. *Numer. Math.*, 81(1):1–29, (1998).

[3] K. Jbilou, L. Reichel, and H. Sadok. Vector extrapolation enhanced TSVD for linear discrete ill-posed problems. *Numer. Algorithms*, 51(2):195–208, (2009).

Applications of trace estimation techniques

Yousef Saad [1]

[1] *Dpt. Computer Science and Engineering, University of Minnesota, USA*
E-mail: saad@umn.edu

Abstract

We discuss various applications of trace estimation techniques for evaluating functions of the form $\mathrm{Tr}(f(A))$ where f is a certain function. The first problem we consider that can be cast in this form is that of approximating the *Spectral density* or *Density of States*(DOS) of a matrix. The DOS is a probability density distribution that measures the likelihood of finding eigenvalues of the matrix at a given point on the real line and it is of enormous importance in solid state physics. Spectral densities can also be very useful in numerical linear algebra where they can help to estimate ranks. Other common linear algebra problems that can be solved with trace estimators are to extract the diagonal of a matrix inverse or its trace $\mathrm{Tr}(A^{-1})$, or to count the number of eigenvalues of a matrix in a real interval or complex region. In particular, estimating eigenvalue counts can be particularly important for methods that rely on rational approximation methods, such as FEAST. Rational approximation methods can in turn be used to estimate eigenvalue counts by exploiting trace estimators. We also discuss a few similar computations that arise in machine learning applications. Two computationally inexpensive methods to compute traces of matrix functions will be highlighted, namely, the Chebyshev expansion and the Lanczos Quadrature methods and a few numerical examples will be presented to illustrate the performances of these methods.

Acknowledgements: Work (partially) supported by NSF

References

[1] S. Ubaru and Y. Saad. *Applications of trace estimation techniques* In Proceedings of The 3rd Intern. Conf. of High Performance Computing in Science and Engineering (HPCSE 2017); Karolinka, Czech Republic, May 22–25, 2017, pp. 19–33 (2018), T. Kozubek et al. eds.

[2] E. Di Napoli, E. Polizzi and Y. Saad, *Efficient estimation of eigenvalue counts in an interval*, Numerical Linear Algebra with Applications, vol 23, pp. 674-692 (2016).

On generalized inverse of a vector, with applications to vector epsilon algorithm

A. Salam

Université du Littoral, LMPA, CS 80699.
50 F. Buisson, 62228 Calais, France.
E-mail: ahmed.salam@univ-littoral.fr

Abstract

The vector Epsilon-algorithm introduced by P. Wynn is a powerful method for accelarating the convergence of vector sequences. It is well known that the algorithm has been derived directly from the rules of scalar Epsilon algorithm, by replacing the inverse of a real number in the scalar case, with Samelson's inverse of a vector, in the vector case.

In this talk, we show that other expressions of interest of generalized inverse of vectors can be designed giving rise to new versions of the vector Epsilon-algorithm. Moreover, all these generalized inverses can be put easily in an unified framework, using Clifford algebra. Also, we give the necessary and sufficient condition for caracterizing the kernel of the new versions of the algorthm, where the kernel states for the set of sequences transformed by the algorithm to stationnnary sequences.

References

[1] C. Brezinski, M. Redivo-Zaglia, and A. Salam, *On the kernel of vector ε-algorithm and related topics*, Numer. Algorithms, **92** (2023) 207- 221.

[2] C. Brezinski, A. Salam, *Matrix and vector sequence transformations revisited*, Proc. Edinburgh Math. Soc., **38** (1995) 495-510.

[3] A. Salam, *An algebraic approach to the vector ε-algorithm*, Numer. Algorithms, **11** (1996) 327- 337.

[4] D. J. H. Garling, *Clifford Algebras: An Introduction*, London Mathematical Society Student Texts 78, 2011, Cambridge University Press.

[5] P. Wynn, *Acceleration techniques for iterated vector and matrix problems*, Math. Comp., **16** (1962) 301322.

Computing the generalized rational minimax approximation

Nir Sharon[1]

[1] *School of Mathematical Sciences, Tel-Aviv University, Israel*
E-mail: nsharon@tauex.tau.ac.il

Abstract

We present a unique optimization approach for estimating the minimax rational approximation. We use the fact that the optimization problem that appears in the uniform rational approximation is quasiconvex to define a bisection solver. This iterative process allows us to estimate the minimax up to a prescribed accuracy while posing additional constraints if needed. Finally, we will conclude the talk with several examples and applications, including evaluating matrix functions and their usage for spectrum-slicing applications.

Acknowledgements: This work was partially supported by the NSF-BSF award 2019752

Error bounds for the approximation of matrix functions with rational Krylov methods

Igor Simunec[1]

[1] *Scuola Normale Superiore di Pisa, Italy*
E-mail: igor.simunec@sns.it

Abstract

In this talk we present and compare some error bounds for the approximation of matrix vector products $f(A)\boldsymbol{b}$ and quadratic forms $\boldsymbol{b}^T f(A)\boldsymbol{b}$ for a Hermitian matrix A and a vector $\boldsymbol{b}$ with a rational Krylov subspace method. The error bounds are obtained by exploiting properties of rational Arnoldi decompositions, the Cauchy integral formula and the residue theorem to link the matrix function error to the residuals of shifted linear systems. This leads to both upper and lower bounds, generalizing the bounds derived in [1] for the Lanczos method. The accuracy of the bounds is demonstrated with several numerical experiments.

References

[1] T. Chen, A. Greenbaum, C. Musco, and C. Musco, *Error bounds for Lanczos-based matrix function approximation*, SIAM J. Matrix Anal. Appl., 43 (2022), pp. 787–811.

[2] I. Simunec, *Error bounds for the approximation of matrix functions with rational Krylov methods*, in preparation.

Perfect shifts for Hessenberg-Hessenberg pencils

N. Mastronardi[1], *M. Van Barel*[2], *R. Vandebril*[3], *P. Van Dooren*[4]

[1] *Istituto Applicazioni Calcolo, CNR, Bari, Italy*
E-mail: n.mastronardi@ba.iac.cnr.it
[2] *Department of Computer Science, KU Leuven, Belgium*
E-mail: marc.vanbarel@kuleuven.be
[3] *Department of Computer Science, KU Leuven, Belgium*
E-mail: raf.vandebril@kuleuven.be
[4] *Department of Mathematical Engineering, UCLouvain, Belgium*
E-mail: vandooren.p@gmail.com

Abstract

In this talk we analyze the stability of the problem of performing a rational QZ step with a shift that is an eigenvalue of a given regular pencil $H - \lambda K$ in unreduced Hessenberg–Hessenberg form. This problem appears when downdating orthogonal rational functions with prescribed poles, i.e., remove a node from the corresponding discrete inner product. In exact arithmetic, the backward rational QZ step moves the eigenvalue to the top of the pencil, while the rest of the pencil is maintained in Hessenberg–Hessenberg form, which then yields a deflation of the given shift. But in finite-precision the rational QZ step gets "blurred" and precludes the deflation of the given shift at the top of the pencil. In this talk we show that when we first compute the corresponding eigenvector to sufficient accuracy, then the rational QZ step can be constructed using this eigenvector, so that the exact deflation is also obtained in finite-precision.

If time permits, we show how the residual can be improved using a scaling procedure and how the method can be applied to general rank structured pencils.

Acknowledgements: The first and fourth authors were partly supported by Gruppo Nazionale Calcolo Scientifico (GNCS) of Istituto Nazionale di Alta Matematica (INdDAM). The second author was supported by the Research Council KU Leuven, C1-project C14/17/073 and by the Fund for Scientific Research–Flanders (Belgium), EOS Project no 30468160. The third author was supported by the Research Council KU Leuven (Belgium), project C16/21/002 and by the Fund for Scientific Research – Flanders (Belgium), project G0A9923N. The second and third author were also supported by the Fund for Scientific Research – Flanders (Belgium), project G0B0123N.

MSC08. In honour of Steve Kirkland's 60th Birthday

Hermie Monterde, Sarah Plosker, and Jane Breen

A Short Survey on the Scrambling Index of Primitive Digraphs
Mahmud Akelbek

Smallest positive eigenvalue of graphs
Sasmita Barik, Subhasish Behera

Markov chains: theory and applications
Jane Breen

Reminiscences of Steve Kirkland
Richard A. Brualdi

Speaker
Minerva Catral

Pretty good state transfer among large set of vertices
Ada Chan, Peter Sin

On Kemeny's constant and its applications
Emanuele Crisostomi, Robert Shorten

Some bounds for the energy of complex unit gain graphs
M. Rajesh Kannan, Aniruddha Samanta

Kemeny's constant and Braess edges
Sooyeong Kim

Minisymposium speaker
Chi-Kwong Li

Perfect state transfer on trees with small diameter
Steve Kirkland, Christopher van Bommel

Extremal Singular Graphs and Nut Graphs
Irene Sciriha

Stochastic Matrices Realising the Boundary of the Karpelevi? Region
Helena Smigoc, Stephen Kirkland

Limit points of Laplacian spectral radii of graphs
Vilmar Trevisan, Francesco Belardo, Elismar Oliveira

Rank one perturbations for cone reachability and holdability
Michael Tsatsomeros, Faith Zhang

Fractional revival on graphs
Xiaohong Zhang

A Short Survey on the Scrambling Index of Primitive Digraphs

Mahmud Akelbek[1]

[1] *Department of Mathematics, Weber State University, USA*
E-mail: makelbek@weber.edu

Abstract

The *scrambling index* of a primitive digraph is the smallest positive integer k such that for every pair of vertices u and v, there exists a vertex w such that from vertices u and v we can get to vertex w by a directed walk of length k. Akelbek and Kirkland [2009] introduced and obtained an upper bound on the scrambling index of a primitive digraph D in terms of the order and girth of D, and gave a characterization of the primitive digraphs with the largest scrambling index. Liu and Huang [2010] generalized the concept of scrambling index, they gave the upper bounds on the generalized scrambling index on for various classes of primitive digraphs. In this talk, I will present some of the recent progress and results related to scrambling index and generalized scrambling index.

References

[1] M. Akelbek, S. Kirkland, *Coefficients of Ergodicity and the Scrambling Index*, Linear Algebra Appl., 430 (2009) 1111-1130.

[2] M. Akelbek, S. Kirkland, *Primitive Digraphs with the Largest Scrambling Index*, Linear Algebra Appl., 430 (2009) 1099-1110.

[3] M. Akelbek, J. Shen, S. Fital, *A bound on the scrambling index of a primitive matrix using Boolean rank*, Linear Algebra Appl., 431 (2010), 1923-1931.

[4] Y Huang, B Liu, *Generalized scrambling indices of a primitive digraph*, Linear Algebra Appl., 433 (2010) 1798-1808.

[5] B Liu, Y Huang, *The scrambling index of primitive digraphs*, Computers & Mathematics with Applications, 2010, Volume 60, Issue 3 (2010) 706-721.

Smallest positive eigenvalue of graphs

Sasmita Barik[1], Subhasish Bahera[2]

[1] School of Basic Sciences, IIT Bhubaneswar, Bhubaneswar, India
E-mail: sasmita@iitbbs.ac.in
[2] School of Basic Sciences, IIT Bhubaneswar, Bhubaneswar, India
E-mail: sb52@iitbbs.ac.in

Abstract

Let G be a simple graph with adjacency matrix $A(G)$ and $\tau(G)$ denote the smallest positive eigenvalue of $A(G)$. This eigenvalue plays an important role in spectral graph theory as well as in chemical graph theory. We discuss about the bounds on the smallest positive eigenvalue of some graph classes. Extremal graphs are also presented in most of the cases.

References

[1] S. Akbari, S. J. Kirkland. On unimodular graphs. *Linear Algebra Appl.*, 421:3–15 (2007).

[2] S. Barik, S. Behera. On the smallest positive eigenvalue of bipartite unicyclic graphs with a unique perfect matching. *Discrete Math.*, 346(2):113252 (2023).

[3] S. Barik, S. Behera. On the smallest positive eigenvalue of bipartite unicyclic graphs with a unique perfect matching II. *Linear Multiliear Alg.*, DOI:10.1080/03081087.2023.2177247, (2023).

[4] C. D. Godsil. Inverses of trees. *Combinatorica*, 5:33–39 (1985).

[5] I. Gutman. Bounds for the smallest positive eigenvalue of a bipartite molecular graph. *MATCH Commun. Math. Comput. Chem.*, 11:75–86 (1981).

[6] A. J. Hoffman. On limit points of spectral radii of nonnegative symmetric integral matrices, in: Y. Alavi et al. (Eds). *Graph Theory and its Applications*, Lecture Notes in Mathematics, Springer, New York, 303:165–172 (1972).

[7] J. H. Smith, Some properties of the spectrum of a graph, in: Richard Guy (Editions). *Combinatorial Structures and Their Applications*, 403–406 (1970)

[8] S. Pavlíková, J. Krč-jediný. On the inverse and dual index of a tree. *Linear Multilinear Alg.*, 28:93–109 (1990).

Markov chains: theory and applications

Jane Breen[1]

[1] *Faculty of Science, Ontario Tech University, Canada*
E-mail: jane.breen@ontariotechu.ca

Abstract

In this talk, I will discuss some results in the area of Markov chains that are inspired by or in collaboration with Steve Kirkland. These results incorporate techniques from spectral graph theory and combinatorial matrix theory, as well as pull from a wide range of applications in a variety of domains.

Reminiscences of Steve Kirkland

Richard A. Brualdi[1]

[1] *Department of Mathematics, UW-Madison, USA*
E-mail: brualdi@math.wisc.edu

Abstract

I shall recount some of my interactions and work with Steve Kirkland.

References

[1] R.A. Brualdi and S. Kirkland, Aztec diamonds and digraphs, and Hankel determinants of Schröder numbers. *J. Combin. Theory, Ser. B*, 94: 334-351 (2005). (40)

[2] R.A. Brualdi and S. Kirkland, Totally nonnegative $(0,1)$-matrices. *Linear Algebra Appl.*, 432: 1650-1662 (2010). (3)

Refined inertias of full and hollow positive sign patterns

Adam H. Berliner[1], *Minerva Catral*[2], *D. D. Olesky*[3], *P. van den Driessche*[4]

[1] *Department of Mathematics, Statistics, and Computer Science, St. Olaf College, USA*
E-mail: berliner@stolaf.edu
[2] *Department of Mathematics, Xavier University, USA*
E-mail: catralm@xavier.edu
[3] *Department of Computer Science, University of Victoria, Canada*
E-mail: dolesky@cs.uvic.ca
[4] *Department of Mathematics and Statistics, University of Victoria, Canada*
E-mail: vandendr@uvic.ca

Abstract

For an $n \times n$ matrix A, the *inertia* of A is the 3-tuple $\mathrm{i}(A) = (n_+, n_-, n_0)$ where $n_+ + n_- + n_0 = n$ and n_+, n_-, n_0 equal the number of eigenvalues of A with positive, negative, and zero real parts (respectively). The *refined inertia* of A is the 4-tuple $\mathrm{ri}(A) = (n_+, n_-, n_z, 2n_p)$ where n_z is the number of eigenvalues of A equal to zero and $2n_p$ is the number of nonzero pure imaginary eigenvalues of A (note that $n_0 = n_z + 2n_p$). We investigate inertias and refined inertias of full positive sign patterns, and of sign patterns that have positive off-diagonal entries and zero diagonal entries, i.e., hollow positive sign patterns. For positive sign patterns, we prove that every refined inertia $(n_+, n_-, n_z, 2n_p)$ with $n_+ \geq 1$ can be realized. For hollow positive sign patterns, we prove that every refined inertia with $n_+ \geq 1$ and $n_- \geq 2$ can be realized. Constructions of matrix realizations illustrating these results are given.

Pretty good state transfer among large set of vertices

Ada Chan [1], *Peter Sin*[2]

[1] *Dept. of Mathematics and Statistics, York University, Canada*
E-mail: ssachan@yorku.ca
[2] *Dept. of Mathematics, University of Florida, U.S.A.*
E-mail: sin@ufl.edu

Abstract

For the continuous-time quantum walk on graphs, it is well known that perfect state transfer can occur from a vertex to at most one other vertex. By constrast, there is pretty good state transfer between any two vertices of degree two in the cartesian product of a path of length two and a path of length three [2]. Building on Godsil, Kirkland, Severini and Smith's characterization of paths with pretty good state transfer [1], we look for arbitrarily large sets of vertices with pairwise pretty good state transfer in cartesian products of paths.

This is joint work with Peter Sin.

Acknowledgements: Work (partially) supported by a grant from the Simons Foundation (#633214 to Peter Sin) and a Discovery Grant from the Natural Sciences and Engineering Research Council of Canada (RGPIN-2021-03609 to Ada Chan).

References

[1] C. D. Godsil, S. Kirkland, S. Severini, and J. Smith, *Number-theoretic nature of communication in quantum spin systems*, Phys. Rev. Lett. 109(5), 050502 (2012).

[2] H. Pal, B. Bhattacharjya, *Pretty good state transfer on some NEPS*, Discrete Math. 340 (2017), no. 4, 746-752.

On Kemeny's constant and its applications

Emanuele Crisostomi[1], Robert Shorten[2]

[1] Department of Energy, Systems, Territory and Constructions Engineering, University of Pisa, Italy
E-mail: emanuele.crisostomi@unipi.it
[2] Dyson School of Design Engineering, Imperial College London, UK
E-mail: r.shorten@imperial.ac.uk

Abstract Kemeny's constant has for long attracted the interest of the research community, looking for practical explanations regarding its constancy. Nowadays, Kemeny's constant is recognized as a peculiar measure of the connectivity of a network, and has found several applications in different fields, ranging from urban networks to epidemiology, from robotic surveillance to pollution mitigation. This talk aims at providing a quick overview of such applications, and at identifying some distinguishing characteristics of Kemeny's constant.

Kemeny's constant of a finite discrete-time Markov chain had been originally defined as the expected time to reach a randomly-chosen state - selected with probability proportional to its stationary distribution - starting from a fixed initial state [1]. This parameter is surprisingly independent from the choice of the initial state, and several papers have been devoted to motivate or interpret the meaning of Kemeny's constant, as in terms of the expected number of steps before a "lost" surfer reaches the desired destination [2], or in terms of the expected number of steps before a Markov chain becomes "close" to its stationary distribution [3]. In general, Kemeny's constant exhibits the ability to provide a peculiar connectivity measure of a network, and significantly differs from other connectivity indicators.

It was proved in [4] that for an irreducible stochastic matrix T, there exists another stochastic matrix T^* with the property that its non-zero entries are a subset of the non-zero entries of T (and therefore it can be obtained without adding extra links to the original graph), such that its Kemeny's constant is the minimum among the matrices with the same property. This property has a nice interpretations in terms of traffic networks, where the nodes of the graph correspond to roads, as this implies to stating that - in terms of random walks - a *ring road* is the most convenient way to plan an urban network. Kemeny's constant has been also used as a ranking indicator to assess the importance of a node, or an edge, in a graph, in terms of the loss - or increase - of connectedness of the network after that node, or edge, is removed from the graph. Such a possibility was explored again in the context of road networks [5] to evaluate the

importance of single roads. Kemeny's constant has found useful applications also in other fields than road networks. For instance, it was shown that the minimization of Kemeny's constant is convenient to design stochastic surveillance strategies for quickest detection of anomalies in network environments. Also, the ability of Kemeny's constant to identify critical nodes in a network was also used as a means to identify the individuals in a population more likely to facilitate the spreading of a virus.

This talk will review the aforementioned applications of Kemeny's constant, and will try to extract and identify the peculiar characteristics of this connectivity indicator which make it different from the many other existing ones.

References

[1] J. G. Kemeny, J. L. Snell *Finite Markov Chains*. Princeton: Van Nostrand. 1960.

[2] M. Levene, G. Loizou. Kemeny's Constant and the Random Surfer. *American Mathematical Monthly*, 109: 741–745 (2002).

[3] J. J. Hunter. Mixing times with applications to perturbed Markov chains. *Linear Algebra Appl.*, 417(1): 108–123 (2006).

[4] S. Kirkland. Fastest Expected Time to Mixing for a Markov Chain on a Directed Graph. *Linear Algebra and its Applications*, 433:1988–1996 (2010).

[5] E. Crisostomi, S. Kirkland, R. Shorten. A Google-like model of road network dynamics and its application to regulation and control. *International Journal of Control*, 84(3): 633–651 (2011).

Some bounds for the energy of complex unit gain graphs

Aniruddha Samanta[1], *M. Rajesh Kannan*[2]

[1] *School of Mathematical Sciences, National Institute of Science Education and Research, Bhubaneswar, Odisha 752050, India.*
E-mail: aniruddha.sam@gmail.com
[2] *Department of Mathematics, Indian Institute of Technology Hyderabad, Kandi, Sangareddy- 502 285, Telangana State, India.*
E-mail: rajeshkannan@math.iith.ac.in, rajeshkannan1.m@gmail.com

Abstract

A complex unit gain graph ($\mathbb{T}$-gain graph), $\Phi = (G, \varphi)$ is a graph where the gain function φ assigns a unit complex number to each orientation of an edge of G and its inverse is assigned to the opposite orientation. The associated adjacency matrix $A(\Phi)$ is defined canonically. The energy $\mathcal{E}(\Phi)$ of a $\mathbb{T}$-gain graph Φ is the sum of the absolute values of all eigenvalues of $A(\Phi)$. In this talk, we shall discuss some of the bounds for the energy of complex unit gain graphs in terms of the minimum vertex degree, maximum edge degree and matching number of the underlying graph G.

Kemeny's constant and Braess edges

Sooyeong Kim[1]

[1] *Department of Mathematics and Statistics, York University, Canada*
E-mail: kimswim@yorku.ca

Abstract

It is a great pleasure to give a talk in this special session as I am greatful to Steve for his supervision during my Ph.D. This talk will be about one of his contributions in Markov chains.

Kemeny's constant can measure the average travel time for a random walk between two randomly chosen vertices. So, it can serve as a proxy for identifying an edge whose insertion increases the average travel time, as opposed to one's anticipation that the more edges a graph has, the less travel time a random walker on the graph takes. Such an edge is called a *Braess edge*, which was introduced by Kirkland and Zeng [1] in 2016. I will summarize what has been studied in this area since then, and present some recent results.

Acknowledgements: This is supported by York-Fields Postdoctoral Fellowship grant.

References

[1] S. Kirkland and Z. Zeng. Kemeny's constant and an analogue of Braess' paradox for trees. *Electronic Journal of Linear Algebra*, 31:444–464, 2016.

Quantum computing and graph theory

Chi-Kwong Li

Department of Mathematics, College of William & Mary, Williamsburg, VA 23187, USA

E-mail: ckli@math.wm.edu

Abstract

We discuss some matrix problems related to quantum computing and graph theory. For example, we consider bounds for the spectral gaps of Hamiltonian in using adiabatic quantum computing method to study graph theory problems. We also discuss graph parameters of operator systems by considering their underlying quantum channels.

Acknowledgements: Work (partially) supported by the Simons Foundation

Perfect state transfer on trees with small diameter

Steve Kirkland[1], *Christopher van Bommel*[2]

[1] *Department of Mathematics, University of Manitoba, Canada*
E-mail: stephen.kirkland@umanitoba.ca
[2] *Department of Mathematics and Statistics, University of Guelph, Canada*
E-mail: cvanbomm@uoguelph.ca

Abstract

The transfer of quantum information in a quantum computer can be modelled by a spin network, which in turn is represented by a graph. There is particular interest in perfect state transfer (PST), the scenario in which quantum information is transferred with perfect fidelity. PST has been extensively investigated using the techniques of spectral graph theory. The paths on 2 and 3 vertices are known to exhibit PST, and there is a long–standing open question as to whether there are any other trees that do so. In this talk we report on some progress on that question, showing that PST is impossible for trees of diameter 4, as well as for mirror–symmetric trees on an even number of vertices.

Extremal Singular Graphs and Nut Graphs

Irene Sciriha[1]

[1] *Dpt.of Mathematics, Faculty of Science, University of Malta, Malta*
E-mail: irene.sciriha-aquilina@um.edu.mt

Abstract

The *rank* of the 0–1–adjacency matrix $\mathbf{A}$ of a graph is the dimension of the orthogonal complement of the nullspace of $\mathbf{A}$. A graph G is singular of *nullity* η if the nullspace of $\mathbf{A}$ has dimension $\eta \geq 1$. Such a graph contains η *cores* determined by the non–zero entries of the vectors in a basis for the nullspace of $\mathbf{A}$. These cores are induced subgraphs of *singular configurations*, the latter occurring as induced subgraphs of G. A *minimal basis* for the nullspace of $\mathbf{A}$, corresponding to a *fundamental system* $\mathcal{F}$ of cores, has the minimum sum of the vector weights which are strictly bounded above by the *core–width* τ. We explore how η and τ control the order of the singular configurations. A graph is *extremal singular* if $\eta + \tau$ reaches the maximum possible. We show that among graphs of nullity one, extremal graphs are *nut graphs*, that is the nullspace of $\mathbf{A}$ is generated by a vector with no zero entries.

Acknowledgements: Work supported by UM for project GCGN2023 (*Spectra of Nanostructures*)

References

[1] I. Sciriha, A. Farrugia, *From Nut Graphs to Molecular Structure and Conductivity*, University of Kragujevac, Serbia, 2021.

[2] I. Sciriha, M. Debono, M. Borg, P.W. Fowler, and B. T. Pickup. Interlacing-extremal graphs. Ars Math. Contemp., 6—(2):261–278, 2013.

[3] I. Sciriha. Maximal and extremal singular graphs II, Journal of Mathematical Sciences-Springer (JMS), 182-2:117–125, 2012.

[4] I. Sciriha. Maximal and extremal singular graphs I, Sovremennaya Matematika i Ee Prilozheniya-Contemporary Mathematics and its Applications, 71:1–9, 2011.

[5] I. Sciriha. Extremal non-bonding orbitals. MATCH Commun. Math. Comput. Chem. http://www.pmf.kg.ac.rs/match/, 63(3):751–768, 2010.

[6] I. Sciriha. Maximal core size in singular graphs. Ars Math. Contemp., 2(2):217–229, 2009

[7] I. Sciriha and I. Gutman. Nut graphs: maximally extending cores. Util. Math., 54:257–272, 1998.

Stochastic Matrices Realising the Boundary of the Karpelevič Region

Stephen Kirkland[1], *Helena Šmigoc*[2]

[1] *Department of Mathematics, University of Manitoba, Canada*
E-mail: stephen.kirkland@umanitoba.ca
[2] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: helena.smigoc@ucd.ie

Abstract

A celebrated result of Karpelevič [1] describes Θ_n, the collection of all eigenvalues arising from the stochastic matrices of order n. The boundary of Θ_n consists of roots of certain one-parameter families of polynomials. Johnson and Paparella [2] construct, for each λ on the boundary of the Θ_n, a stochastic matrix of order n having λ as an eigenvalue. In this talk we present the results from [3] where all possible stochastic realizations of an eigenvalue on the border of Θ_n are considered.

Acknowledgements: This work was supported by University College Dublin: Grant SF1588 and NSERC Discovery: Grant RGPIN-2019-05408.

References

[1] F. Karpelevič. On the characteristic roots of matrices with nonnegative elements. *in Eleven Papers Translated from the Russian, Amer. Math. Soc. Transl. (2),* 140:79-100, (1988).

[2] C. Johnson and P. Paparella. A matricial view of the Karpelevič theorem. *Linear Algebra Appl.*, 520 (2017), 115.

[3] S. Kirkland and H. migoc. Stochastic matrices realising the boundary of the Karpelevi region. *Linear Algebra Appl.*, 635 (2022), 116138.

Limit points of Laplacian spectral radii of graphs

Francesco Belardo[1], *Elismar R. Oliveira*[2], *Vilmar Trevisan*[2]

[1] *Department of Mathematics and Applications, University of Naples Federico II, Italy*
E-mail: fbelardo@unina.it
[2] *Instituto de Matemática e Estatística, UFRGS, Porto Alegre, Brazil*
E-mail: elismar.oliveira@ufrgs.br, trevisan@mat.ufrgs.br

Abstract

In 1972 A. J. Hoffman [1] proposed to determine which numbers are limit points of spectral radii of matrices. Let $\mathcal{A}$ be the set of all symmetric matrices of all orders, every entry of which is a non-negative integer and $R = \{\rho : \rho = \rho(A) \text{ for some } A \in \mathcal{A}\}$ where $\rho(A)$ is the largest eigenvalue of A. He showed that it is sufficient to consider matrices of $\mathcal{A}$ having only entries in $\{0, 1\}$ and 0 diagonal, e.g. adjacency matrices of graphs. Additionally, he determined all limit points of $R \leq \sqrt{2+\sqrt{5}}$. More precisely, let $\tau = \frac{1+\sqrt{5}}{2}$ (the golden mean). For $n = 1, 2, \ldots$, let $\overline{\beta}_n$ be the positive root of

$$Q_n(x) = x^{n+1} - \left(1 + x + x^2 + \cdots + x^{n-1}\right).$$

Let $\bar{\alpha}_n = \bar{\beta}_n^{1/2} + \bar{\beta}_n^{-1/2}$. Then $2 = \bar{\alpha}_1 < \bar{\alpha}_2 < \cdots$ are all the limit points of R smaller than $\lim_{n\to\infty} \bar{\alpha}_n = \tau^{1/2} + \tau^{-1/2} = \sqrt{2+\sqrt{5}}$ $(= 2.05+)$.

Then, in 1989, J. B. Shearer [2] extended this result. He showed that every real number larger than $\sqrt{2+\sqrt{5}}$ is a limit point of R.

Both Hoffman and Shearer found sequences of trees whose spectral radii were limit points. It seems remarkable that the set composed only by the largest eigenvalue of the adjacency matrix of trees is dense in the interval $[\sqrt{2+\sqrt{5}}, \infty)$.

In this talk, we are interested in a problem originated by Hoffman's question. More specifically, we want to study the Laplacian version of Hoffman and Shearer's results, that is, what real numbers are limit points of the spectral radii of Laplacian matrices of graphs. The converse of this problem may also be viewed as *which sequence of graphs have limit points*.

We recall the work of Guo [3], which may be seen as the analogous of Hoffman's. Let $\omega = \frac{1}{3}(\sqrt[3]{19+3\sqrt{33}} + \sqrt[3]{19-3\sqrt{33}} + 1)$, $\beta_0 = 1$ and β_n, $n \geq 1$ be the largest positive root of

$$P_n(x) = x^{n+1} - \left(1 + x + \cdots + x^{n-1}\right)(\sqrt{x}+1)^2.$$

Let $\alpha_n = 2 + \beta_n^{\frac{1}{2}} + \beta_n^{-\frac{1}{2}}$. Then $4 = \alpha_0 < \alpha_1 < \alpha_2 < \cdots$ are all of the limit points of Laplacian spectral radii of graphs smaller than $\lim_{n\to\infty} \alpha_n = 2 + \omega + \omega^{-1}$ $(= 4.38+)$.

By analogy to the adjacency case, it is natural to ask whether any real number $\mu \geq 2 + w + w^{-1} = 4.38+$ is the limit point of the Laplacian spectral radii of graphs. We refer to Figure 1 for an illustration of the current state of knowledge.

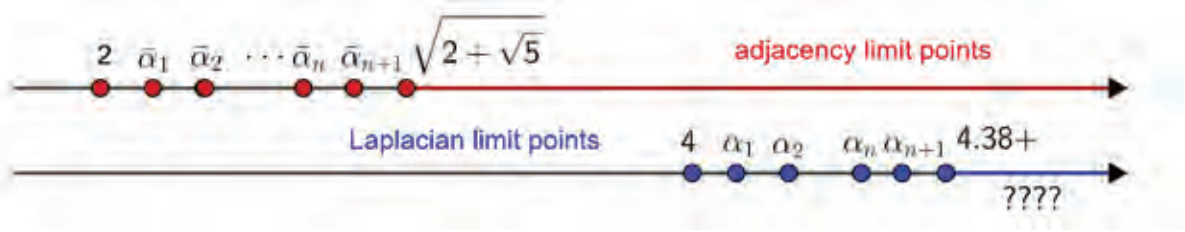


Figure 1: Conjecture about Laplacian limit points

In this talk we explain the development of some analytical tools allowing one to study the density of Laplacian spectral radius in $[4.38+, \infty)$. We first adapt Shearer's method to the Laplacian case, verifying that it is not sufficient to prove density. In spite of the fact that the method produces sequence of caterpillars having a limit point, we show that this limit point is not the *desired* number. As a consequence, we find a whole interval where the method produces no Laplacian limit points. We then extend Shearer's method to the class of linear trees, e.g. we define sequences of linear trees whose Laplacian spectral radius has limit points. This provides a generalization of Shearer's process, since caterpillars is a subclass of linear trees. Our generalization improves Shearer's process in the sense that we are able to find a larger set of limit points.

Acknowledgements: This work is partially supported by MATH-AMSUD under project GSA, brazilian team financed by CAPES process 88881.694479/2022-01. V. Trevisan acknowledges partial support of CNPq grants 409746/2016-9 and 310827/2020-5, and FAPERGS grant PqG 17/2551-0001.

References

[1] Hoffman, A. J. On limit points of spectral radii of non-negative symmetric integral matrices. In Graph theory and Applications. Springer, 1972, pp. 165–172.

[2] Shearer, J. B. On the distribution of the maximum eigenvalue of graphs. Linear Algebra and its Applications 114 (1989), 17–20.

[3] Guo, J.-M. On limit points of laplacian spectral radii of graphs. Linear algebra and its applications 429, 7 (2008),1705–1718.

Rank one perturbations for cone reachability and holdability

Michael Tsatsomeros[1], *Faith Zhang*[2],

[1] *Mathematics & Statistics, Washington State University, USA*
E-mail: tsat@wsu.edu
[1] *Mathematics & Statistics, University of Massachusetts Amherst, USA*
E-mail: yzhang@math.umass.edu

Abstract

A choice of feedback control in a linear control system is proposed in order to achieve that a trajectory eventually enters the nonnegative orthant and remains therein for all time thereafter. This is achieved by imposing the strong Perron-Frobenius property and involves altering the eigenvalues, as well as the left eigenvectors via rank one perturbations.

Acknowledgements: In celebration of Steve Kirkland's 60th birthday, his contributions to linear algebra and his unwavering friendship over many years.

References

[1] Michael J. Tsatsomeros and Yueqiao Faith Zhang, Recursive rank one perturbations for pole placement and cone reachability, *Applied Mathematics and Computation*, Vol. 416, Issue C, March 2022.

Fractional revival on graphs

Xiaohong Zhang[1]

[1] *Centre de Recherches Mathématiques, Université de Montréal, Canada*
E-mail: xiaohong.zhang@umontreal.ca

Abstract

Let M be the adjacency matrix or Laplacian matrix of a graph X. The transition matrix of the continuous time quantum walk at time t is $U(t) = e^{itM}$. Let u, v be vertices of X. If there is a time t such that $U(t)e_u = \alpha e_u + \beta e_v$, then we say that X admits fractional revival at time t. In this talk, we present some recent developments on fractional revival on graphs.

MSC09. Polynomial and rational matrices and applications

Maria del Carmen Quintana Ponce,
Vanni Noferini, Paul M. Van Dooren

Linearization of meromorphic matrix-valued functions
Rafikul Alam, Jibrail Ali

On the Rellich eigendecomposition of para-Hermitian matrices on the unit circle
Giovanni Barbarino, Vanni Noferini

Error representation of block rational Krylov methods by means of rational matrices
Angelo Alberto Casulli, Leonardo Robol

Computing the nearest (structured) singular matrix polynomial
Miryam Gnazzo, Nicola Guglielmi

Randomized sketching of nonlinear eigenvalue problems
Daniel Kressner, Stefan Guettel, Bart Vandereycken

Spectral Localization and the Infinite Elementary Divisor Structure of Matrix Polynomials
Steve Mackey, Richard Hollister

Filters connecting spectrally equivalent nonsingular polynomial matrices
Silvia Marcaida, Agurtzane Amparan, Ion Zaballa

Rational approximation and linearisation for nonlinear eigenvalue problems and nonlinear systems
Karl Meerbergen

Nearest singular pencil via Riemannian optimization
Lauri Nyman, Froilán Dopico, Vanni Noferini

Error analysis of compact Arnoldi methods for linearized polynomial eigenvalue problems
Javier Perez

Isomorphisms between Ansatz Spaces over Classical Polynomial Bases
Vasilije Perovic, D. Steven Mackey

Rectangular multiparameter eigenvalue problems
Bor Plestenjak, Michiel E. Hochstenbach, Tomaž Košir

Computing zeros of rational functions and matrices
Maria del Carmen Quintana Ponce, Yuji Naktsukasa, Vanni Noferini

Eigenvector error bounds and perturbation for nonlinear eigenvalue problems
Francoise Tisseur, Yuji Nakatsukasa, Javier Perez

Computing a compact local Smith McMillan form
Paul Van Dooren, Vanni Noferini

DiagonalizableMatrix Polynomials
Ion Zaballa

Linearization of meromorphic matrix-valued functions

Rafikul Alam [1], Jibrail Ali [2]

[1] Department of Mathematics, IIT Guwahati, India
E-mail: rafik@iitg.ac.in
[2] Department of Mathematics, IIT Guwahati, India
E-mail: jibrail@iitg.ac.in

Abstract Let $M : \Omega \longrightarrow \mathbb{C}^{n\times n}$ be meromorphic and regular, where $\Omega \subset \mathbb{C}$ is a bounded domain. Then $\lambda \in \Omega$ is said to be a *zero* of $M(s)$ if there exists a holomorphic function $v : \Omega \longrightarrow \mathbb{C}$ such that $v(\lambda) \neq 0$ and $\lim_{s\to\lambda} M(s)v(s) = 0$. Consider the spectrum of M given by

$$\sigma(M) := \{\lambda \in \Omega : \lambda \text{ is a zero of } M(s)\}.$$

When $M(s)$ is a rational matrix (entries are rational functions), it is possible to construct a regular matrix pencil $A - sB$, called a linearization of $M(s)$, such that $M(s)$ and $A - sB$ are "equivalent" and hence $\sigma(M) = \sigma(A, B)$, where $\sigma(A, B)$ is the spectrum of $A - sB$. We extend the concept of linearization of a rational matrix to the case of a meromorphic matrix-valued function. We show that it is possible to construct a regular operator pencil $A - sB$, which we refer to as a linearization of $M(s)$, such that $M(s)$ and $A - sB$ are "equivalent" and that $\sigma(M) = \sigma(A, B)\cap\Omega$. Thus, the eigenvalue problem $M(\lambda)v = 0$ is subsumed by the generalized eigenvalue problem $(A - \lambda B)u = 0$.

Acknowledgements: Work of the second author supported by the Govt. of India in the form of an Institute Fellowship.

References

[1] R. Alam, N. Behera. Linearizations for rational matrix functions and Rosenbrock system polynomials. *SIAM J. Matrix Anal. Appl.*, 37: 354–380 (2016).

[2] R. K. Das, R. Alam. Affine spaces of strong linearizations for rational matrix functions and the recovery of eigenvectors and minimal bases. *Linear Algebra Appl.*, 569: 335–368 (2019).

On the Rellich eigendecomposition of para-Hermitian matrices on the unit circle

Giovanni Barbarino[1], *Vanni Noferini*[1]

[1] *Department of Mathematics and Systems Analysis, Aalto University, Helsinki, Finland*
E-mail: giovanni.barbarino@aalto.fi
E-mail: vanni.noferini@aalto.fi

Abstract

The eigendecompositions of holomorphic para-Hermitian matrices, matrix-valued functions that are Hermitian on the unit circle, is a fundamental key in signal processing and decorrelation methods. In this presentation, the existence of an eigendecomposition through paraunitary base change is discussed, showing that in general there does not exist a holomorphic decomposition, but it can be performed in the field of Puiseux series. This generalizes the celebrated theorem of Rellich for matrix-valued functions that are analytic and Hermitian on the real line. In fact, a version of Rellich's theorem can be stated for matrix-valued function that are analytic and Hermitian on any line or any circle on the complex plane. Moreover, these results can be extended to para-Hermitian matrices whose entries are Puiseux series, and in particular can be used to examine the singular value decomposition of rectangular matrices whose entries are Puiseux series. Finally, the same results allow an analysis of the stability for the finite eigenvalues of *-palindromic matrix polynomials through their associated sign characteristics and features.

Acknowledgements: Work supported by the Alfred Kordelinin säätiö Grant No. 210122, in collaboration with professors Ian Proudler, Jennifer Pestana and Stephan Weiss of Strathclyde University.

Error representation of block rational Krylov methods by means of rational matrices

Angelo A. Casulli[1], *Leonardo Robol*[2]

[1] *Scuola Normale Superiore, Pisa (PI), Italy*
E-mail: angelo.casulli@sns.it
[2] *Department of Mathematics, University of Pisa, Pisa (PI), Italy*
E-mail: leonardo.robol@unipi.it

Abstract

We present an algorithm for the solution of Sylvester equations with right-hand side of low rank, based on projection onto a block rational Krylov subspace. Extending the convergence analysis in [2] to the block case, we link the convergence with the problem of minimizing the norm of a small rational matrix over the spectra or field-of-values of the involved matrices. This is in contrast with the non-block case, where the minimum problem is scalar, instead of matrix-valued. Replacing the norm of the objective function with an easier to evaluate function yields several adaptive pole selection strategies, providing a theoretical analysis for known heuristics, as well as effective novel techniques.

References

[1] A. Casulli, L. Robol, *An efficient block rational Krylov solver for Sylvester equations with adaptive pole selection*. arXiv:2301.08103, 2023.

[2] B. Beckermann. *An error analysis for rational Galerkin projection applied to the Sylvester equation*. SIAM Journal on Numerical Analysis, 2011.

Computing the nearest (structured) singular matrix polynomial

Miryam Gnazzo[1], *Nicola Guglielmi*[1]

[1] *Division of Mathematics, Gran Sasso Science Institute, Italy*
E-mail: miryam.gnazzo@gssi.it, nicola.guglielmi@gssi.it

Abstract

Given a set of matrices $A_i \in \mathbb{C}^{n\times n}$, we consider a regular matrix polynomial $P(\lambda) = \sum_{i=0}^{d} \lambda^i A_i$ of degree d. An interesting problem consists in the computation of the nearest singular polynomial in the form $\widetilde{P}(\lambda) = \sum_{i=0}^{d} \lambda^i (A_i + \Delta A_i)$. For instance, this can be important when $\det(P(\lambda))$ represents the characteristic equation of a system of differential-algebraic equations, to robustly guarantee its well-posedness. We extend the idea presented in [2], which consists of imposing that the determinant vanishes on a prescribed set of complex points $\{\mu_j\}_{j=1}^{m}$, with m larger than the degree of $\det(P(\lambda))$. This is obtained by a two-level procedure, in which for a fixed perturbation size $\varepsilon = \|[\Delta A_0, \ldots, \Delta A_d]\|_F$ we minimize

$$F_\varepsilon(\Delta A_0, \ldots, \Delta A_d) = \frac{1}{2}\sum_{j=1}^{m} \sigma_{\min}^2\left(\widetilde{P}(\mu_j)\right),$$

where $\sigma_{\min}$ denotes the smallest singular value, and then we find the smallest value ε such that the functional F_ε vanishes.

Whenever the singularity of the polynomial is determined by the property that the perturbed matrices $A_i + \Delta A_i$ have a common (left/right) kernel, we can show that the perturbations have a low rank property, which can be exploited by the algorithm.

An additional constraint that can be addressed by the method in a natural way is to include given structures to perturbation matrices, like a certain sparsity pattern determined by the original matrices or structures involving the whole matrix polynomial, like palindromic properties.

Acknowledgements: Work (partially) supported by Italian MUR within the PRIN-2017 Project "Discontinuous dynamical systems: theory, numerics and applications" and by the INdAM Research group GNCS

References

[1] M. Gnazzo, N. Guglielmi. Computing the closest singular matrix polynomial. *arXiv preprint arXiv:2301.06335*, (2023).

[2] N. Guglielmi, C. Lubich, and V. Mehrmann. On the nearest singular matrix pencil. *SIAM Journal on Matrix Analysis and Applications*, 38: 776–806 (2017).

Randomized sketching of nonlinear eigenvalue problems

Stefan Güttel[1], *Daniel Kressner*[2], *Bart Vandereycken*[3]

[1] *Department of Mathematics, The University of Manchester, M139PL, Manchester, UK*
[2] *Institute of Mathematics, EPFL, CH-1015 Lausanne, Switzerland*
E-mail: daniel.kressner@epfl.ch
[3] *Section of Mathematics, University of Geneva, CH-1205 Geneva, Switzerland*

Abstract

Rational approximation is a powerful tool to obtain accurate surrogates for nonlinear functions that are easy to evaluate and linearize. The interpolatory adaptive Antoulas–Anderson (AAA) method is one approach to construct such approximants numerically. For large-scale vector- and matrix-valued functions, however, the direct application of the set-valued variant of AAA becomes inefficient. We propose and analyze a new sketching approach for such functions called sketchAAA that, with high probability, leads to much better approximants than previously suggested approaches while retaining efficiency. The sketching approach works in a black-box fashion where only evaluations of the nonlinear function at sampling points are needed. Numerical tests with nonlinear eigenvalue problems illustrate the efficacy of our approach, with speedups above 200 for sampling large-scale black-box functions without sacrificing on accuracy.

References

[1] S. Güttel, D. Kressner, and B. Vandereycken. *Randomized sketching of nonlinear eigenvalue problems*, arXiv:2211.12175, 2022.

Spectral Localization and the Infinite Elementary Divisor Structure of Matrix Polynomials

Steve Mackey[1], *Richard Hollister*[2]

[1] *Department of Mathematics, Western Michigan University, USA*
E-mail: steve.mackey@wmich.edu
[2] *Department of Mathematics, University at Buffalo, USA*
E-mail: rahollis@buffalo.edu

Abstract

The underlying motivation for this work is the search for transparent solutions of inverse problems for matrix polynomials. More specifically, given a collection of structural data and a choice of degree (or grade), we aim to construct a matrix polynomial of the given degree (grade) with exactly the given structural data, from which that data can be recovered in a purely combinatorial fashion, without any numerical computation.

In the strictly regular case, where the structural data consists solely of finite elementary divisors, this has been achieved in a way that can reasonably be called a canonical form (see the contributed talk by R. Hollister). But for the general regular case, the possible presence of infinite elementary divisors must now also be considered. This talk focuses on that aspect of the inverse problem, i.e., how to incorporate infinite elementary divisors in a controlled fashion, without spoiling the desired finite elementary divisor structure. Our strategy for achieving this exploits the notion of "spectral localization", a property possessed by all of the matrix polynomials in our transparent solution of the strictly regular inverse problem. Describing the spectral localization property, and how it can be used in the transparent solution of the regular inverse problem, is then the main goal of this talk.

Filters connecting spectrally equivalent nonsingular polynomial matrices

Agurtzane Amparan[1], *Silvia Marcaida*[2], *Ion Zaballa*[3]

[1] *Dpt. Matemáticas, University of the Basque Country (UPV/EHU), Spain*
E-mail: agurtzane.amparan@ehu.eus
[2] *Dpt. Matemáticas, University of the Basque Country (UPV/EHU), Spain*
E-mail: silvia.marcaida@ehu.eus
[3] *Dpt. Matemáticas, University of the Basque Country (UPV/EHU), Spain*
E-mail: ion.zaballa@ehu.eus

Abstract

Filters connecting two polynomial matrices $D_1(s), D_2(s)$ are polynomial matrices $F_1(s), F_2(s)$ that satisfy the equation $F_2(s)D_1(s) = D_2(s)F_1(s)$. Filters that connect square quadratic matrix polynomials with nonsingular leading coefficients and the same finite elementary divisors were called coprime filters in [2]. The notion of coprime filters has been extended for matrix polynomials of possibly different sizes, ranks or degrees that share the same spectral structure, i.e., the same finite and infinite elementary divisors. Such filters have been named spectral filters in [1] and they completely characterize when two polynomial matrices are spectrally equivalent, that is, when they have the same spectral structure. Spectrally equivalent nonsingular polynomial matrices have the same degree. Given two spectrally equivalent nonsingular polynomial matrices of degree d we first parametrize the set of their spectral filters of degree $d-1$, and then show how to obtain the spectral filters of any other degree. The parameter space is the subset of invertible matrices of the centralizer of any linearization of the reversals with respect to a scalar that is not an eigenvalue of the given matrices.

Acknowledgements: Work supported by grant PID2021-124827NB-I00 funded by MCIN/AEI/ 10.13039/501100011033 and by "ERDF A way of making Europe" by the "European Union", and by grant GIU21/020 funded by UPV/EHU.

References

[1] A. Amparan, S. Marcaida, I. Zaballa. Spectral filters connecting high order systems. *Applied Mathematic and Computation*, 391: 125672 1–14 (2021).

[2] S. D. Garvey, P. Lancaster, A. A. Popov, U. Prells, I. Zaballa. Filters connecting isospectral quadratic systems. *Linear Algebra and its Applications*, 438: 1497–1516 (2013).

Rational approximation and linearisation for nonlinear eigenvalue problems and nonlinear systems

Karl Meerbergen[1]

[1] *Department of Computer Science. KU Leuven, Belgium*
E-mail: Karl.Meerbergen@kuleuven.be

Abstract

Polynomial and rational eigenvalue problems are spectrally equivalent to linear eigenvalue problems, called linearizations. The prototype example is the companion matrix pencil for the polynomial eigenvalue problem. Other nonlinear eigenvalue problems are approximated by a polynomial or rational eigenvalue problem. In [9], a Padé approximation is suggested. Potential theory was used in NLEIGS [3]. A rational approximation based on contour integration is proposed by [8] and uses a basis of rational monomials. In this work, we use the AAA method for rational approximation [7] and the AAA-least squares variant [1]. The set-valued AAA method and variants are efficient approaches for approximating all entries of the matrix and obtaining a small size linearization [4][6][5][2].

In this talk, we present results related to the linearization of nonlinear eigenvalue problems. Classical vibro-acoustic analysis relies on a description of the model in the frequency domain. Passive damping and absorptive materials, such as visco-elastic, porous and poro-elastic materials exhibit viscous and thermal damping mechanisms, which result in complex, frequency-dependent behaviour. Many different descriptions to account for their complex behaviour can be found in the literature. Time domain analysis of such systems has gained quite some attention in the context of auralisation, virtual sensing and inverse characterization. Starting from the corresponding time-domain description of these materials is not always so straightforward as convolutions are required to account for the constitutive relationships. One solution is to make use of a linearization of the frequency dependent system matrix.

In its most general form, the frequency dependent model can be expressed as

$$A(\omega)\hat{x} \quad = \quad \hat{b}(\omega), \tag{1}$$

where x is the state vector and ω is the angular frequency. In classical vibration analysis, the system matrix is quadratic in ω. In this talk, we consider $A(\omega)$ that is not polynomial or rational in ω. In fact, we assume that the system in

the frequency domain can be written as the holomorphic decomposition or split form

$$(A_0 + \imath\omega A_1 - \omega^2 A_2 + A_{-1}g_1(\omega) + \cdots + A_{-m}g_m(\omega))\hat{x} \quad = \quad \hat{b}(\omega) \qquad (2)$$

where g_i is a scalar function, and the number of nonlinear terms, m, is not large, i.e., a few dozen at most.

For the simulation in the time domain of (1), we aim to find a linear model

$$\begin{aligned} -\mathbf{E}\frac{d\mathbf{x}}{dt} + \mathbf{A}\mathbf{x} &= \mathbf{b}(t) \quad , \quad t \geq 0, \\ \mathbf{x}(0) &= \mathbf{x}_0, \end{aligned} \qquad (3)$$

such that $\hat{x}$ is related to the Laplace transform of $\mathbf{x}$. First, we will derive a linear model in the frequency domain that approximates (1). The link with the time domain is then straightforward. In order to form a linear model, we will use ideas from the solution of nonlinear eigenvalue problems. For the relation with the time domain, we introduce the Laplace variable $s = i\omega$ and represent A as a function of s. When A is a matrix polynomial or a rational matrix, i.e., the entries of A are polynomials or rational functions, there always are $\mathbf{E}$ and $\mathbf{A}$, $\mathbf{b}$ and $\mathbf{c}$ so that

$$-\imath\omega\mathbf{E}\hat{\mathbf{x}} + \mathbf{A}\hat{\mathbf{x}} \quad = \quad \hat{\mathbf{b}} \qquad (4)$$

for $\omega \in \mathbb{R}$, and a way to extract $\hat{x}$ from $\hat{\mathbf{x}}$. Eq. (4) is called a *linearization* of (1).

The linearization should satisfy the following properties:

- having real matrices $\mathbf{E}$ and $\mathbf{A}$,
- having eigenvalues with negative real parts only (stable system).

The standard AAA approximations do not satisfy these properties. Obtaining real matrices is straightforward by exploiting that $g_1, \ldots, g_m$ are 'real' functions, i.e., $g_i(\overline{z}) = \overline{g_i(z)}$ for $i = 1, \ldots, m$. Obtaining stable systems is less obvious. The eigenvalues of the nonlinear eigenvalue problem $\det(A(\lambda)) = 0$ are stable, otherwise the physical problem is not stable. However, it appears that a bad choice of the approximation region for AAA can lead to unstable spurious eigenvalue estimates. The main difficulty is that the linearization from [6] is not strong, in that it has an eigenvalue at infinity and the poles of the rational approximation can be eigenvalues too. We therefore propose a new linearization that does not have the eigenvalue at infinity. The poles of the rational function are more difficult to eliminate from the spectrum. For most cases that we have tried, we found that the poles lie in the left half plane, i.e., the functions can be well approximated by the transfer function of a stable linear system. However, it does happen that poles are unstable. We show how we can deal with unstable poles by playing with the AAA approximation and AAA least squares.

Acknowledgements: Work (partially) supported by KU Leuven Research Council and Research Foundation Flanders (FWO)

References

[1] S. Costa and N. Trefethen. AAA-least squares rational approximation and solution of Laplace problems. arXiv:2107.01574 [math.NA], 2021. Submitted for publication.

[2] S. Güttel, G. M. Negri Porzio, and F. Tisseur. Robust rational approximations of nonlinear eigenvalue problems. MIMS Preprint 2796, 2020.

[3] S. Güttel, R. Van Beeumen, K. Meerbergen, and W. Michiels. NLEIGS: A class of fully rational Krylov methods for nonlinear eigenvalue problems. *SIAM Journal on Scientific Computing*, 36(6):A2842–A2864, 2014.

[4] Amit Hochman. FastAAA: A fast rational-function fitter. In *2017 IEEE 26th Conference on Electrical Performance of Electronic Packaging and Systems (EPEPS)*, pages 1–3, 2017.

[5] P. Lietaert and K. Meerbergen. Comparing Loewner and Krylov based model order reduction for time delay systems. In *Proceedings of the European Control Conference 2018*, 2018.

[6] P. Lietaert, K. Meerbergen, J. Pérez, and B. Vandereycken. Automatic rational approximation and linearization of nonlinear eigenvalue problems. *IMA Journal on Numerical Analysis*, (draa098), 2021.

[7] Yuji Nakatsukasa, Olivier Sète, and Lloyd N. Trefethen. The AAA algorithm for rational approximation. *SIAM Journal on Scientific Computing*, 40(3):A1494–A1522, 2019.

[8] Y. Saad, M. El Guide, and A. Miedlar. A rational approximation method for the nonlinear eigenvalue problem. Preprint https://arxiv.org/abs/1901.01188, 2019.

[9] Y. Su and Z. Bai. Solving rational eigenvalue problem via linearization. *SIAM Journal on Matrix Analysis and Applications*, 32(1):201–216, 2011.

Nearest singular pencil via Riemannian optimization

Froilán Dopico[1], *Vanni Noferini*[2], *Lauri Nyman*[2]

[1] *Departamento de Matemáticas, Universidad Carlos III de Madrid, Leganés, Spain*
E-mail: dopico@math.uc3m.es
[2] *Department of Mathematics and Systems Analysis, Aalto University, Aalto, Finland*
E-mail: vanni.noferini@aalto.fi,lauri.s.nyman@aalto.fi

Abstract

The problem of finding the nearest singular pencil to a given regular, complex or real, $n \times n$ matrix pencil $A + \lambda B$ is a long-standing problem in Numerical Linear Algebra that was originally posed in [1]. This problem turned out to be very difficult and, so far, just a few numerical algorithms are available in the literature for its solution [2, 3], though they may be very expensive from a computational point of view. For instance, the one in [3] has a cost of $O(n^{12})$ flops per iteration. In this talk, we introduce two new algorithms for solving this problem based on Riemannian optimization. The first one looks for the closest complex singular pencil via the minimization of an objective function over the cartesian product of the unitary group by itself. The second one considers a regular *real* pencil and looks for the nearest singular *real* pencil to it via the minimization of a different objective function over the cartesian product of the special orthogonal group by itself. Moreover, we present a collection of numerical experiments that show that the new algorithms can deal effectively with pencils of larger sizes than those considered by previous algorithms and find minimizers of, at least, the same quality than previous algorithms.

Acknowledgements: F. Dopico is supported by grant PID2019-106362GB-I00 funded by MCIN/AEI/10.13039/501100011033. V. Noferini and L. Nyman are supported by an Academy of Finland grant (Suomen Akatemian päätös 331240).

References

[1] R. Byers, C. He, V. Mehrmann, *Where is the nearest non-regular pencil?*, Linear Algebra and its Applications, 285(1-3), 81-105, (1998).

[2] N. Guglielmi, C. Lubich, V. Mehrmann, *On the nearest singular matrix pencil*, SIAM Journal on Matrix Analysis and Applications, 38(3), 776-806, (2017).

[3] M. Giesbrecht, J. Haraldson, G. Labahn, *Computing the nearest rank-deficient matrix polynomial*, in Proceedings of the 2017 ACM on International Symposium on Symbolic and Algebraic Computation (pp. 181-188) (2017, July).

Error analysis of compact Arnoldi methods for linearized polynomial eigenvalue problems

Javier Pérez[1]

[1] *Dpt. Mathematical Sciences, University of Montana, USA*
E-mail: javier.perez-alvaro@mso.umt.edu

Abstract

The talk considers polynomial eigenvalue problems that involve matrix polynomials expressed in various bases. The standard approach to solving these problems is to embed the matrix polynomial coefficients in a larger matrix pencil, denoted as $A - \lambda B$, through a process called linearization. When only a few eigenvalues of the matrix polynomial are needed, the rational Krylov procedure applied to $A - \lambda B$ is the preferred method for solving the linearized problem. To reduce computational and storage costs, memory-efficient versions of the Arnoldi method have recently been proposed. This work aims to analyze the numerical stability of the compact Arnoldi method for a broad range of linearizations, including the well-known Frobenious companion form and the colleague and comrade linearizations.

References

Isomorphisms between Ansatz Spaces over Classical Polynomial Bases

D. Steven Mackey[1], *Vasilije Perović*[2]

[1] *Dept. of Mathematics, Western Michigan University, USA*
E-mail: steve.mackey@wmich.edu
[2] *Dept. of Mathematics and Applied Mathematical Sciences, Univ. of Rhode Island, USA*
E-mail: perovic@uri.edu

Abstract

One classical approach to solving an eigenvalue problem associated with a matrix polynomial $P(\lambda)$ is to solve an eigenvalue problem associated with a matrix pencil $L(\lambda)$ that is spectrally equivalent to $P(\lambda)$; such an $L(\lambda)$ is called a *linearization* of $P(\lambda)$. During the last two decades much research has been done on developing various ways of constructing linearizations, e.g., ansatz spaces [3], Fiedler pencils [1] and their variations, block minimal basis pencils [2], etc.

We start by revisiting the notion of ansatz spaces of matrix pencils [3] associated with an $n \times n$ matrix polynomial $P(\lambda)$, regular or singular, and discuss generalizations of this notion that are more in alignment with polynomials $P(\lambda)$ that are expressed in non-monomial bases. We then explore some non-obvious isomorphic relationships between the various generalized ansatz spaces, and show how these isomorphisms can be exploited to easily produce linearizations of matrix polynomials expressed in many of the classical (non-monomial) polynomial bases, all in a single unified framework. One of the distinctive features in our development is a systematic use of non-standard representations of ansatz pencils in the form $XU(\lambda) + YD(\lambda)$, where X, Y are constant matrices associated with pencils in a "classical" ansatz space $\mathbb{L}_1(P)$ [3] and $U(\lambda)$, $D(\lambda)$ are matrix pencils whose structures are chosen to be better adapted to the polynomial basis that $P(\lambda)$ is expressed in. We conclude with several concrete examples of $U(\lambda)$ and $D(\lambda)$ associated with $P(\lambda)$ expressed in non-monomial bases such as Newton, Bernstein, Lagrange, and Chebyshev, and observe that here $U(\lambda)$ and $D(\lambda)$ are often diagonal or have a low bandwidth.

References

[1] E. N. Antoniou, S. Vologiannidis. A new family of companion forms of polynomial matrices, *Electron. J. Linear Algebra*, 11:78–87 (2004).

[2] F. M. Dopico, P. W. Lawrence, J. Pérez, P. Van Dooren. Block Kronecker linearizations of matrix polynomials and their backward errors, *Numerische Math.*, 140:373–426 (2018).

[3] D. S. Mackey, N. Mackey, C. Mehl, V. Mehrmann. Vector spaces of linearizations for matrix polynomials, *SIAM J. Matrix Anal. Appl.*, 27:821–850 (2006).

Rectangular multiparameter eigenvalue problems

Michiel E. Hochstenbach[1], *Tomaž Košir*[2], *Bor Plestenjak*[2]

[1] *Department of Mathematics and Computer Science, TU Eindhoven*
Homepage: www.win.tue.nl/~hochsten
[2] *Faculty of Mathematics and Physics, University of Ljubljana*
E-mail: tomaz.kosir@fmf.uni-lj.si, bor.plestenjak@fmf.uni-lj.si

Abstract

The *multiparameter eigenvalue problem* (MEP) has the form

$$W_i(\boldsymbol{\lambda})\,\mathbf{x}_i := (V_{i0} + \lambda_1 V_{i1} + \cdots + \lambda_k V_{ik})\,\mathbf{x}_i = \mathbf{0}, \quad i = 1, \ldots, k, \tag{1}$$

where $V_{ij} \in \mathbb{C}^{n_i \times n_i}$, $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_k) \in \mathbb{C}^k$, and $\mathbf{x}_i \in \mathbb{C}^{n_i}$ is nonzero. In the generic case, (1) has $n_1 \cdots n_k$ *eigenvalues* that are roots of the polynomial system $\det(W_i(\boldsymbol{\lambda})) = 0$ for $i = 1, \ldots, k$. A generalization of (1) are polynomial MEPs, where $W_1, \ldots, W_k$ are multivariate matrix polynomials. For instance, a *quadratic two-parameter eigenvalue problem* has the form

$$(V_{i00} + \lambda V_{i10} + \mu V_{i01} + \lambda^2 V_{i20} + \lambda\mu V_{i11} + \mu^2 V_{i02})\,\mathbf{x}_i = \mathbf{0}, \quad i = 1, 2, \tag{2}$$

where $V_{ipq} \in \mathbb{C}^{n_i \times n_i}$. A generic problem (2) has $4n_1n_2$ eigenvalues (λ, μ).

Recently, a new type of eigenvalue problems with $k \geq 2$ parameters has appeared with applications in ARMA and LTI models [1, 2]. A general form is

$$M(\boldsymbol{\lambda})\,\mathbf{x} := \Big(\sum_{\boldsymbol{\omega}} \boldsymbol{\lambda}^{\boldsymbol{\omega}} A_{\boldsymbol{\omega}}\Big)\,\mathbf{x} = \mathbf{0}, \tag{3}$$

where $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_k)$ is a multi-index, $\boldsymbol{\lambda}^{\boldsymbol{\omega}} = \lambda_1^{\omega_1} \cdots \lambda_k^{\omega_k}$, and $\mathbf{x} \in \mathbb{C}^n$ is nonzero. The key properties of (3) are that there is just *one equation* and $A_{\boldsymbol{\omega}} = A_{\omega_1,\ldots,\omega_k} \in \mathbb{C}^{(n+k-1)\times n}$ are *rectangular* matrices. To distinguish it from (1) and (2), we call (3) a *rectangular MEP*. We will show how we can efficiently numerically solve (3) using methods for (1).

Acknowledgements: T. Košir and B. Plestenjak have been supported by the Research Agency of the Republic of Slovenia, Research Grant N1-0154.

References

[1] C. Vermeersch, B. De Moor. Two complementary block Macaulay matrix algorithms to solve multiparameter eigenvalue problems. *Linear Algebra Appl.*, 654:177-209, 2022.

[2] M. E. Hochstenbach, T. Košir, B. Plestenjak. On the solution of rectangular multiparameter eigenvalue problems. *arXiv* 2212.01867, 2022.

Computing zeros of rational functions and matrices

Yuji Nakatsukasa[1], *Vanni Noferini*[2], *María C. Quintana*[3]

[1] *Mathematical Institute, University of Oxford, UK*
E-mail: nakatsukasa@maths.ox.ac.uk
[2] *Dpt. Mathematics and Systems Analysis, Aalto University, Finland*
E-mail: vanni.noferini@aalto.fi
[3] *Dpt. Mathematics and Systems Analysis, Aalto University, Finland*
E-mail: maria.quintanaponce@aalto.fi

Abstract

One of the most commonly used method to solve polynomial equations $p(x) = 0$ relies on considering the companion matrix of the polynomial $p(x)$, whose eigenvalues coincides with the roots of $p(x)$. The roots of a rational function $r(x)$ are the roots of its numerator when $r(x)$ is written as an irreducible fraction $p(x)/q(x)$. Then, the first idea to solve rational equations $r(x) = 0$ could be simply to consider the companion matrix of the numerator $p(x)$. However, not always rational functions are given as the ratio of two coprime polynomials and, in addition, the computation of $p(x)$ may be a source of numerical instabilities. In this work we transform rational equations $r(x) = 0$ into eigenvalue problems without explicitly computing the numerator $p(x)$. For that, rational functions will be given by considering different representations. Finally, we will see how these ideas can be extended to compute zeros of rational matrices, i.e., matrices whose entries are rational functions.

Acknowledgements: Work (partially) supported by an Academy of Finland grant (Suomen Akatemian päätös 331230) and by the Agencia Estatal de Investigación of Spain through grant PID2019-106362GB-I00 MCIN / AEI / 10.13039/501100011033/.

Eigenvector error bounds and perturbation for nonlinear eigenvalue problems

Yuji Nakatsukasa[1], *Javier Pérez*[2], *Françoise Tisseur*[3]

[1] *Mathematical Institute, University of Oxford, UK*
E-mail: nakatsukasa@maths.ox.ac.uk
[2] *Department of Mathematical Sciences, University of Montana, USA*
E-mail: javier.perez-alvaro@mso.umt.edu
[3] *Department of Mathematics, The University of Manchester, UK*
E-mail: francoise.tisseur@manchester.ac.uk

Abstract

A computed approximate eigenpair $(\widehat{\lambda}, \widehat{x})$ of a matrix-valued function $F(\lambda)$ that is analytic on a nonempty open set $\Omega \subseteq \mathbb{C}$ is usually considered acceptable if the normalized residual $F(\widehat{\lambda})\widehat{x}$ is small since it indicates that the computed solution has a small backward error. When $F(\lambda)$ satisfies a one-sided factorization of the form

$$H(\lambda)F(\lambda) = L(\lambda)G(\lambda) \tag{1}$$

with $L(\lambda)$ a linear pencil, and $G(\lambda)$, $H(\lambda)$ of full column rank for each $\lambda \in \Omega$ then the error in the eigenvalue $\widehat{\lambda}$ can be bounded as the error of $\widehat{\lambda}$ as an eigenvalue of $L(\lambda)$. We can then invoke well-established eigenvalue perturbation results for linear problems. For approximate eigenvectors, however, no previous result exists that rigorously bound the error in the computed eigenvector $\widehat{x}$.

In this talk we present a posteriori upper bounds for the angle between $\widehat{x}$ and an exact eigenvector x of $F(\lambda)$ when the latter satisfies a one-sided factorization of the form (1). We also present first-order error bounds that do not require a one-sided factorization of $F(\lambda)$. One implication of our result is that an eigenvector can be computed accurately even when it corresponds to several distinct eigenvalues.

Computing a compact local Smith McMillan form

Vanni Noferini[1], *Paul Van Dooren*[2]

[1] *Dpt. Mathematics and Systems Analysis, Aalto University, Finland*
E-mail: vanni.noferini@aalto.fi
[2] *Dpt. Mathematical Engineering, Catholic University of Louvain, Belgium*
E-mail: vandooren.p@gmail.com

Abstract We define a compact local Smith-McMillan form of a rational matrix $R(\lambda)$ as the diagonal matrix whose diagonal elements are the nonzero entries of a local Smith-McMillan form of $R(\lambda)$. We show that a recursive rank search procedure, applied to a block-Toeplitz matrix built on the Laurent expansion of $R(\lambda)$ around an arbitrary complex point λ_0, allows us to compute a compact local Smith-McMillan form of that rational matrix $R(\lambda)$ at the point λ_0, provided we keep track of the transformation matrices used in the rank search. It also allows us to recover the root polynomials of a polynomial matrix and root vectors of a rational matrix, at an expansion point λ_0. Numerical tests illustrate the promising performance of the resulting algorithm.

Acknowledgements: Work (partially) supported by the Academy of Finland and by an Aalto Science Institute Visitor Programme

References

[1] V. Noferini and P. Van Dooren, Computing a compact local Smith McMillan form, *arXiv*:2303.10403, 2023.

Diagonalizable Matrix Polynomials

Ion Zaballa[1]

[1] *Departamento de Matemáticas, Universidad del País Vasco (UPV/EHU), Spain*
E-mail: ion.zaballa@ehu.eus

Abstract

An $n \times n$ matrix polynomial $P(\lambda) = P_\ell \lambda^\ell + P_{\ell-1}\lambda^{\ell-1} + \ldots + P_1\lambda + P_0$ will be said to be *diagonalizable* if it is (*unimodularly*) equivalent to a diagonal matrix $D(\lambda) = D_\ell \lambda^\ell + D_{\ell-1}\lambda^{\ell-1} + \ldots + D_1\lambda + D_0$ with D_j diagonal for each $j = 0, 1, \ldots, \ell$ and $D_\ell \neq 0$. When $P(\lambda)$ is diagonalizable, it is also said that it can be *decoupled* by equivalence transformations. This terminology originated in the study of quadratic systems; that is to say, matrix polynomials of degree $\ell = 2$, where the problem of finding physically implementable transformations allowing any given quadratic matrix polynomial to be decoupled has a long tradition.

Every matrix polynomial $P(\lambda)$ is equivalent to a diagonal matrix (its Smith normal form) but, in general, this normal form does not have the property of having the same degree as $P(\lambda)$. The first step in the diagonalization of a matrix polynomial $P(\lambda)$ is the characterization of the elementary divisors that are admissible for decoupling (see [2]). Such a characterization was given in [3] for real and complex matrices, $\ell = 2$, and $\det P_2 \neq 0$.The results in [3] were extended in [6] to quadratic matrix polynomials with non-zero but singular leading coefficient. The case $\ell = 3$ was handled in [4], where implicit necessary and sufficient conditions were given for a matrix polynomial of degree $\ell = 3$ to be diagonalizable. In this presentation, explicit necessary and sufficient conditions will be exhibited for $\ell = 3$ and $\ell = 4$, and a methodology for dealing with higher degree matrix polynomials will be proposed. The highly combinatorial nature of the problem was made clear in [5], where the problem was connected with the existence of a *supply-demand flow* on a *capacity-constrained, supply-demand network* and to the existence of nonnegative integral matrices with prescribed row and column sums. (see [1, Ch. 6] for a detailed account of this type of problem).

Acknowledgements: Work supported by grant PID2021-124827NB-I00 funded by MCIN/AEI/ 10.13039/501100011033 and by "ERDF A way of making Europe" by the "European Union", and by grant GIU21/020 funded by UPV/EHU.

References

[1] R. A. Brualdi and H. J. Ryser. *Combinatorial Matrix Theory*. Cambridge University Press, Cambridge, 1992.

[2] D. T. Kawano, R. G. Salsa, and F. Ma. Decoupling of second-order linear systems by isospectral transformation. *Zeitschrift für angewandte Mathematik und Physik*, 69(6):1–19, 2018.

[3] P. Lancaster and I. Zaballa. Diagonalizable quadratic eigenvalue problems. *Mechanical Systems and Signal Processing*, 23(4):1134–1144, 2009.

[4] P. Yu, G. Zhang, and Y. Zhang. Decoupling of cubic polynomial matrix systems. *Numerical Algebra, Control & Optimization*, 11(1):13–26, 2021.

[5] I. Zaballa. On the jordan sctructure of diagonal matrix polynomials of fixed degree. *Proceedings of the 18th International Conference on Computational and Mathematical Methods in Science and Engineering, CMMSE 2018*, 2018.

[6] J. C. Zúñiga. Diagonalization of quadratic matrix polynomials. *Systems and Control Letters*, 59:105–113, 2010.

MSC10. Numerical linear algebra applications in data science

James Nagy, Sirani M. Perera

Mixed precision randomized Nyström approximation
Erin Carson, Ieva Dauzickaite

Probabilistic Rounding Error Analysis in Numerical Linear Algebra
Nicholas Higham, Michael Connolly

Advanced Krylov Subspace methods with applications to Bayesian inverse problems
Malena Sabaté Landman, Jiahua Jiang, Jianru Zhang, Ren Wuwei

Matrix-Free Hyperparameter Optimization for Gaussian Processes
Theresa Wagner, Martin Stoll, Franziska Nestler

Mixed precision randomized Nyström approximation

Erin C. Carson[1] *and Ieva Daužickaitė*[1]

[1] *Department of Numerical Mathematics, Faculty of Mathematics and Physics, Charles University, Czech Republic*
E-mail: {carson, dauzickaite}@karlin.mff.cuni.cz

Abstract

Support for floating point arithmetic in multiple precisions is becoming increasingly common in emerging architectures. Mixed precision capabilities are already included in a quarter of the machines on the TOP500 list and are expected to be a crucial hardware feature in exascale machines. In this talk, we consider the Nyström method for approximating a positive semidefinite matrix. The computational cost of its single-pass version can be decreased by running it in mixed precision, where the expensive matrix products are computed in a lower precision. Our finite precision analysis supports intuition: the lower the rank of the approximation desired, the lower the precision that can be used without significant detriment. We further develop a heuristic for determining a suitable precision and discuss the use of such approximations within preconditioners for Krylov subspace methods.

Acknowledgements: Work partially supported by ERC Starting Grant No. 101075632 and the Exascale Computing Project (17-SC-20-SC), a collaborative effort of the U.S. Department of Energy Office of Science and the National Nuclear Security Administration.

Probabilistic Rounding Error Analysis in Numerical Linear Algebra

Michael P. Connolly[1], *Nicholas J. Higham*[2]

[1] *Department of Mathematics, University of Manchester, Manchester, M13 9PL, UK*
E-mail: michael.connolly-3@manchester.ac.uk
[2] *Department of Mathematics, University of Manchester, Manchester, M13 9PL, UK*
E-mail: nick.higham@manchester.ac.u

Abstract

Traditional rounding error analysis in numerical linear algebra provides worst-case bounds that tend to be very pessimistic. We describe results that make assumptions about the statistical distribution of the rounding errors and provide sharper bounds that hold with certain probabilities. These results apply to both inner product-based algorithms and orthogonal factorization-based algorithms. The statistical assumptions that we make are satisfied by a form of rounding called stochastic rounding and we explain why there is much current interest in stochastic rounding in machine learning and other areas.

Acknowledgements: This work was supported by the Royal Society, Engineering and Physical Sciences Research Council grant EP/P020720/1, and the Exascale Computing Project (17-SC-20-SC), a collaborative effort of the U.S. Department of Energy Office of Science and the National Nuclear Security Administration.

References

[1] M. P. Connolly, N. J. Higham, and T. Mary. Stochastic rounding and its probabilistic backward error analysis. *SIAM J. Sci. Comput.*, 43(1):A566–A585, 2021.

[2] M. P. Connolly and N. J. Higham. Probabilistic rounding error analysis of Householder QR factorization. MIMS EPrint 2022.5, Manchester Institute for Mathematical Sciences, The University of Manchester, UK, February 2022.

Advanced Krylov Subspace methods with applications to Bayesian inverse problems

Malena Sabaté Landman[1], *Jiahua Jiang*[2], *Jianru Zhang* [3], *Wuwei Ren* [4]

[1] *Department of Mathematics, Emory University, US*
E-mail: msabat3@emory.edu
[2] *School of Mathematics, University of Birmingham, UK*
[3] *Institution of Mathematical Science, ShanghaiTech University, China*
[4] *School of Information Science and Technology, ShanghaiTech University, China*

Abstract

In this talk I will presents a new class of algorithms to compute solutions of large-scale linear discrete ill-posed problems that can be modeled as the sum of two independent random variables. Following a Bayesian modelling approach, this corresponds to adding a different regularization term for each component in the original least-squares minimization scheme (assuming Gaussian noise). In particular, following smoothness and sparsity priors, this involves adding a covariance-weighted quadratic term and a sparsity enforcing ℓ_1 term (with additional smoothing to ensure differentiability at the origin) applied to the different parts of the solution. The approach proposed in this paper consists in constructing a sequence of approximated quadratic problems that are partially solved using augmented flexible Krylov–Tikhonov methods. Compared to other traditional methods, the new algorithms have the advantage of building a single (augmented, flexible) approximation (Krylov) subspace that encodes regularization through variable 'preconditioning' and that is expanded as soon as a new problem in the sequence is defined. This also allows for the regularization parameters to be chosen on-the-fly at each iteration. The performance of these algorithms is shown through a variety of numerical experiments.

Matrix-Free Hyperparameter Optimization for Gaussian Processes

Franziska Nestler[1], *Martin Stoll*[2], *Theresa Wagner*[3]

[1] *Department of Mathematics, Chair of Applied Functional Analysis, TU Chemnitz, Germany*
E-mail: franziska.nestler@mathematik.tu-chemnitz.de
[2] *Department of Mathematics, Chair of Scientific Computing, TU Chemnitz, Germany*
E-mail: martin.stoll@mathematik.tu-chemnitz.de
[3] *Department of Mathematics, Chair of Scientific Computing, TU Chemnitz, Germany*
E-mail: theresa.wagner@mathematik.tu-chemnitz.de

Abstract

Gaussian processes (GPs) are a crucial tool in machine learning and their use across different areas of science and engineering has increased given their ability to quantify the uncertainty in the model. The covariance matrices of GPs arise from kernel functions, which are crucial in many learning tasks and the matrices are typically dense and large-scale. Depending on their dimension even computing all their entries is challenging and the cost of matrix-vector products scales quadratically with the dimension, if no customized methods are applied. We present a matrix-free approach that exploits the computational power of the non-equispaced fast Fourier transform (NFFT) and is of linear complexity for fixed accuracy. With this, we cannot only speed up matrix-vector multiplications with the covariance matrix but also take care of the derivatives needed for the gradient method avoiding Hadamard products of the Euclidean distance matrix and the kernel matrix. This arises when differentiating kernels as the squared-exponential kernel with respect to the length-scale parameter in the denominator of the exponential expression. Our method introduces a derivative kernel which is then well suited for multiplying with the Hadamard product. Since NFFT's efficiency is limited to feature dimensions smaller than 4, we decompose the features into and learn on multiple kernels, allowing our method to be used on high-dimensional data. By applying our preconditioned NFFT-based fast summation technique, fitting the kernel and the derivative kernel will allow for fast tuning of the hyperparameters.

Acknowledgements: Work supported by the BMBF grant 01—S20053A (project SAℓE).

References

[1] F. Nestler, M. Stoll, T. Wagner. *Learning in High-Dimensional Feature Spaces Using ANOVA-Based Fast Matrix-Vector Multiplication*. Foundations of Data Science, 4.3 (2022): 423-440.

MSC11. Eigenvalue applications and optimization in numerical linear algebra

Sara Grundel, Tim Mitchell,
Julio Moro

Locating Eigenvalues of Quadratic Matrix Polynomials
Shreemayee Bora, Nandita Roy

Optimal Rational Matrix Function Approximation Using the Arnoldi Algorithm
Anne Greenbaum, Tyler Chen, Natalie Wellen

Model Order Reduction in Gas Network Simulation and the Role of Eigenvalues
Sara Grundel

Structured eigenvalue optimization via rank-1 ODEs
Nicola Guglielmi, Christian Lubich, Stefano Sicilia

Large-Scale Minimization of the Pseudospectral Abscissa
Emre Mengi, Nicat Aliyev

Root-Max Problems, Hybrid Expansion-Contraction, and Optimization of Passive Systems
Tim Mitchell, Paul Van Dooren

Locating Eigenvalues of Quadratic Matrix Polynomials

Shreemayee Bora[1], *Nandita Roy*[2]

[1] *Department of Mathematics, Indian Institute of Technology Guwahati, India*
E-mail: shbora@iitg.ac.in
[2] *Department of Mathematics, Tinsukia College, India*
E-mail: scr.nandita@gmail.com

Abstract

The location of the roots of a quadratic scalar polynomial may be identified from its coefficients. This paper shows that when the coefficients of the polynomial are square matrices, then appropriate generalizations of some of these statements hold for the eigenvalues of the resulting quadratic matrix polynomial. The locations of the eigenvalues are described with respect to the imaginary axis, the unit circle or the real line. The results lead to upper bounds on some important distances associated with quadratic matrix polynomials. The principal tool used is an eigenvalue localization technique using block Geršgorin sets applied to certain linearizations of these polynomials.

Acknowledgements: The work of the first author is supported by SERB MATRICS Project (MTR/2019/000383), Department of Science and Technology (DST), Government of India. The work of the second author is supported by the Ministry of Education, Government of India.

Optimal Rational Matrix Function Approximation Using the Arnoldi Algorithm

Tyler Chen[1], *Natalie Wellen*[2], *Anne Greenbaum*[3]

[1] *Computer Science Dept., New York University, USA*
E-mail: tyler.chen@nyu.edu
[2] *Applied Math Dept., University of Washington, USA*
E-mail: nwellen@uw.edu
[3] *Applied Math Dept., University of Washington, USA*
E-mail: greenbau@uw.edu

Abstract

Given an n by n matrix A and an n-vector b, along with a rational function $R(z) := D(z)^{-1}N(z)$, we show how to find the optimal approximation to $R(A)b$ from the Krylov space, $\text{span}(b, Ab, \ldots, A^{k-1}b)$, using the basis vectors produced by the Arnoldi algorithm. Here *optimal* is taken to mean optimal in the $D(A)^*D(A)$-norm. Similar to the case for linear systems, we show that eigenvalues alone cannot provide information about the convergence behavior of this algorithm and we discuss other possible error bounds for highly nonnormal matrices.

Acknowledgements: Work (partially) supported by NSF Grant No. DGE-2140004.

References

[1] T. Chen, A. Greenbaum, C. Musco, and C. Musco, *Error bounds for Lanczos-based matrix function approximation*, ArXiv https://arxiv.org/abs/2106.09806, to appear in SIAM J. Matrix Anal. Appl..

[2] T. Chen, A. Greenbaum, C. Musco, and C. Musco, *Low-memory Krylov subspace methods for optimal rational matrix function approximation*, ArXiv https://arxiv.org/abs/2202.11251, submitted to SIAM J. Matrix Anal. Appl..

Model Order Reduction in Gas Network Simulation and the Role of Eigenvalues.

Sara Grundel[1]

[1] *Computational Methods in Systems and Control Theory, Max Planck Institute Magdeburg, Sandtorstr. 1,39106 Magdeburg, Germany E-mail:* grundel@mpi-magdeburg.mpg.de

Abstract

Modeling gas flow through pipelines, and even more so whole pipeline networks as well as gas networks also including control elements requires to go through several steps. The resulting spatially discretized system is a so-called Differential Algebraic Equation (DAE) of large dimension. This DAE is typically nonlinear, however admits a special structure, which allows for structure preserving reduced order modelling techniques. We will present the structure and how to preserve it and discuss the concept of tractability index for DAEs, which is also preserved in the reduced model. We furthermore investigate the eigenvalues of the corresponding ordinary differential equations for the reduced as well as the full order model.

Acknowledgements: Work (partially) supported by BMWi mathenergy 0324019B.

References

[1] Banagaaya, N., Alì, G., Grundel, S., Benner, P. *Index-Aware Model-Order Reduction for a Special Class of Nonlinear Differential-Algebraic Equations* . Journal of Dynamics and Differential Equations, 2022

[2] Benner, P., Braukmüller, M., Grundel, S. *A direct index 1 DAE model of gas networks*. Reduced-Order Modeling (ROM) for Simulation and Optimization: Powerful Algorithms as Key Enablers for Scientific Computing, 2018

Structured eigenvalue optimization via rank-1 ODEs

Nicola Guglielmi[1], *Christian Lubich*[2] *and Stefano Sicilia*[1]

[1] *Gran Sasso Science Institute, Italia*
E-mail: nicola.guglielmi@gssi.it, stefano.sicilia@gssi.it
[2] *Universität Tübingen, Germany*
E-mail: lubich@na.uni-tuebingen.de

Abstract

Stability and robustness analysis of linear continuous and discrete dynamical systems is a vast and active interdisciplinary research area. The word stability in this talk is meant to indicate the broad spectrum of issues that arise in the analytical and numerical study of dynamical and control systems, resulting from the need to have stable and reliable representations robustly preserving essential qualitative properties of the underlying physical model. The analysis of these features is often based on eigenvalue optimization of a certain structured matrix A (e.g., sparse, Hamiltonian, nonnegative, Toeplitz, etc.). The main goal of this talk is to show how robustness of spectral properties can be computed. We propose a 2-level iterative algorithm. In an inner iteration, a *quasi gradient flow* in the manifold of rank-1 matrices drives perturbations to the original matrix of a fixed size into a minimum of a functional that depends on eigenvalues (and possibly eigenvectors). In an outer iteration, the perturbation size is optimized such that the functional reaches some target value [1]. A key point is related to certain rank-related properties of extremizers, which were first used in the pioneering work by Guglielmi and Overton [2] and subsequently developed to exploit gradient flows on low-rank manifolds (see [3]).

Acknowledgements: Work (partially) supported by the Italian MUR (Ministero dell'Università e della Ricerca) within the PRIN Project "Discontinuous dynamical systems: theory, numerics and applications" (2018–2023) .

References

[1] N. Guglielmi, C. Lubich and S. Sicilia. Rank-1 matrix differential equations for structured eigenvalue optimization. *SIAM J. Numer. Anal. Appl*, submitted (2023).

[2] N. Guglielmi and M. Overton. Fast algorithms for the approximation of the pseudospectral abscissa and pseudospectral radius of a matrix. *SIAM J. Matrix Anal. Appl.*, 32: 1166–1192 (2011)

[3] N. Guglielmi and C. Lubich. *Matrix nearness problems and eigenvalue optimization*. Monograph in preparation, 2023.

Large-Scale Minimization of the Pseudospectral Abscissa

Nicat Aliyev[1], *Emre Mengi*[2]

[1] *Czech Technical University, Department of Instrumentation and Control, Technicka 4, 16607, Prague, Czech Republic*
E-mail: nijat.aliyev@cvut.cz
[2] *Koç University, Department of Mathematics, Rumeli Feneri Yolu 34450, Sarıyer, Istanbul, Turkey*
E-mail: emengi@ku.edu.tr

Abstract

The minimization of the spectral abscissa of a matrix dependent on parameters has drawn interest in the last couple of decades [1]. The problem is motivated especially by the stability considerations for the associated linear control system.

The major difficulty in the minimization of the spectral abscissa is that its dependence on the parameters is non-Lipschitz. Especially, if the rightmost eigenvalue whose real part is the spectral abscissa is not simple, the spectral abscissa can change rapidly under small perturbations. A remedy to this is, for a prescribed $\varepsilon > 0$, minimizing the ε-pseudospectral abscissa [2], the real part of the rightmost point in the set consisting of eigenvalues of all matrices at a distance of ε, formally defined for a square matrix A by

$$\begin{aligned} \alpha_\epsilon(A) &:= \max\{\mathrm{Re}(z) \mid z \in \Lambda_\varepsilon(A)\}, \quad \text{where} \\ \Lambda_\varepsilon(A) &:= \left\{z \in \mathbb{C} \mid z \in \Lambda(A+\Delta) \ \exists \Delta \in \mathbb{C}^{n\times n} \ \text{ s.t. } \|\Delta\|_2 \le \varepsilon\right\}. \end{aligned}$$

Unlike the spectral abscissa, the pseudospectral abscissa $\alpha_\varepsilon(A)$ is a locally Lipschitz continuous function of the entries of the matrix A [3]. Moreover, minimizing $\alpha_\varepsilon(A)$ for a matrix A dependent on parameters affects stability of not only A but also all nearby matrices, which is significant in the presence of uncertainties.

We present a subspace framework to minimize the ε-pseudospectral abscissa of a large matrix dependent on parameters analytically. At every subspace iteration, a one-sided subspace restriction on the parameter-dependent matrix yields a small rectangular pseudospectral abscissa minimization problem. We expand the restriction subspace based on the minimizer of this small problem. We prove in theory for the proposed subspace framework that, assuming the global minimizers of the small problems are retrieved, convergence to the global minimizer of the original large-scale pseudospectral abscissa minimization problem occurs in the infinite dimensional setting, and that the rate-of-convergence is superlinear when there is only one parameter [4]. Our theoretical findings are illustrated on real large-scale examples that concern the stabilization by static output feedback of benchmark linear control systems from [5].

References

[1] J. V. Burke, A. S. Lewis and M. L. Overton. Two numerical methods for optimizing matrix stability. *Linear Algebra Appl.*, 351-352: 117–145 (2002).

[2] L. N. Trefethen and M. Embree. *Spectra and Pseudospectra: The Behavior of Nonnormal Matrices and Operators*. Princetion University Press, Princeton, NJ, 2005.

[3] A. S. Lewis and J. P. Pang. Variational analysis of pseudospectra. *SIAM J. Optim.*, 19: 1048-1072 (2008).

[4] N. Aliyev and E. M. Large-scale minimization of the pseudospectral abscissa. *submitted*, preprint: arXiv:2208.07540 (2023).

[5] F. Leibfritz. COMP*leib:* CO*nstraint* M*atrix-optimization* P*roblem* lib*rary* – *a collection of test examples for nonlinear semidefinite programs, control system design and related problems*. Tech. Report, Department of Mathematics, University of Trier, 2005.

Root-Max Problems, Hybrid Expansion-Contraction, and Optimization of Passive Systems

Tim Mitchell[1], *Paul Van Dooren*[2]

[1] *Department of Computer Science, Queens College / CUNY, USA*
E-mail: tim.mitchell@qc.cuny.edu
[2] *Department of Mathematical Engineering, Université catholique de Louvain, Belgium*
E-mail: paul.vandooren@uclouvain.be

Abstract

Given a state-space representation $\mathcal{M} = \{A, B, C, D\}$ of a continuous-time linear dynamical system, the system is strictly passive if the eigenvalues of A are in the open left half-plane and the Hermitian part of its transfer function is positive definite on the imaginary axis. The passivity radius tells us the smallest perturbation that can be made to matrices in $\mathcal{M}$ such that strict passivity will be lost. In this talk, we discuss how to compute a closely related quantity, namely, the extremal scalar value Ξ for which a certain parametric system with one real parameter loses strict passivity. This particular quantity is important because the value of the passivity radius of a general system depends upon the value of Ξ, and it allows one to construct certificates for the passivity of parametric passive systems. As it turns out, computing Ξ involves solving a potentially nonsmooth eigenvalue optimization problem in two real variables. Inspired by the efficient level-set techniques for computing the $\mathcal{H}_\infty$ norm [1, 2], Mehrmann and Van Dooren recently proposed the first algorithm to compute the continuous-time version of Ξ in [3] and shortly thereafter addressed computing its discrete-time version in [4], although they did not analyze the efficiency of their methods. By showing that the value of Ξ can be recast as what we call a root-max problem and then generalizing the Hybrid Expansion-Contraction (HEC) algorithm [5] from its original purpose of approximating the $\mathcal{H}_\infty$ norm of large-scale systems to root-max problems, we have devised new methods that compute successively better locally optimal approximations to Ξ with a local quadratic rate of convergence. Our analysis also demonstrates that the earlier methods of Mehrmann and Van Dooren have at least a superlinear local rate of convergence. However, in practice, we also demonstrate that the performance gap between their methods and our faster new HEC-based approach can be significantly larger than what our convergence rate results suggest.

References

[1] S. Boyd, V. Balakrishnan. A regularity result for the singular values of a transfer matrix and a quadratically convergent algorithm for computing its L_∞-norm. *Systems Control Lett.*, 15(1): 1–7 (1990).

[2] N. A. Bruinsma, M. Steinbuch. A fast algorithm to compute the H_∞-norm of a transfer function matrix. *Systems Control Lett.*, 14(4): 287–293 (1990).

[3] V. Mehrmann, P. M. Van Dooren. Optimal robustness of port-Hamiltonian systems. *SIAM J. Matrix Anal. Appl.*, 41(1): 134–151 (2020).

[4] V. Mehrmann, P. Van Dooren. Optimal robustness of passive discrete-time systems. *IMA J. Math. Control Inform.*, 37(4): 1248–1269 (2020).

[5] T. Mitchell, M. L. Overton. Hybrid expansion-contraction: a robust scaleable method for approximating the H_∞ norm. *IMA J. Numer. Anal.*, 36(3): 985–1014 (2016).

MSC12. Model reduction and learning reduced models through the lens of linear algebra and of optimization

Ion Victor Gosea,
Zoran Tomljanovic

On the Loewner framework for model reduction
Athanasios C. Antoulas

H2 optimal model reduction for simply connected domains
Alessandro Borghi, Tobias Breiten

Parametric Linearization of Nonlinear Dynamical Systems Subject to Periodic Inputs
Giovanni Conni, Karl Meerbergen, Frank Naets

Numerical linear algebra aspects of the Dynamic Mode Decomposition
Zlatko Drmac

One can hear the impedance and loss profiles of a string: from the discrete to continuum dissipative inverse problem
Vladimir Druskin, Jorn Zimmerling, Rob Remis, Murthy Guddati, Elena Cherkaev

Randomized POD-Beyn algorithm for nonlinear eigenvalue problems -- analysis and perspectives
Luka Grubišic

Data-driven balancing: what to sample for different types of balanced reduced models
Serkan Gugercin, Sean Reiter, Ion Victor Gosea

Optimal reduced-order modeling for structured linear systems
Petar Mlinarić, Serkan Güğercin, Peter Benner

On multi-objective optimization of model reduction for port-Hamiltonian systems
Jonas Nicodemus, Paul Schwerdtner, Benjamin Unger

An Eigensystem Realization Algorithm for Continuous-Time Systems and Its Connection with the Hankel Operator
Igor Pontes Duff, Ion Victor Gosea; Serkan Gugercin, Christopher Beattie

From matrix equations to surrogate models
Jens Saak

On the Loewner framework for model reduction

Athanasios C. Antoulas[1]

[1] *Department of Electrical and Computer Engineering, Rice University, Houston, TX, USA 1*
E-mail: aca@rice.edu

Abstract

The Loewner framework has proven to be arguably one of the most successful data-driven model order reduction techniques in recent decades. It was originally proposed in [1] as a solution for the generalized realization problem, including also the tangential interpolation case. In the last 16 years, it was successfully extended, developed and applied to various data-driven model order reduction scenarios, using frequency-domain data, such as samples of the frequency response. The first step in the Loewner framework consists in putting together data matrices, i.e., the Loewner and shifted Loewner matrices based on a chosen partition into right and left data. The latter step is of particular importance, since it greatly influences the quality of the fitted models. Then, the singular value decomposition (SVD) is employed to a linear combination of the Loewner matrices; by forming the projection bases using the dominant singular values and vectors, a compressed model is computed. We present new insights into the Loewner framework for the case of discretized linear systems, its application to approximation of some irrational functions, and if time permits, some recent extensions to the parametric case (by avoiding the use of complicated barycentric forms of the interpolant, in the case of multiple parameters).

References

[1] A. J. MAYO, A. C. ANTOULAS, A framework for the solution of the generalized realization problem, Linear Algebra and its Applications, vol. 425, pp. 634–662, 2007.

$\mathcal{H}_2$ optimal model reduction for simply connected domains

Alessandro Borghi[1], *Tobias Breiten*[1]

[1]*Institut für Mathematik, Technische Universität Berlin, Straße des 17. Juni 136, Berlin, 10623, Berlin, Germany*
E-mail: borghi@tu-berlin.de, tobias.breiten@tu-berlin.de

Abstract

Optimal $\mathcal{H}_2$ approximation has been widely used for the development of efficient model order reduction algorithms. Well known examples are IRKA [1] and MIRIAm [2] respectively adopted in continuous and discrete time state-space systems. These methods rely on the definition of a norm on a Hardy space from which the $\mathcal{H}_2$ optimal interpolation conditions follow. One of the main assumptions for IRKA and MIRIAm is that the transfer function of the full-order model needs to be analytic on the right half complex plane and on the outside of the unit disk respectively. However, there can be cases in which the transfer function is analytic in domains that differ from the ones above. Hence, we propose a framework to derive first-order interpolation conditions for $\mathcal{H}_2$ optimality in a simply connected set. The theoretical background relies on conformal maps and generalizes Hardy spaces to functions that are analytic on specific domains. The objective is to eventually develop algorithms that can be used to find a reduced order transfer function that satisfies $\mathcal{H}_2$ optimality conditions in the chosen set.

References

[1] S. Gugercin, A.C. Antoulas, C. Beattie. $\mathcal{H}_2$ model reduction for large-scale linear dynamical systems, *SIAM Journal on Matrix Analysis and Applications*, 30(2): 609–638 (2008).

[2] A. Bunse-Gerstner, D. Kubalinska, G. Vossen, D. Wilczek. h_2-norm optimal model reduction for large scale discrete dynamical MIMO systems, *Journal of Computational and Applied Mathematics*, 233: 1202–1216 (2010).

Parametric Linearization of Nonlinear Dynamical Systems Subject to Periodic Inputs

Giovanni Conni[1], *Karl Meerbergen*[2], *Frank Naets*[3]

[1] *Computer Science Dpt., KU Leuven, Belgium*
E-mail: giovanni.conni@kueuven.be
[2] *Computer Science Dpt., KU Leuven, Belgium*
E-mail: karl.meerbergen@kuleuven.be
[3] *Mechanical Engineering Dpt., KU Leuven, Belgium*
E-mail: frank.naets@kuleuven.be

Abstract

Mathematical models of dynamical systems have seen a gradual increase both in size and complexity, and together with these the necessity of computational power. Moreover, these models are often nonlinear, which makes them more computationally expensive and limits their real-world applications. To allow for faster simulations, Model Order Reduction (MOR) techniques in the frequency and in the time domain have been developed.
In the frequency domain, these methods are usually related to the Volterra series [1]. This is an extension of the transfer function to nonlinear system, where the input-output relationship is expanded through an infinite series of so called Volterra kernels. The Volterra series is a powerful tool for the analysis of nonlinear systems, but its application can be complicated: research is still ongoing regarding convergence conditions, and the identification of the kernels is not straightforward.
On the other hand, the interest for time-domain MOR methods is raising, due to their data-driven nature. We have developed a variant of the AAA MOR method [2], called tLS-AAA, which employs time domain data to generate a linear Reduced Order Model (ROM). This reduction method regards the system as a black box and does not require any information on the input to generate the ROM. tLS-AAA approximates the input-output relationship of the nonlinear system, and it can be related to the Volterra series. By exploiting this equivalence, tLS-AAA can be modified to generate a parametric linear ROM which approximates the system output for periodic input functions. Moreover, other techniques which employ the Volterra series in the context of condition monitoring and system analysis can be redefined for the tLS-AAA method in an efficient way.

Acknowledgements: This research is partly founded by internal KU Leuven funds as part of the C2 project 3E190371.

References

[1] Cheng, C.M., Peng, Z.K., Zhang, W.M., & Meng, G. (2017) Volterra-series-based nonlinear system modeling and its engineering applications: A state-of-the-art review. Mechanical Systems and Signal Processing, 87, 340–364.

[2] Nakatsukasa, Y., Sète, O., & Trefethen, L.N. (2018) The AAA algorithm for rational approximation. SIAM J. Sci. Comput., 40, A1494–A1522.

Numerical linear algebra aspects of the Dynamic Mode Decomposition

Zlatko Drmač[1]

[1] *Department of Mathematics, Faculty of Science, University of Zagreb, Croatia*
E-mail: drmac@math.hr

Abstract

The Dynamic Mode Decomposition (DMD) is a method for computational analysis of nonlinear dynamical systems in data driven scenarios. Based on high fidelity numerical simulations and/or experimental data, the DMD can be used to reveal latent structures in the dynamics or as a forecasting or a model order reduction tool. Theoretical underpinning of the DMD is the Koopman operator on a Hilbert space of observables of the dynamics under study.

The two main computational tasks in DMD analysis are: (i) data driven Rayleigh-Ritz extraction of eigenvalues and eigenvectors using the subspace spanned by the data snapshots; (ii) spatio-temporal representation of the snapshots using a subset of the computed eigenpairs, which amounts to solving a structured least squares problem. The solutions of (i), (ii) allow for an analysis of the structure of the dynamics, forecasting and control. The numerical realization of the method is in the framework of dense numerical linear algebra.

This talk presents a LAPACK implementation of new variants of the DMD for the task (i), and it offers a numerical analysis with insights that provide a better understanding of the accuracy of the method and of its limits. It shows that the state of the art dense numerical linear algebra (perturbation theory, numerical algorithms, software) is the tool of the trade for computational analysis of complex nonlinear dynamics, in particular in data driven scenarios.

Acknowledgements:

The work of the author is supported by the DARPA SBIR Program Office under Contract No. W31P4Q-21-C-0007 to AIMdyn, Inc., the Croatian Science Foundation (CSF) grant IP-2019-04-6268, and the University of Zagreb.

References

[1] Z. Drmač, I. Mezić, and R. Mohr. Data driven modal decompositions: analysis and enhancements. *SIAM Journal on Scientific Computing*, 40(4):A2253–A2285, 2018.

[2] Z. Drmač, A LAPACK implementation of the Dynamic Mode Decomposition I., LAPACK Working Note 298, 2022.

[3] Z. Drmač, A LAPACK implementation of the Dynamic Mode Decomposition II. (Symmetric/Hermitian DMD), LAPACK Working Note 300, 2022.

ONE CAN HEAR THE IMPEDANCE AND LOSS PROFILES OF A STRING: FROM THE DISCRETE TO CONTINUUM DISSIPATIVE INVERSE PROBLEM

Jörn Zimmerling[1], *Roberto Remis* [2], *Murthy Guddati*[3], *Vladimir Druskin*[4], *Elena Cherkaev*[5]

[1] *Uppsala University, Sweden*
E-mail: jtzimmerling@gmail.com
[2] *Delft University, Netherlands*
E-mail: R.F.Remis@tudelft.nl
[3] *North Carolina State University, USA*
E-mail: mnguddat@ncsu.edu [4] *Worcester Polytechnic Institute 3, USA*
E-mail: vdruskin@wpi.edu [5] *University of Utah, USA*
E-mail: elena@math.utah.edu

Abstract

It is well known a lossless discrete mass-spring string can be identified from its spectrum via the Lanczos algorithm assuming symmetry of the string with respect to its center[1]. The impedance distribution of the continuum symmetric string can be obtained via the finite-difference quadrature rules a.k.a.the optimal grids as the limit of the discrete string [2]. A discrete damped string can be also identified from the spectrum via the bi-Lanczos algorithm. However the transition to the continuum damped problem might fail due to possibly negative discrete dumpers that can be obtained even from dissipative spectral data. We show how this problem can be circumvented in the reduced order Lippmann-Schwinger framework a.k.a. the Lippmann-Schwinger-Lanczos algorithm. It allows simultaneous estimation of both the losses and impedance profiles.

Acknowledgements: D ruskin was partially supported by AFOSR grant FA9550-20-1-0079, also Cherkaev, Druskin and Guddati were partially supported by NSF grant 11008-GR.

References

[1] S. Cox, M. Embree, J. Hokanson *One Can Hear the Composition of a String: Experiments with an Inverse Eigenvalue Problem*, SIAM Rev., DOI:10.1137/080731037, 2012

[2] L. Borcea, V.Druskin, L. Knizhnerman. *On the continuum limit of a discrete inverse spectral problem*, Comm. Pure Appl. Math., doi.org/10.1002/cpa.20073, 2005

Randomized POD-Beyn algorithm for nonlinear eigenvalue problems – analysis and perspectives

Luka Grubišić[1]

[1] *Department of Mathematics, Faculty of Science, University of Zagreb, Croatia*
E-mail: luka.grubisic@math.hr

Abstract

We propose a method to accelerate the solution of 3D FEM-discretized nonlinear eigenvalue problems by utilizing a reduced order model (ROM) via a randomized projection onto a suitable subspace, with eigenpairs identical to the full problem in a region of the complex plane (Beyn approach). The subspace is automatically constructed by solving the full problem at a few random points inside the region of interest. The obtained method is suitable for any nonlinear eigenvalue problem given in the separable (Affine like) form. We test our theory on a family of thermoacoustic application, and show how does the method generalize to applications dealing with other vibrational problems.

This is a joint work with Georg A. Mensah, Alessandro Orchini and Philip E. Buschmann.

Acknowledgements: Work (partially) supported by Croatian Science Foundation Grant IP-2019-04-6268.

Data-driven balancing: what to sample for different types of balanced reduced models

Sean Reiter[1], *Serkan Gugercin*[2], *Ion Victor Gosea*[3]

[1] *Department of Mathematics, Virginia Tech, United States*
E-mail: seanr7@vt.edu
[2] *Department of Mathematics and Division of Computational Modeling and Data Analytics, Academy of Data Science, Virginia Tech, United States*
E-mail: gugercin@vt.edu
affilnum1 Max Planck Institute for Dynamics of Complex Technical Systems, Germany
E-mail: gosea@mpi-magdeburg.mpg.de

Abstract

In this work we present extensions of the Quadrature-based Balanced Truncation (Quad-BT) framework of [1] to other types of balancing. Quad-BT, a "non-intrusive" (data-driven) reformulation of the classical projection-based balanced truncation for linear systems [2, 3], builds reduced-order models entirely from system input/output response data (e.g., using transfer function samples) without the need to access an explicit state-space realization of the underlying system. We extend this data-driven framework to include other types of balancing; namely, balanced stochastic truncation, positive-real balancing, bounded-real balancing, and frequency weighted balanced truncation. This is accomplished by sampling certain spectral factors associated with the system of interest. We verify this approach with several numerical examples.

Acknowledgements: Work partially supported by US National Science Foundation grant AMPS-1923221

References

[1] I. V. Gosea, S. Gugercin, and C. Beattie, *Data-driven balancing of linear dynamical systems*. SIAM Journal on Scientific Computing, Vol. 44, No. 1, pp. A554–A582, 2022

[2] B. Moore, *Principal component analysis in linear systems: Controllability, observability, and model reduction*. IEEE Transactions on Automatic Control, Vol. 26, No. 1, pp. 17–32, 1981.

[3] C. Mullis and R. Roberts, *Synthesis of minimum roundoff noise fixed point digital filters*. IEEE Trans. Circuits and Systems, Vol. 23, No. 9, pp. 551–562, 1976.

Optimal reduced-order modeling for structured linear systems

Peter Benner[1], *Serkan Güğercin*[2], *Petar Mlinarić*[3]

[1] *Computational Methods in Systems and Control Theory, Max Planck Institute for Dynamics of Complex Technical Systems, Germany*
E-mail: benner@mpi-magdeburg.mpg.de
[2] *Department of Mathematics and Division of Computational Modeling and Data Analytics, Academy of Data Science, Virginia Tech, United States*
E-mail: gugercin@vt.edu
[3] *Department of Mathematics, Virginia Tech, United States*
E-mail: mlinaric@vt.edu

Abstract

There are various approaches to $\mathcal{H}_2$-optimal reduced-order modeling of (unstructured) linear time-invariant dynamical systems, such as the iterative rational Krylov algorithm [4] which uses the rational interpolation framework. Interpolatory model order reduction was also extended to structured linear systems [2]. We are interested in extending $\mathcal{H}_2$-optimal reduced-order modeling to structured linear systems and investigating whether it necessitates interpolation. Interpolatory $\mathcal{H}_2$-optimality conditions have been established for certain second-order systems and port-Hamiltonian systems [1], as well as special time-delay system [6]. Furthermore, for a special class of parametric linear time-invariant systems, interpolatory $\mathcal{H}_2 \otimes \mathcal{L}_2$-optimality conditions are known [3]. In this talk, we present some generalizations of these results using the work on $\mathcal{L}_2$-optimal reduced-order modeling [5] and present them on a few numerical examples.

References

[1] C. A. Beattie and P. Benner. $\mathcal{H}_2$-optimality conditions for structured dynamical systems. Preprint MPIMD/14-18, MPI Magdeburg, 2014.

[2] C. A. Beattie and S. Gugercin. Interpolatory projection methods for structure-preserving model reduction. *SCL*, 58(3):225–232, 2009.

[3] A. R. Grimm. *Parametric Dynamical Systems: Transient Analysis and Data Driven Modeling*. PhD thesis, Virginia Tech, 2018.

[4] S. Gugercin, A. C. Antoulas, and C. Beattie. $\mathcal{H}_2$ model reduction for large-scale linear dynamical systems. *SIMAX*, 30(2):609–638, 2008.

[5] P. Mlinarić and S. Gugercin. A unifying framework for interpolatory $\mathcal{L}_2$-optimal reduced-order modeling. arXiv preprint 2209.00714, 2022.

[6] I. Pontes Duff, S. Gugercin, C. Beattie, C. Poussot-Vassal, and C. Seren. $\mathcal{H}_2$-optimality conditions for reduced time-delay systems of dimensions one. *IFAC-PapersOnLine*, 49(10):7–12, 2016.

On multi-objective optimization of model reduction for port-Hamiltonian systems

Jonas Nicodemus[1], *Paul Schwerdtner*[2], *Benjamin Unger*[1]

[1] *Stuttgart Center for Simulation Science (SC SimTech), University of Stuttgart, Universitätsstr. 32, 70569 Stuttgart, Germany*
E-mail: {jonas.nicodemus,benjamin.unger}@simtech.uni-stuttgart.de
[2] *Institut für Mathematik, Technische Universität Berlin, Straße des 17. Juni 136, 10623 Berlin, Germany*
E-mail: schwerdt@math.tu-berlin.de

Abstract

Conventional structure preserving *model order reduction* (MOR) for *port-Hamiltonian* (pH) systems focus on approximating the input-output dynamics by (approximately) minimizing classical system norms, such as the Hardy $\mathcal{H}_2$ norm. Nevertheless, the definition of a pH system consists of two objects: the input-output dynamics and an energy function, typically called the Hamiltonian. If we thus measure the approximation quality only with respect to the input-output dynamics, then the approximation of the Hamiltonian is not reflected at all. This is particularly relevant since recent results [1] demonstrate that modifying the Hamiltonian of the *full order model* (FOM) may yield better *reduced order models* (ROMs), at least if only the input-output dynamics are analyzed. In this talk, we take a first step towards the dual-objective optimization problem for optimal approximating both objectives: the input-output dynamics and the Hamiltonian, by noticing that in the pH ROM, we can modify the Hamiltonian without changing the input-output dynamics. Thus, for a given pH ROM, we can search for the reduced Hamiltonian that best approximates the FOM Hamiltonian. We prove that the resulting optimization problem is strictly convex and uniquely solvable. Moreover, we propose a numerical algorithm to solve the optimization problem and demonstrate its applicability with two academic toy examples. This talk describes joint work with Paul Schwerdtner (TU Berlin) and Benjamin Unger (U Stuttgart).

Acknowledgements: P. Schwerdtner acknowledges funding from the DFG within the project 424221635. J. Nicodemus and B. Unger acknowledge funding from the DFG under Germany's Excellence Strategy – EXC 2075 – 390740016 and are thankful for support by the Stuttgart Center for Simulation Science (SimTech).

References

[1] T. Breiten and B. Unger. Passivity preserving model reduction via spectral factorization. *Automatica J. IFAC*, 142:110368, 2022.

An Eigensystem Realization Algorithm for Continuous-Time Systems and Its Connection with the Hankel Operator

Igor Pontes Duff[1], *Ion Victor Gosea*[2], *Serkan Gugercin*[3], *Christopher Beattie*[4]

[1] *Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany*
E-mail: pontes@mpi-magdeburg.mpg.de
[2] *Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany*
E-mail: gosea@mpi-magdeburg.mpg.de
[3] *2Department of Mathematics, Virginia Tech, Blacksburg, USA*
E-mail: gugercin@vt.edu
[4] *2Department of Mathematics, Virginia Tech, Blacksburg, USA*
E-mail: beattie@vt.edu

Abstract

The Eigensystem Realization Algorithm (ERA) is a commonly employed system-theoretical data-driven method to identify the underlying dynamic behavior of a discrete-time system [1]. The algorithm works by first collecting a the system's impulse response data and then constructing the Hankel matrix. Finally an SVD of the Hankel matrix allows to identify the underlying discrete-time dynamical systems. Additionally, if enough data is available, the algorithm is theoretically equivalent to discrete-time balanced truncation [2].

In this work, we propose a continuous-time eigensystem realization algorithm (ctERA). To this end, we introduce new continuous-time Hankel matrices that are constructed using the impulse response of the underlying dynamical system. Based on those matrices, we can determine the McMillan degree of the underlying dynamical system and construct a parsimonious realization. Additionally, we show the connections between ctERA and the new data-driven methodology to compute balanced truncatioin via quadrature rules (quadBT) [3]. Moreover, we show how both algorithms are related to the Hankel operator and allow to construct balanced reduced models for infinite dimensional systems.

References

[1] Juang, J. N., & Pappa, R. S. (1985). An eigensystem realization algorithm for modal parameter identification and model reduction. *Journal of guidance, control, and dynamics*, 8(5), 620-627.

[2] Ma, Z., Ahuja, S., & Rowley, C. W. (2011). Reduced-order models for control of fluids using the eigensystem realization algorithm. *Theoretical and Computational Fluid Dynamics*, 25, 233-247.

[3] Gosea, I. V., Gugercin, S., & Beattie, C. (2022). Data-driven balancing of linear dynamical systems. *SIAM Journal on Scientific Computing*, 44(1), A554-A582.

From matrix equations to surrogate models

Jens Saak[1]

[1] *Computational Methods in Systems and Control Theory, Max Planck Institute for Dynamics of Complex Technical Systems, Germany*
E-mail: saak@mpi-magdburg.mpg.de

Abstract

The dynamics of a linear time-invariant dynamical system can be expressed by its transfer function H in the Laplace domain. For a given Laplace variable s this can be written as $H(s) = C(sE - A)^{-1}B$.

In the most simple case, i.e., when the system is single-input-single-output, this is a scalar rational function. A natural question to ask, thus, is how to compute much smaller matrices $\hat{E}$, $\hat{A}$, $\hat{B}$, $\hat{C}$, such that $\hat{H}(s) = \hat{C}(s\hat{E} - \hat{A})^{-1}\hat{B}$ interpolates H in a given set of $s_j, j = 1, \ldots, r$.

The literature provides two well-known answers to this question. To compute a surrogate via (intrusive) projection-based reduced-order modeling, one collects an orthogonal basis of a rational Krylov subspace in a tall matrix V to form $\hat{E} = V^{\mathsf{T}}EV$, $\hat{A} = V^{\mathsf{T}}AV$, $\hat{B} = V^{\mathsf{T}}B$, $\hat{C} = CV$. For (non-intrusive) data-driven surrogate modeling, when one has measurements of H available, an alternative is given by the Loewner framework. There, the surrogate model is derived as a realization formed from Loewner and shifted Loewner matrices.

A common feature of both approaches is that the matrices X of interest can be computed as the solutions of certain matrix Sylvester equations of the form $MXP + NXQ = R$, where the eigenvalues of the pair (Q, P) are the interpolation points s_j. For the Loewner framework P, Q, M and N are chosen diagonal, while the right hand side encodes the measurements. In the moment matching case, e.g., $M = A$, $N = E$ and $R = B\tilde{R}$, for an $\tilde{R}$ such that $(P^{-1}Q, \tilde{R})$ is observable.

Now, performing excessive oversampling, i.e., drawing so many samples that the encoded information becomes (almost) redundant, the solutions of the Sylvester equations will have low (numerical) rank. In this contribution, we investigate how iterative low-rank Sylvester solvers can be used to compute the solution factors directly, rather than performing rank-truncation on the matrix X.

Acknowledgements: The author would like to thank Hussam Al Daas for many fruitful discussions about extensions of this work to parametric systems and consequently tensor equations

MSC13. Linear algebra and quantum information theory

Ángela Capel, Fernando Lledó,
Julio de Vicente

Entangled subspaces and their characterization
Remigiusz Augusiak

Free spectrahedra in quantum information theory
Andreas Bluhm

Mutually unbiased measurements and their applications in quantum information
Máté Farkas, Máté Farkas, Armin Tavakoli, Denis Rosset, Jean-Daniel Bancal, Jędrzej Kaniewski, María Prat Colomer, Luke Mortimer, Irénée Frérot, Antonio Acín, Ashwin Nayak

Refuting spectral compatibility of quantum marginals
Felix Huber, Nikolai Wyderka

Recoverability of quantum channels via hypothesis testing
Anna Jenčová

Recoverability of quantum Fisher information
Haojian Li, Li Gao, Iman Marvian, Cambyse Rouze

A complete hierarchy of linear systems for certifying quantum entanglement of subspaces
Benjamin Lovitz, Nathaniel Johnston, Aravindan Vijayaraghavan

Spectral gap for AKLT models on arbitrary decorated graphs
Angelo Lucia, Amanda Young

Diagonal unitary symmetry in quantum theory
Ion Nechita, Satvik Singh

Thermalization in quantum spin systems
Antonio Pérez Hernández, Angelo Lucia, David Pérez-García

Monogamy of entanglement between cones and DPS-like hierarchies
Martin Plávala, Guillaume Aubrun, Alexander Müller-Hermes

Positive maps and entanglement in real Hilbert spaces
Mizanur Rahaman

When are quantum states antidistinguishable?
Jamie Sikora, Nathaniel Johnston, Vincent Russo

Quantum concentration inequalities
Daniel Stilck Franca

Abstract Cone Systems
Mirte van der Eyden, Tim Netzer, Gemma De les Coves

Entangled subspaces and their characterization

Remigiusz Augusiak[1]

[1] *Center for Theoretical Physics, Polish Academy of Sciences, Al. Lotników 32/46, 02-668 Warsaw, Poland*
E-mail: augusiak@cft.edu.pl

Abstract

Genuinely entangled subspaces are a class of subspaces in the multipartite Hilbert spaces that are composed of only genuinely entangled states and are thus a natural generalization of the completely entangled subspaces to the multipartite regime. They are thus an interesting object of study in the context of multipartite entanglement. In this talk I will present some of our recent results concerning characterization of this type of subspaces (such as criteria for being genuinely entangled or self-testing statements), putting a particular emphasis on the stabilizer subspaces known in the context of quantum error stabilizer codes. In particular, I will discuss a conjecture that there are no genuinely entangled multipartite stabilizer states (mixed states defined on stabilizer subspaces) with positive partial transpositions. This talk is based on a series of papers [1, 2, 3, 4].

Acknowledgements: The research presented in this talk has been supported by the Polish National Science Centre through the SONATA BIS project No. 2019/34/E/ST2/00369.

References

[1] M. Demianowicz and R. Augusiak, Phys. Rev. A **100**, 062318 (2019).

[2] F. Baccari, R. Augusiak, I. Šupić, and A. Acín, Phys. Rev. Lett. **125**, 260507 (2020).

[3] O. Makuta, R. Augusiak, New J. Phys. **23**, 043042 (2021).

[4] O. Makuta, B. Kuzaka, R. Augusiak, Quantum **7**, 915 (2023).

Free spectrahedra in quantum information theory

Andreas Bluhm[1]

[1] *Univ. Grenoble Alpes, CNRS, Grenoble INP, LIG, France*
E-mail: andreas.bluhm@univ-grenoble-alpes.fr

Abstract

In this talk, I will explain how some problems in quantum information theory can be rephrased as inclusion problems of certain free spectrahedra. Free spectrahedra typically arise as matricial relaxations of linear matrix inequalities. A well-known example is the matrix cube. On the quantum side, I will focus on measurement incompatibility: Two quantum measurements are compatible if there exists a third one which implements both measurements at the same time. The best known example of incompatible measurements are the position and the momentum of a particle. Using the connection to free spectrahedra, we can show that inclusion constants correspond to the robustness of measurement incompatibility to white noise. This part of the talk will be based on [1], [2], and [3].

References

[1] A. Bluhm, I. Nechita. Joint measurability of quantum effects and the matrix diamond. *Journal of Mathematical Physics*, 59: 112202 (2018).

[2] A. Bluhm, I. Nechita. Compatibility of quantum measurements and inclusion constants for the matrix jewel. *SIAM Journal on Applied Algebra and Geometry*, 4: 255–296 (2020).

[3] A. Bluhm, I. Nechita. A tensor norm approach to quantum compatibility. *Journal of Mathematical Physics*, 63: 062201 (2022).

Mutually unbiased measurements and their applications in quantum information

Máté Farkas[1]

Armin Tavakoli[2]

Denis Rosset[3]

Jean-Daniel Bancal[4]

Jędrzej Kaniewski[5]

Maria Prat Colomer[1]

Luke Mortimer[1]

Irénée Frérot[6]

Antonio Acín[1]

Ashwin Nayak[7]

[1] *ICFO-Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels, Spain*
E-mail: mate.farkas@icfo.eu
[2] *Physics Department, Lund University, Box 118, 22100 Lund, Sweden*
[3] *Department of Applied Physics University of Geneva, 1211 Geneva, Switzerland*
[4] *Université Paris-Saclay, CEA, CNRS, Institut de Physique Théorique, 91191, Gif-sur-Yvette, France*
[5] *Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*
[6] *Univ Grenoble Alpes, CNRS, Grenoble INP, Institut Néel, 38000 Grenoble, France*
[7] *Department of Combinatorics and Optimization, and Institute for Quantum Computing, University of Waterloo, 200 University Ave. W., Waterloo, ON, N2L 3G1, Canada*

Abstract

Mutually unbiased bases (MUBs) are highly symmetric pairs of orthonormal bases in finite-dimensional complex Hilbert space. Specifically, the modulus square of the overlap between any two vectors from the two different bases is a constant, the reciprocal of the dimension. In quantum information theory, every orthonormal basis has an associated quantum measurement, and measurements corresponding to MUBs are particularly useful in quantum information processing tasks due to the symmetric nature of MUBs.

MUBs can also be defined using the terminology of quantum measurements: MUBs are d-outcome projective measurements on d-dimensional complex Hilbert space such that if a quantum state yields a definite outcome on one of the measurements, then it yields a uniformly random outcome on the other one. In this talk I will introduce mutually unbiased measurements (MUMs) that are a generalisation of this latter definition. In particular, the MUM definition coincides with the MUB definition without the assumption on the Hilbert space dimension, making it a "device-independent" definition.

We provide a fully algebraic characterisation of MUMs in terms of the measurement operators. Then, we devise a family of Bell inequalities, parametrised by an integer d, such that the maximal Bell inequality violation certifies MUM measurements with d outcomes in the associated Bell experiment. We show that MUMs have the same entropic uncertainty relations and incompatibility robustness as MUBs. On the other hand, MUMs are strictly more general than MUBs: there exist MUMs that cannot be mapped to MUBs by any completely positive unital map, and the number of d-outcome measurements that are pairwise unbiased is unbounded, in stark contrast with the number of pairwise MUBs in dimension d, which is bounded by $d + 1$.

References

[1] A. Tavakoli, M. Farkas, D. Rosset, J.-D. Bancal, J. Kaniewski, Mutually unbiased bases and symmetric informationally complete measurements in Bell experiments. *Science Advances* 7, eabc3847 (2021)

[2] M. Prat Colomer, L. Mortimer, I. Frérot, M. Farkas, A. Acín, Three numerical approaches to find mutually unbiased bases using Bell inequalities. *Quantum* 6, 778 (2022)

[3] M. Farkas, J. Kaniewski, A. Nayak, Mutually Unbiased Measurements, Hadamard Matrices, and Superdense Coding. `arXiv`:2204.11886 (2022)

Refuting spectral compatibility of quantum marginals

Felix Huber[1], *Nikolai Wyderka*[2]

[1] *Faculty of Physics, Astronomy and Applied Computer Science, Institute of Theoretical Physics, Jagiellonian University, ul. Łojasiewicza 11, 30-348 Kraków, Poland*
E-mail: felix.huber@uj.edu.pl
[2] *Institut für Theoretische Physik III, Heinrich-Heine-Universität Düsseldorf, Universitätsstraße 1, DE-40225 Düsseldorf, Germany*
E-mail: wyderka@hhu.de

The spectral variant of the quantum marginal problem asks: Given prescribed spectra for a set of quantum marginals, does there exist a compatible joint state? The main idea of this work is a symmetry-reduced semidefinite programming hierarchy for detecting incompatible spectra. The hierarchy can provide refutations that are dimension-free, certifying incompatibility in all local dimensions. It equally applies to the compatibility of local unitary invariants, to the sums of Hermitian matrices problem, to optimize trace polynomials on the positive cone, and to certify vanishing Kronecker coefficients.

Acknowledgements: FH was supported by the FNP through TEAM-NET (POIR.04.04.00-00-17C1/18-00). NW acknowledges support by the QuantERA grant QuICHE and the German Ministry of Education and Research (BMBF grant no. 16KIS1119K). Computational infrastructure and support were provided by the Centre for Information and Media Technology at Heinrich Heine University Düsseldorf.

Recoverability of quantum channels via hypothesis testing

Anna Jenčová

Mathematical Institute, Slovak Academy of Sciences, Slovakia
E-mail: jenca@mat.savba.sk

Abstract

A fundamental property of the quantum relative entropy is the data processing inequality (DPI), stating that relative entropy cannot be increased by the action of a quantum channel. The question when the DPI becomes an equality was first answered by Petz [1], who showed that this occurs if and only if there exists a channel that fully recovers both states. Moreover, the recovery channel can be chosen to be universal, depending only on one of the states. Meanwhile, it has been proved that the same is true for equality in the DPI for a number of quantum distinguishability measures.

A much stronger approximate result, first proved by Fawzi and Renner [2] in the special case of the quantum Markov chains, shows that there exists a recovery map which recovers one of the states perfectly and the other state is recovered up to an error bounded by the decrease in the relative entropy. Again, the recovery map can be chosen universal [3]. (Approximate) recoverability has found a number of applications in quantum information theory and different areas of physics.

We use a recent integral formula for the relative entropy due to Frenkel [4] for an easy proof of recoverability in terms of preservation of error probabilities in hypothesis testing, or, equivalently, the L_1-distance. We further discuss relations between recoverability and some restricted settings of broadcasting.

Acknowledgements: Work supported by the grants VEGA 1/0142/20 and the Slovak Research and Development Agency grant APVV-20-0069.

References

[1] D. Petz. Sufficient subalgebras and the relative entropy of states of a von Neumann algebra. *Commun. Math. Phys.*, 105: 123–131 (1986).

[2] O. Fawzi, R. Renner. Quantum conditional mutual information and approximate Markov chains. *Commun. Math. Phys.*, 340: 575–611 (2015).

[3] M. Junge, R. Renner, D. Sutter, M. M. Wilde, A. Winter. Universal recovery maps and approximate sufficiency of quantum relative entropy. *Annales H. Poincaré* 19: 2955–2978 (2018).

[4] P. E. Frenkel. Integral formula for quantum relative entropy implies data processing inequality. arXiv:2208.12194 (2022).

Recoverability of quantum Fisher information

Li Gao [1],
Haojian Li[2],
Iman Marvian [3],
Cambyse Rouzé[4]

[1] *Department of Mathematics, University of Houston, USA*
E-mail: lgao12@uh.edu
[2] *Zentrum Mathematik, Technische Universität München, Germany*
E-mail: haojian.li@tum.de
[3] *Department of Electrical and Computer Engineering and Department of Physics, Duke University, USA*
E-mail: iman.marvian@duke.edu
[4] *Zentrum Mathematik, Technische Universität München, Germany*
E-mail: cambyse.rouze@tum.de

Abstract

Recovery maps were first introduced by Petz and later found rich applications in quantum information theory. In this talk, I will present a recoverability result of Riemannian monotone metrics on the quantum state space. For two quantum states, the monotone metric gives the corresponding quantum χ^2-divergence. We obtain a universal recovery bound for a special χ^2 divergence. I will also discuss applications to quantum metrology and quantum asymmetry.

Acknowledgements: LG is partially supported by NSF grant DMS-2154903. IM is supported by NSF grants FET-1910571, Phy-2046195, FET-2106448. C.R. acknowledges the support of the Munich Center for Quantum Sciences and Technology, funding from the Humboldt Foundation as well as by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germanys Excellence Strategy EXC-2111 390814868. H.L. acknowledges support by the DFG cluster of excellence 2111 (Munich Center for Quantum Science and Technology). This work was supported by the DOE QSA program.

References

[BC94] Samuel L Braunstein and Carlton M Caves. Statistical distance and the geometry of quantum states. *Physical Review Letters*, 72(22):3439, 1994.

[BCM96] Samuel L Braunstein, Carlton M Caves, and Gerard J Milburn. Generalized uncertainty relations: theory, examples, and lorentz invariance. *annals of physics*, 247(1):135–173, 1996.

[BNG00] OE Barndorff-Nielsen and RD Gill. Fisher information in quantum statistics. *Journal of Physics A: Mathematical and General*, 33(24):4481, 2000.

[Čen78] NN Čencov. Algebraic foundation of mathematical statistics. *Statistics: A Journal of Theoretical and Applied Statistics*, 9(2):267–276, 1978.

[CHP19] Jordan Cotler, Patrick Hayden, Geoffrey Penington, Grant Salton, Brian Swingle, and Michael Walter. Entanglement wedge reconstruction via universal recovery channels. *Physical Review X*, 9(3):031011, 2019.

[CL19] Yu Cao and Jianfeng Lu. Tensorization of the strong data processing inequality for quantum chi-square divergences. *Quantum*, 3:199, 2019.

[Cra16] Harald Cramér. Mathematical methods of statistics (pms-9), volume 9. In *Mathematical Methods of Statistics (PMS-9), Volume 9*. Princeton university press, 2016.

[CS22] Sam Cree and Jonathan Sorce. Approximate Petz recovery from the geometry of density operators. *Communications in Mathematical Physics*, pages 1–13, 2022.

[CV20a] Eric A Carlen and Anna Vershynina. Recovery map stability for the data processing inequality. *Journal of Physics A: Mathematical and Theoretical*, 53(3):035204, 2020.

[CV20b] Eric A Carlen and Anna Vershynina. Recovery map stability for the data processing inequality. *Journal of Physics A: Mathematical and Theoretical*, 53(3):035204, 2020.

[Fis22] Ronald A Fisher. On the mathematical foundations of theoretical statistics. *Philosophical transactions of the Royal Society of London. Series A, containing papers of a mathematical or physical character*, 222(594-604):309–368, 1922.

[FR15] Omar Fawzi and Renato Renner. Quantum conditional mutual information and approximate markov chains. *Communications in Mathematical Physics*, 340(2):575–611, 2015.

[Gir14] Davide Girolami. Observable measure of quantum coherence in finite dimensional systems. *Physical review letters*, 113(17):170401, 2014.

[GR22] Li Gao and Cambyse Rouzé. Complete entropic inequalities for quantum markov chains. *Archive for Rational Mechanics and Analysis*, pages 1–56, 2022.

[GW21] Li Gao and Mark M Wilde. Recoverability for optimized quantum f-divergences. *Journal of Physics A: Mathematical and Theoretical*, 54(38):385302, 2021.

[Hay02] Masahito Hayashi. Two quantum analogues of fisher information from a large deviation viewpoint of quantum estimation. *Journal of Physics A: Mathematical and General*, 35(36):7689, 2002.

[HM17] Fumio Hiai and Milán Mosonyi. Different quantum f-divergences and the reversibility of quantum operations. *Reviews in Mathematical Physics*, 29(07):1750023, 2017.

[Hol11] Alexander S Holevo. *Probabilistic and statistical aspects of quantum theory*, volume 1. Springer Science & Business Media, 2011.

[HP19] Patrick Hayden and Geoffrey Penington. Learning the alpha-bits of black holes. *Journal of High Energy Physics*, 2019(12):1–55, 2019.

[HPS21] Patrick Hayden, Onkar Parrikar, and Jonathan Sorce. The markov gap for geometric reflected entropy. *Journal of High Energy Physics*, 2021(10):1–58, 2021.

[Jen12] Anna Jenčová. Reversibility conditions for quantum operations. *Reviews in Mathematical Physics*, 24(07):1250016, 2012.

[JP06] Anna Jenčová and Dénes Petz. Sufficiency in quantum statistical inference: A survey with examples. *Infinite Dimensional Analysis, Quantum Probability and Related Topics*, 9(03):331–351, 2006.

[JRS18] Marius Junge, Renato Renner, David Sutter, Mark M Wilde, and Andreas Winter. Universal recovery maps and approximate sufficiency of quantum relative entropy. In *Annales Henri Poincaré*, volume 19, pages 2955–2978. Springer, 2018.

[KJJ18] Hyukjoon Kwon, Hyunseok Jeong, David Jennings, Benjamin Yadin, and MS Kim. Clock–work trade-off relation for coherence in quantum thermodynamics. *Physical review letters*, 120(15):150602, 2018.

[Kom66] Hikosaburo Komatsu. Fractional powers of operators. *Pacific Journal of Mathematics*, 19(2):285–346, 1966.

[Kos05] Hideki Kosaki. Matrix trace inequalities related to uncertainty principle. *International Journal of Mathematics*, 16(06):629–645, 2005.

[KS05] Abram Kagan and Lawrence A Shepp. A sufficiency paradox: an insufficient statistic preserving the fisher information. *The American Statistician*, 59(1):54–56, 2005.

[LR74] Elliott H Lieb and Mary Beth Ruskai. Some operator inequalities of the Schwarz type. *Advances in Mathematics*, 12(2):269–273, 1974.

[LR99] Andrew Lesniewski and Mary Beth Ruskai. Monotone riemannian metrics and relative entropy on noncommutative probability spaces. *Journal of Mathematical Physics*, 40(11):5702–5724, 1999.

[Mar22] Iman Marvian. Operational interpretation of quantum fisher information in quantum thermodynamics. *Physical Review Letters*, 129(19):190502, 2022.

[MC89] Elena Aleksandrovna Morozova and Nikolai Nikolaevich Chentsov. Markov invariant geometry on state manifolds. *Itogi Nauki i Tekhniki. Seriya" Sovremennye Problemy Matematiki. Noveishie Dostizheniya"*, 36:69–102, 1989.

[ML16] Iman Marvian and Seth Lloyd. From clocks to cloners: Catalytic transformations under covariant operations and recoverability. *arXiv preprint arXiv:1608.07325*, 2016.

[MS14] Iman Marvian and Robert W Spekkens. Extending noether's theorem by quantifying the asymmetry of quantum states. *Nature communications*, 5(1):1–8, 2014.

[Pet86] Dénes Petz. Sufficient subalgebras and the relative entropy of states of a von neumann algebra. *Communications in mathematical physics*, 105(1):123–131, 1986.

[Pet88] Dénes Petz. Sufficiency of channels over von neumann algebras. *The Quarterly Journal of Mathematics*, 39(1):97–108, 1988.

[Pet96] Dénes Petz. Monotone metrics on matrix spaces. *Linear algebra and its applications*, 244:81–96, 1996.

[Pet02] Dénes Petz. Covariance and fisher information in quantum mechanics. *Journal of Physics A: Mathematical and General*, 35(4):929, 2002.

[PG11] Dénes Petz and Catalin Ghinea. Introduction to quantum fisher information. In *Quantum probability and related topics*, pages 261–281. World Scientific, 2011.

[PH96] Dénes Petz and Hiroshi Hasegawa. On the riemannian metric of α-entropies of density matrices. *Letters in Mathematical Physics*, 38(2):221–225, 1996.

[Pol13] David Pollard. A note on insufficiency and the preservation of fisher information. *Probability to Statistics and Back: High-Dimensional Models and Processes-A Festschrift in Honor of Jon A. Wellner,. Beachwood: Institute of Mathematical Statistics*, 2013.

[Rao45] CR Rao. Information and accuracy attainable in the estimation of statistical parameters. kotz s & johnson nl (eds.), breakthroughs in statistics volume i: Foundations and basic theory, 235–248, 1945.

[SBT17] David Sutter, Mario Berta, and Marco Tomamichel. Multivariate trace inequalities. *Communications in Mathematical Physics*, 352(1):37–58, 2017.

[Sch12] Mark J Schervish. *Theory of statistics*. Springer Science & Business Media, 2012.

[SFR16] David Sutter, Omar Fawzi, and Renato Renner. Universal recovery map for approximate markov chains. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 472(2186):20150623, 2016.

[Tak19] Ryuji Takagi. Skew informations from an operational view via resource theory of asymmetry. *Scientific reports*, 9(1):1–12, 2019.

[TKR10] Kristan Temme, Michael James Kastoryano, Mary Beth Ruskai, Michael Marc Wolf, and Frank Verstraete. The χ 2-divergence and mixing times of quantum markov processes. *Journal of Mathematical Physics*, 51(12):122201, 2010.

[Wol12] M. M. Wolf. Quantum channels & operations: Guided tour. *Lecture notes available at http://www-m5. ma. tum. de/foswiki/pub M*, 5, 2012.

[YV16] Benjamin Yadin and Vlatko Vedral. General framework for quantum macroscopicity in terms of coherence. *Physical Review A*, 93(2):022122, 2016.

A complete hierarchy of linear systems for certifying quantum entanglement of subspaces

Nathaniel Johnston[1], *Benjamin Lovitz*[2], *Nathaniel Johnston*[3]

[1] *Mount Allison University, Canada*
E-mail: author1@
[2] *Northeastern University, USA*
E-mail: benjamin.lovitz@gmail.com
[3] *Northwestern University, USA*
E-mail: author3@

Abstract

We introduce a hierarchy of linear systems for showing that a given subspace of pure quantum states is entangled (i.e., contains no product states). This hierarchy outperforms known methods already at the first level, and it is complete in the sense that every entangled subspace is shown to be so at some finite level of the hierarchy. It generalizes straightforwardly to the case of higher Schmidt rank, as well as the multipartite cases of completely and genuinely entangled subspaces. These hierarchies work extremely well in practice even in very large quantum systems, as they can be implemented via elementary linear algebra techniques rather than the semidefinite programming techniques that are required by previously-known hierarchies.

Spectral gap for AKLT models on arbitrary decorated graphs

Angelo Lucia[1], *Amanda Young*[2]

[1] *Universidad Complutense de Madrid and ICMAT, Madrid, Spain*
E-mail: anglucia@ucm.es
[2] *Munich Center for Quantum Science and Technology and Zentrum Mathematik, TU München, Germany*
E-mail: young@ma.tum.de

We consider AKLT models on decorated versions of simple, connected graphs G, defined by replacing all edges of G with a chain of n sites, and show that they have a spectral gap if the n is larger than a linear function of the maximal vertex degree.

The AKLT model [Aff+87; Aff+88; KLT88] is a $SU(2)$-invariant quantum spin model which has played a major role in the development of connections between quantum information and quantum many body theory. A major open problem is to understand for which lattices the model has a spectral gap, a non-vanishing difference between the first excited energy level of the Hamiltonian and the ground state energy. While originally only the case for the 1D spin chain was shown to be gapped, it has been recently been shown that the model on the hexagonal lattice is gapped [LSW20; PW19]; on the other hand, the AKLT model on Cayley graphs with coordination number greater than 5 is known to be gapless [FNW92].

In this talk, I will consider the AKLT model defined on on the decorated version $G^{(n)}$ of a (potentially infinite) simple graph G with maximum degree $\Delta(G) < \infty$, which is defined by replacing each original edge in G by a 1D chain of length n (called the decoration number). I will then show that the AKLT model on $G^{(n)}$ is **gapped** if the decoration parameter n is at least $n(\Delta(G))$, where

$$n(d) = \begin{cases} d & d \leq 4 \\ \frac{\ln(2)}{\ln(3)} d + \frac{\ln(f(d))}{\ln(3)} & d > 4 \end{cases} \tag{1}$$

where $f(d)$ is the decreasing function

$$f(d) = 3 \cdot \frac{2 + (1 + \frac{1}{4^d})^{d-1}}{[4 - (1 + \frac{\sqrt{3}}{2^d})^{d-1}]^2}. \tag{2}$$

satisfying $1 \leq f(d) \leq f(5) \approx 1.17851$, i.e., our result shows that the AKLT model on $G^{(n)}$ has a positive uniform gap when n is greater than a linear function of $\Delta(G)$.

Acknowledgements:
A. L. was supported by MCIN/AEI/10.13039/501100011033 and by "ESF Investing in your future" (grant RYC2019-026475-I). A. Y. was supported by the DFG under EXC-2111–390814868.

References

[Aff+87] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki. "Rigorous results on valence-bond ground states in antiferromagnets". In: *Phys. Rev. Lett.* 59.7 (Aug. 1987), pp. 799–802.

[Aff+88] I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki. "Valence bond ground states in isotropic quantum antiferromagnets". In: *Comm. Math. Phys.* 115.3 (1988), pp. 477–528.

[KLT88] T. Kennedy, E.H. Lieb, and H. Tasaki. "A Two-Dimensional Isotropic Quantum Antiferromagnet with Unique Disordered Ground State". In: *J. Stat. Phys.* 53 (1988), pp. 383–415.

[LSW20] M. Lemm, A. W. Sandvik, and L. Wang. "Existence of a Spectral Gap in the Affleck-Kennedy-Lieb-Tasaki Model on the Hexagonal Lattice". In: *Phys. Rev. Lett.* 124.17 (Apr. 2020).

[PW19] N. Pomata and T.-C. Wei. "AKLT models on decorated square lattices are gapped". In: *Phys. Rev. B* 100 (2019), p. 094429.

[FNW92] M. Fannes, B. Nachtergaele, and R. F. Werner. "Ground states of VBS models on cayley trees". In: *J Stat Phys* 66.3-4 (Feb. 1992), pp. 939–973.

Diagonal Unitary and Orthogonal Symmetries in Quantum Theory

Ion Nechita[1], *Satvik Singh*[2]

[1] *CNRS, LPT Toulouse, France*
E-mail: ion.nechita@univ-tlse3.fr
[2] *Cambridge University, UK*
E-mail: ss2821@cam.ac.uk

Abstract

We study bipartite operators which stay invariant under the local action of the diagonal unitary and orthogonal groups. We investigate structural properties of these operators, arguing that the diagonal symmetry makes them suitable for analytical study, and that they are a rich source of (couter-)examples in the theory of quantum information. We focus on positive semi-definite operators, and relate their separability to completely positive matrices and some generalizations of this notion.

Thermalization in quantum spin systems

Angelo Lucia[1], *David Pérez-García*[2], *Antonio Pérez-Hernández*[3]

[1] *Departamento de Análisis y Matemática Aplicada, Universidad Complutense de Madrid, Spain*
and Instituto de Ciencias Matemáticas (ICMAT), Spain
E-mail: anglucia@ucm.es
[2] *Departamento de Análisis y Matemática Aplicada, Universidad Complutense de Madrid, Spain*
and Instituto de Ciencias Matemáticas (ICMAT), Spain
E-mail: dperezga@ucm.es
[3] *Departamento de Matemática Aplicada I, Universidad Nacional de Educación a Distancia, Spain*
E-mail: antperez@ind.uned.es

Abstract

A relevant open problem in the area of quantum computation is the existence problem of quantum self-correcting memories at finite temperature, that is, quantum systems that keep quantum information protected against thermal errors without the need of active error correction. The general conjecture is that there are no self-correcting quantum memories in 2D. However, despite compelling evidence and aside from some particular cases, a formal proof has remained elusive. In order to formally solve the problem, one has to consider the thermal evolution operator modeled by the Davies master equation. Self-correction would not be possible if the noise operator relaxes fast to the Gibbs ensemble, where all information is lost. The key quantity that controls this relaxation time is the spectral gap of the Davies Lindbladian generator, so that having a gap independent of the system size excludes self-correction. Our main result [3] shows that the conjecture holds for one of the most prominent families of topologically ordered systems and original candidates: *Kitaev's Quantum Double Models* [4], complementing previous works by Alicki et al. [1] for the toric code, and by Kómár et al. [2] for the Abelian case.

Acknowledgements: Work (partially) supported by Ministerio de Ciencia e Innovación (grants MTM2017-88385-P, PID2020-113523GB-I00, SEV-2015-0554 and CEX2019-000904-S); Comunidad de Madrid (QUITEMAD-CM, ref. S2018/TCS-4342); European Research Council (ERC) under the EU's Horizon 2020 programme (grant agreement No 648913); Walter Burke Institute for Theoretical Physics (Sherman Fairchild Fellowship); Spanish "Ramón y Cajal" Programme (grant RYC2019-026475-I); "Juan de la Cierva Formación" Programme (FJC2018-036519-I).

References

[1] R. Alicki, M. Fannes, M. Horodecki. *On thermalization in Kitaev's 2D models*. Journal of Physics A: Mathematical and Theoretical, 42 (6), 065303 (2009).

[2] A. Kómár, O. Landon-Cardinal, K. Temme (2007). *Necessity of an energy barrier for self-correction of Abelian quantum doubles*. Physiscal Review A, 93(5), 052337 (2016).

[3] A. Lucia, D. Pérez-García, A. Pérez-Hernández. *Thermalization in Kitaev's quantum double models via Tensor Network techniques* https://arxiv.org/abs/2107.01628

[4] A.Y. Kitaev *Fault-tolerant quantum computation by anyons*, Annals Phys. 303 2–30 (2003).

Monogamy of entanglement between cones and DPS-like hierarchies

Guillaume Aubrun[1], *Alexander Müller-Hermes*[2], *Martin Plávala*[3]

[1] *Institut Camille Jordan, Université Claude Bernard Lyon 1, 43 boulevard du 11 novembre 1918, 69622 Villeurbanne cedex, France*
E-mail: aubrun@math.univ-lyon1.fr
[2] *Department of Mathematics, University of Oslo, P.O. box 1053, Blindern, 0316 Oslo, Norway*
E-mail: muellerh@math.uio.no
[3] *Naturwissenschaftlich-Technische Fakultät, Universität Siegen, 57068 Siegen, Germany*
E-mail: martin.plavala@uni-siegen.de

Abstract A separable quantum state shared between parties A and B can be symmetrically extended to a quantum state shared between party A and parties $B_1, \ldots, B_k$ for every $k \in \mathbb{N}$. This phenomenon is known as "monogamy of entanglement"; quantum states that are not separable, i.e., entangled, do not have this property. We show that monogamy is not only a feature of quantum theory, but that it characterizes the minimal tensor product of general pairs of convex cones C_A and C_B: The elements of the minimal tensor product $C_A \otimes_{\min} C_B$, which is the cone generated by all elements of the form $x_A \otimes x_B$, $x_A \in C_A$, $x_B \in C_B$, are precisely the tensors that can be symmetrically extended to elements in the maximal tensor product $C_A \otimes_{\max} C_B^{\otimes_{\max} k}$ for every $k \in \mathbb{N}$, where $C_A \otimes_{\max} C_B = (C_A^* \otimes_{\min} C_B^*)^*$ and C_A^*, C_B^* are the dual cones. We also present applications of the result, such as to semi-device-independent characterization of high-dimensional steering.

Acknowledgements: Work (partially) supported by by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation, project numbers 447948357 and 440958198), the Sino-German Center for Research Promotion (Project M-0294), the ERC (Consolidator Grant 683107/TempoQ), the German Ministry of Education and Research (Project QuKuK, BMBF Grant No. 16KIS1618K) and the Alexander von Humboldt Foundation.

Positive maps and entanglement in real Hilbert spaces

Mizanur Rahaman

Laboratoire de l'Informatique du Parallélisme, École Normale Supérieure de Lyon (ENS de Lyon), France
E-mail: mizanur.rahaman@ens-lyon.fr

Abstract

Even though quantum theory uses complex Hilbert spaces and they play a key "tidying" role in the theory, it is only fairly recently that physicists have started to ask if the quantum world is inherently complex. Very recently a Bell-like experiment based on a network scenario is proposed that numerically separates complex from real quantum theory. In brief, it has now been shown that the real-world is not!

In this talk we discuss the similarities/differences between the real and complex case for various concepts like, entanglement and separability, positive versus completely positive maps and the various characterizations of entanglement breaking maps, thereby pointing out a number of fundamental differences in these two scenarios and discussing their implications in quantum information. We also discuss the real version of the *PPT-squared conjecture*.

When are quantum states antidistinguishable?

Nathaniel Johnston[1], *Vincent Russo*[2], *Jamie Sikora*[3]

[1] *Mount Allison University*
[2] *PsiQuantum*
[3] *Virginia Tech*

Abstract

We examine the problem of characterizing which pure state we are holding if promised to be given one from a finite set of (known) quantum states. It is well-known that the states must be pair-wise orthogonal to learn which state we are holding with certainty. In this talk we examine the antidistinguishing problem where you want to guess a state you are *not* holding. We present almost optimal bounds characterizing when a set of pure states can be perfectly antidistinguished.

Quantum concentration inequalities

Daniel Stilck França[1]

[1] *Univ Lyon, ENS Lyon, UCBL, CNRS, Inria, LIP, F-69342, Lyon Cedex 07, France*
E-mail: dsfranca@protonmail.com

Abstract

In this talk I will discuss recent advances on undertanding concentration properties of quantum states. We will present an elementary proof that outputs of shallow circuits satisfy a Gaussian concentration inequality and extend it to other classes of physically relevant states. We will discuss additional functional inequalities for such states, including a Poincaré inequality for outputs of shallow circuits. We will then discuss the application of such inequalities to study the limitations of shallow quantum circuits to solve certain optimization problems and a new algorithm to simulate quantum circuits.

This is based on joint work with Cambyse Rouzé.

Abstract Cone Systems

Gemma De les Coves[1], *Mirte van der Eyden*[2], *Tim Netzer*[3]

[1] *University of Innsbruck, Institute for Theoretical Physics, Austria*
E-mail: gemma.delescoves@uibk.ac.at
[2] *University of Innsbruck, Institute for Theoretical Physics, Austria*
E-mail: mirte.van-der-eyden@uibk.ac.at
[3] *University of Innsbruck, Institute for Mathematics, Austria*
E-mail: tim.netzer@uibk.ac.at

Abstract

An Abstract Operator Systems (AOS) is a collection of a proper convex cones inside the vector space of square matrices of dimension s tensored with a vector field V, for all $s \in \mathbb{N}$ [1]. There are many examples of interesting AOS, in particular when V is also a matrix space and we fix the cone of positive semidefinite matrices on the first level. Well-studied examples are the operator system of separable matrices (arising from the *minimal* tensor product), positive simidefinite matrices, and block-positive matrices (from the *maximal* tensor product) [2]. In the higher levels, the interaction between the positive cone and the tensor product becomes particularly insightful.

We propose a generalized version of AOS, called the Astract Cone System (ACS). We no longer restrict to matrix spaces (and their positive semidefinite cones), but instead look at tensor products of V with arbitrary finitedimensional vector spaces with involution. We reprove the most important theorems about AOS in the general case: the existence of minimal and maximal ACS, the existence of a finite dimensional realization as a concrete cone system, and that free dual of a finitely realizable ACS is finitely generated. This generalization not only sheds a new light on AOS and allows us to define them in a coordinate free way, it also allows to rephrase recent studies [3, 4] on entanglement between convex cones as ACS, offering a new toolbox and perspective.

Acknowledgements: MVDE acknowledges support of the Stand Alone Project P33122-N of the Austrian Science Fund (FWF)

References

[1] T. Fritz, T. Netzer, A. Thom *Spectrahedral containment and operator systems with finite-dimensional realization*, SIAM Journal on Applied Algebra and Geometry 1, no. 1, 2017

[2] M. Berger, T. Netzer *Abstract operator systems over the cone of positive semidefinite matrices*, arXiv:2109.14453, 2021

[3] G. Aubrun, A. Müller-Hermes, M. Plávala *Monogamy of entanglement between cones*, arXiv:2206.11805, 2022

[4] G. Aubrun, A. Müller-Hermes *Annihilating entanglement between cones*, arXiv:2110.11825, 2021

MSC14. Advances in cospectrality

Carolyn Reinhart, Kate Lorenzen

The Degree-Distance and Transmission-Adjacency Matrices
Carlos Alfaro, Octavio Zapata

Cospectral graphs by edge deletion
Chris Godsil, Wanting Sun, Xiaohong Zhang

Phantom mates of strongly cospectral vertices
Krystal Guo, Hanmeng Zhan, Ada Chan

Coalescing sets for a cospectral construction
Joel Jeffries, Steve Butler, Elana D'Avanzo, Rachel Heikkinen, Alyssa Kruczek, Harper Niergarth

The Degree-Distance and Transmission-Adjacency Matrices

Carlos A. Alfaro[1], *Octavio Zapata*[2]

[1] *Banco de México, Mexico City, Mexico*
E-mail: alfaromontufar@gmail.com
[2] *Departamento de Matemáticas, Facultad de Ciencias, Universidad Nacional Autónoma de México, Mexico City, Mexico*
E-mail: octavioz@ciencias.unam.mx

Abstract

Let G be a connected graph with adjacency matrix $A(G)$ and distance matrix $D(G)$. Let $\mathrm{dist}(u,v)$ denote the distance between the pair of vertices $u,v \in V(G)$, then the transmission $\mathrm{trs}(u)$ of vertex u is defined as $\sum_{v\in V(G)} \mathrm{dist}(u,v)$. Let $\mathrm{trs}(G)$ be the diagonal matrix whose diagonal elements are the transmissions of the vertices of G. And, let $\deg(G)$ be the diagonal matrix whose diagonal elements are the degrees of the vertices of G. In this paper we investigate the Smith normal form (SNF) and the spectrum of the matrices $D_+^{\deg}(G) := \deg(G) + D(G)$, $D^{\deg}(G) := \deg(G) - D(G)$, $A_+^{\mathrm{trs}}(G) := \mathrm{trs}(G) + A(G)$ and $A^{\mathrm{trs}}(G) := \mathrm{trs}(G) - A(G)$. In particular, we explore how good the SNF and the spectrum of these matrices are for determining graphs up to isomorphism. We found that the SNF of A^{trs} has an interesting behaviour when compared with other classical matrices. We note that the SNF of A^{trs} can be used to compute the structure of the sandpile group of certain graphs. We compute the SNF of $D_+^{\deg}$, $D^{\deg}$, A_+^{trs} and A^{trs} for several graph families. We prove that the SNF of $D_+^{\deg}$, $D^{\deg}$, A_+^{trs} and A^{trs} determine complete graphs. Finally, we derive some results about the spectrum of $D^{\deg}$ and A^{trs}.

Acknowledgements: The research of C.A. Alfaro is partially supported by the Sistema Nacional de Investigadores grant number 220797. The research of O. Zapata is partially supported by the Sistema Nacional de Investigadores grant number 620178.

References

[1] C. A. Alfaro and O. Zapata. *The degree-distance and transmission-adjacency matrices*. Preprint arXiv:2212.05297 [math.CO].

Cospectral graphs by edge deletion

Chris Godsil[1], *Wanting Sun*[2], *Xiaohong Zhang*[3]

[1] *C&O, University of Waterloo, Canada*
E-mail: cgodsil@uwaterloo.ca
[2] *Mathematics and Statistics, Central China Normal University, China*
E-mail: wtsun2018@sina.com
[3] *C&O, University of Waterloo, Canada*
E-mail: xiaohong.zhang@uwaterloo.ca

Abstract

If X is a 1-walk regular graph (e.g. any distance-regular graph) we show that by deleting edges inside a clique of X we obtain families of graphs that are not necessarily isomorphic, but are cospectral with respect to four types of matrices: the adjacency matrix, Laplacian matrix, unsigned Laplacian matrix, and normalized Laplacian matrix.

This has lead us to consider a stronger version of cospectrality. Let the degree matrix D of a graph X be the diagonal matrix with $i-th diagonal entry equal to the valency of the i-th vertex of X. We say that graphs X_1 and X_2 are degree similar if there is an invertible matrix L such that $L^{-1}A_1L = A_2$ and $L^{-1}D_1L = D_2$. If X_1 and X_2 are degree similar then X_1 and X_2 are cospectral with respect to all four types of matrices mentioned above. We have shown that the converse is not true, and have constructed families of degree-similar graphs.

Acknowledgements: Work supported by NSERC

References

[1] Chris Godsil, Wanting Sun. Xiaohong Zhang. Cospectral graphs obtained by edge deletion. *arXiv:2302.03854*

Phantom mates of strongly cospectral vertices

A. Chan[1], *Krystal Guo*[2], *H. Zhan*[3]

[1] *Department of Mathematics, York University, Canada*
E-mail: ssachan@yorku.ca @
[2] *Korteweg-de Vries Institute for Mathematics, University of Amsterdam, The Netherlands*
E-mail: k.guo@uva.nl
[3] *Department of Mathematics, Simon Fraser University, Canada*
E-mail: hanmeng_zhan@sfu.ca

Abstract

Two vertices, u and v, of a graph X are *cospectral* if the subgraphs $X \setminus u$ and $X \setminus v$ are cospectral. A stronger version of this property arises in the study of continuous-time quantum walks; two vertices, u and v are *strongly cospectral* if the orthogonal projections of the elementary basis vectors e_u and e_v are either equal up to sign, over each eigenspace of $A(X)$. We relax this condition and also allow a vector v to be strongly cospectral to a vertex u if e_u and v are either equal up to sign, over each eigenspace of $A(X)$. In some restricted conditions, we call such a vector v a phantom mate of vertex u. The property of being strongly cospectral is an equivalence relation and partitions the vertices of the graph into equivalence classes. We show that these equivalence classes are the orbits of some 2-group acting on the graphs and use the existence of phantom mates to investigate classes containing exactly three strongly cospectral vertices.

Coalescing sets for a cospectral construction

Steve Butler[1], *Elena D'Avanzo*[2], *Rachel Heikkinen*[3], *Joel Jeffries*[1], *Alyssa Kruczek*[4], *Harper Niergarth*[5]

[1] *Iowa State University, Ames, IA 50011, USA*
E-mail: butler,joeljef@iastate.edu
[2] *Carleton College, Northfield, MN 55057, USA*
E-mail: davanzoe@carleton.edu
[3] *Augustana College, Rock Island, IL, 61201, USA*
E-mail: rachelheikkinen19@augustana.edu
[4] *Susquehanna University, Selinsgrove, PA 17870, USA*
E-mail: a.kruczek0413@gmail.com
[5] *University of Minnesota–Twin Cities, Minneapolis, MN 55455, USA*
E-mail: nierg001@umn.edu

Abstract

We consider matrices of the form $qD + A$, with D being the diagonal matrix of degrees, A being the adjacency matrix, and q a fixed value. Given a graph H and $B \subseteq V(G)$, which we call a coalescent pair (H, B), we derive a formula for the characteristic polynomial where a copy of same rooted graph G is attached by the root to *each* vertex of B. Moreover, we establish if (H_1, B_1) and (H_2, B_2) are two coalescent pairs which are cospectral for any possible rooted graph G, then $(H_1, V(H_1) \setminus B_1)$ and $(H_2, V(H_2) \setminus B_2)$ will also always be cospectral for any possible rooted graph G.

Acknowledgements: This research was conducted primarily at the 2022 Iowa State University Math REU which was supported through NSF Grant DMS-1950583.

MSC15. Connection between rational function/ polynomial approximation and structured matrices for solving differential equations

Olivier Sète, Niel Van Buggenhout

Rational approximations of BEM systems for the 3D scalar Helmholtz equation
Simon Dirckx, Karl Meerbergen, Daan Huybrechs

A *-product solver for linear nonautonomous fractional differential equations
Fabio Durastante, Stefano Pozza

Rational approximation with minimal sampling for Helmholtz-like problems
Davide Pradovera, Fabio Nobile

Rational Krylov for Stieltjes matrix functions with Kronecker structure
Leonardo Robol, Stefano Massei

Sketched and truncated polynomial Krylov methods for matrix equations
Marcel Schweitzer, Davide Palitta, Valeria Simoncini

Polynomial preconditioning with Faber polynomials for the Helmhotz equation
Olivier Sète, Luis García Ramos, Reinhard Nabben

Quantum Krylov Methods: What?s the Deal?
Roel Van Beeumen, Daan Camps, Siva Darbha, Katherine Klymko, Yizhi Shen

A new Legendre polynomial approach for computing the matrix exponential
Shazma Zahid, Stefano Pozza

Rational approximations of BEM systems for the 3D scalar Helmholtz equation

S. Dirckx [1], *D. Huybrechs* [2], *K. Meerbergen*[3]

[1] *Dpt. Comp. Sci., KULeuven, Belgium*
E-mail: simon.dirckx@kuleuven.be
[2] *Dpt. Comp. Sci., KULeuven, Belgium*
E-mail: daan.huybrechs@kuleuven.be
[3] *Dpt. Comp. Sci., KULeuven, Belgium*
E-mail: karl.meerbergen@kuleuven.be

Abstract

Galerkin discretization of Boundary integral operators for the scalar Helmholtz equation results in large dense matrices. Owing to the separability of the Green's kernel, these matrices can however be well-approximated using data-compact *hierarchical matrices*. Recently, in [1] we demonstrated that rational approximation in combination with frequency extraction leads to a data-compact tensorial representation of the entire wave-number dependence of these hierarchical BEM matrices. We showed that this construction can be done in almost linear time and that the data requirements of the individual BEM matrices scale only weakly with the wave number.
In this talk we extend our data-compact representation to include discontinuous Galerkin discretization and set-valued rational functions. We introduce a sweeping preconditioner arising almost 'gratis' from the same compact representation and demonstrate the sweeping capabilities of our approach.

Acknowledgements: Work (partially) supported by KU Leuven IF project C14/15/055 and FWO Research Foundation Flanders G0B7818N

References

[1] S. Dirckx, D. Huybrechs, K. Meerbergen. Frequency extraction for BEM-matrices arising from the 3D scalar Helmholtz equation. *SISC*, 44(5): B1282–B1311 (2022)

A ⋆-product solver for linear nonautonomous fractional differential equations

Fabio Durastante[1], *Stefano Pozza*[2]

[1] *Mathematics Department, University of Pisa, Italy*
E-mail: fabio.durastante@unipi.it
[2] *Department of Numerical Mathematics, Charles University, Czech Republic*
E-mail: pozza@karlin.mff.cuni.cz

Abstract

In this talk we consider the solution of the Fractional Ordinary Differential Equation of the form

$$y^{(\alpha)}(t) = \widetilde{f}(t, y(t)), \quad y(0) = y_0, \quad t \in I = [t_0, t_1] \subseteq \mathbb{R}, \quad 0 < \alpha \leq 1,$$

where $y^{(\alpha)}(t)$ is the αth order Caputo derivative of y, i.e.,

$$y^{(\alpha)}(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} (t-\tau)^{-\alpha} y'(\tau)\, \mathrm{d}\tau.$$

For this task we propose extending the newly introduced ⋆-product approach, originally developed for the solution ordinary differential equation case in [1, 2], to treat this new class of problems. We will consider the differences and the additional difficulties encountered in the construction of the matrices associated with the new method; specifically regarding the stable and accurate calculation of their coefficients. We will show some preliminary experiments to demonstrate that this approach is promising and comparable with some of the widely used methods in the literature.

Acknowledgements: This work was partially supported by Charles University Research programs No. PRIMUS/21/SCI/009 and UNCE/SCI/023, and by the Magica project ANR-20-CE29-0007 funded by the French National Research Agency.

References

[1] P.-L. Giscard and S. Pozza, Tridiagonalization of systems of coupled linear differential equations with variable coefficients by a Lanczos-like method, Linear Algebra Appl. **624** (2021), 153–173.

[2] P.-L. Giscard and S. Pozza, Lanczos-like algorithm for the time-ordered exponential: the ∗-inverse problem, Appl. Math. **65** (2020), no. 6, 807–827.

Rational approximation with minimal sampling for Helmholtz-like problems

Davide Pradovera[1], *Fabio Nobile*[2]

[1] *Department of Mathematics, University of Vienna, Austria*
E-mail: davide.pradovera@univie.ac.at
[2] *CSQI, EPF Lausanne, Switzerland*
E-mail: fabio.nobile@epfl.ch

Abstract

Numerical methods for time-harmonic wave propagation phenomena are often computationally intensive, leading to high simulation costs, e.g., in outer-loop applications like design optimization and uncertainty quantification. In this framework, model order reduction methods can be used to obtain cheap and reliable approximations of the expensive high-fidelity problem.

On one hand, intrusive algorithms like implicit moment matching and reduced bases can be applied to obtain a reduced model by (quasi-)optimal projection of the original problem onto a low-dimensional subspace. On the other hand, non-intrusive methods like vector fitting and the Loewner framework build a rational approximation of the solution without the need to access the high-fidelity problem. However, this usually comes at the cost of "oversampling", i.e., solving the expensive high-fidelity problem more times than is necessary with, e.g., intrusive methods.

In this talk, we describe the "minimal rational interpolation" method [1], which combines some advantages of intrusive and non-intrusive methods. We showcase a strategy for adaptive sampling [2, 3], which mimics the weak-greedy reduced basis method without the need for intrusiveness. In this context, we discuss how minimal rational interpolation is able to achieve accuracy and efficiency by leveraging the properties of a certain Gramian matrix, built from high-fidelity data.

References

[1] D. Pradovera, Interpolatory rational model order reduction of parametric problems lacking uniform inf-sup stability. *SIAM Journal on Numerical Analysis* 58 (2020).

[2] D. Pradovera, F. Nobile, Frequency-domain non-intrusive greedy model order reduction based on minimal rational approximation. *Scientific Computing in Electrical Engineering*, 36 (2021).

[3] D. Pradovera, Toward a certified greedy Loewner framework with minimal sampling. Submitted (2023).

Rational Krylov for Stieltjes matrix functions with Kronecker structure

Stefano Massei[1], *Leonardo Robol*[1]

[1] *Department of Mathematics, University of Pisa, Italy*
E-mails: stefano.massei@unipi.it, leonardo.robol@unipi.it

Abstract

Evaluating the action of a matrix function on a vector, that is $x = f(\mathcal{M})v$, is an ubiquitous task in applications. When $\mathcal{M}$ is large, one usually relies on Krylov projection methods [1]. We provide effective choices for the poles of the rational Krylov method for approximating x when $f(z)$ is either Cauchy-Stieltjes or Laplace-Stieltjes (or, which is equivalent, completely monotonic) and $\mathcal{M}$ is a positive definite matrix. A relevant example of such functions is the inverse fractional power $x^{-\alpha}$ for $0 < \alpha < 1$, which appears in the solution of fractional diffusion equations.

Relying on the same tools used to analyze the generic situation, we then focus on the case $\mathcal{M} = I \otimes A - B^T \otimes I$, and v obtained vectorizing a low-rank matrix; this finds application in solving fractional diffusion equation on two-dimensional tensor grids. We see how to leverage tensorized Krylov subspaces to exploit the Kronecker structure and we introduce an error analysis for the numerical approximation of x. Pole selection strategies with explicit convergence bounds are given also in this case.

References

[1] S. Güttel. Rational Krylov approximation of matrix functions: Numerical methods and optimal pole selection. *GAMM-Mitteilungen 36.1 (2013): 8-31.*

[2] S. Massei, L. Robol. Rational Krylov for Stieltjes matrix functions: convergence and pole selection *BIT, 61.1 (2021): 237-273.*

Sketched and truncated polynomial Krylov methods for matrix equations

Davide Palitta[1], *Marcel Schweitzer*[2], *Valeria Simoncini*[1]

[1] *Dipartimento di Matematica, AM*[2], *Alma Mater Studiorum - Università di Bologna, 40126 Bologna, Italy*
E-mail: davide.palitta,valeria.simoncini}@unibo.it
[2] *School of Mathematics and Natural Sciences, Bergische Universität Wuppertal, 42097 Wuppertal, Germany*
E-mail: arcel@uni-wuppertal.de

Abstract

Recently, randomized Krylov subspace methods based on the *sketch-and-solve* paradigm have been successfully employed in the efficient solution of linear systems [3, 4], eigenvalue problems [4] and for approximating the action of matrix functions on vectors [1, 2]. In this setting, combining a basic Krylov method with a suitably constructed (randomized) oblivious subspace embedding allows to greatly reduce the cost of orthonormalization (and thus, in a HPC setting, the amount of communication).

In this talk, we will discuss how to extend these techniques to the matrix equation setting, with particular focus on the Sylvester equation

$$AX + XB = C,$$

which requires nontrivial modifications of the established algorithms, e.g., in order to prevent excessive memory requirements for storing the approximate solution. The efficiency of our proposed Krylov schemes in comparison to other state-of-the art solvers is demonstrated with several numerical examples originating in the discretization of partial differential equations and in control.

In addition to showing experimental results, we also present theoretical connections between sketched Krylov methods, Krylov methods with truncated orthonormalization and Krylov methods working with non-standard (possibly semidefinite) inner products. These allow to better understand and theoretically justify the observed performance and stability properties of sketched Krylov methods, a topic that is still far from being fully understood.

If time permits, we will also discuss implications of our theoretical findings for the case of linear systems and matrix functions.

[2] S. Güttel, M. Schweitzer. *Randomized sketching for Krylov approximations of large-scale matrix functions*, arXiv preprint arXiv:2208.11447 (2022).

[3] O. Balabanov, L. Grigori. *Randomized Gram–Schmidt process with application to GMRES*, SIAM J. Sci. Comput., 44(3), A1450–A1474 (2022).

[4] Y. Nakatsukasa, J. A. Tropp. *Fast & accurate randomized algorithms for linear systems and eigenvalue problems*, arXiv preprint arXiv:2111.00113 (2021)

Polynomial preconditioning with Faber polynomials for the Helmhotz equation

Olivier Sète[1]

[1] *Institute of Mathematics and Computer Science, University of Greifswald, Germany*
E-mail: olivier.sete@uni-greifswald.de

Abstract

We consider polynomial preconditioning of a linear algebraic system $Ax = b$, i.e., $Ap(A)y = b$ and $p(A)y = x$, where the polynomial p is constructed as follows. Let $\Omega \subseteq \mathbb{C}$ be an inclusion set for the spectrum of A, where Ω is compact with $0 \notin \Omega$, simply connected, and possibly non-convex. Then p is chosen as a truncated Faber series of $1/z$ on Ω.

This polynomial preconditioning is applied to the Helmholtz equation preconditioned with the complex shifted Laplacian. In this case, we use a non-convex 'bratwurst'-shaped inclusion set to construct the polynomial p as described above. In numerical experiments, this polynomial preconditioner leads to a reduction of the number of GMRES iterations, and may also reduce the computation time.

This talk is based on joint work [1] with Luis García Ramos and Reinhard Nabben (TU Berlin).

References

[1] Luis García Ramos, Olivier Sète, and Reinhard Nabben. Preconditioning the Helmholtz equation with the shifted Laplacian and Faber polynomials. *Electronic Transactions on Numerical Analysis*, 54: 534–557 (2021). https://doi.org/10.1553/etna_vol54s534

Quantum Krylov Methods: What's the Deal?

Roel Van Beeumen[1], *Daan Camps*[2], *Siva Darbha*[1,2]
Katherine Klymko[2], *Yizhi Shen*[1]

[1] *Applied Mathematics and Computational Research Division, Lawrence Berkeley National Laboratory, United States*
[2] *National Energy Research Super Computing Center (NERSC), Lawrence Berkeley National Laboratory, United States*
E-mail: [rvanbeeumen,dcamps,sdarbha,kklymko,yizhis]@lbl.gov

Abstract

Classical methods based on Krylov subspaces are among the most successful numerical linear algebra algorithms. More recently, quantum subspace methods have become a promising class of hybrid quantum algorithms for computing approximate eigenvalues in condensed matter physics and electronic structure theory. In these quantum Krylov methods we prepare the states that form the subspace basis on the quantum computer and the projected problem is retrieved through quantum measurement. Next, we obtain the Ritz values through classically solving a generalized eigenvalue problem that is often ill-conditioned. In this talk, we present an overview of quantum subspace algorithms from an NLA perspective. We discuss why we believe they are promising, give some details on how they can be implemented on quantum computers, and provide numerical and theoretical evidence of their convergence. We also present strategies to improve the robustness of the convergence in the presence of noise.

Acknowledgements: Work supported by the U.S. Department of Energy (DOE) under Contract No. DE-AC02-05CH11231, through the Office of Advanced Scientific Computing Research (ASCR).

A new Legendre polynomial approach for computing the matrix exponential

Stefano Pozza[1], *Shazma Zahid*[2]

[1] *Faculty of Mathematics and Physics, Charles University, Prague, 186 75, Czech Republic*
E-mail: pozza@karlin.mff.cuni.cz
[2] *Faculty of Mathematics and Physics, Charles University, Prague, 186 75, Czech Republic*
E-mail: shazma.zahid@matfyz.cuni.cz

Abstract

The solution of a linear autonomous ODE system can be expressed by a matrix exponential. An alternative method for solving such ODEs is based on a product that generalizes the convolution, and it is known as the $\star$-product. A new approach [1], based on the Legendre discretization of the $\star$-product has been recently proposed. In this presentation, we propose a new method for computing the matrix exponential based on this approach. It enables us to find the matrix exponential efficiently. Our approach allows the ODEs solution to be expressed through a linear system that can be alternatively formulated as a matrix equation. We use a Krylov subspace model reduction approach to solve the matrix equation and so compute the matrix exponential. We provide numerical results to demonstrate the effectiveness and efficiency of the new method.

Acknowledgements: Work supported by Charles University Research program PRIMUS/21/SCI/009 and by the Magica project ANR-20-CE29-0007 funded by the French National Research Agency.

References

[1] S. Pozza, N. Van Buggenhout, *A new matrix equation expression for the solution of non-autonomous linear systems of ODEs*, https://arxiv.org/abs/2210.07052, 2022.

MSC16. Orthogonal polynomials, matrix analysis and applications

Amílcar Branquinho, Ana Foulquié,
Manuel Mañas, Francisco Marcellán

*I*nverse Darboux transformations and Sobolev inner products
María-José Cantero, Fernando Marcellán, Leandro Moral, Luis Velázquez

Time-and-band limiting for exceptional orthogonal polynomials
Mirta María Castro Smirnova, F. Alberto Grünbaum

Discrete Darboux Transformations Leading to Nonstandard Orthogonality
Maxim Derevyagin

Eigenvalues of infinite Hermitian matrices and Sobolev orthogonal polynomials
Carmen Escribano, Raquel Gonzalo

Spectral theory for bounded banded matrices with positive bidiagonal factorization and mixed multiple orthogonal polynomials
Ana Foulquie, Amílcar Branquinho, Manuel Mañas

Berstein-Szeg/h o measures in the plane
Jeffrey Geronimo

A matrix approach to bounded point evaluation and zeros of Sobolev orthogonal polynomials
Raquel Gonzalo, Carmen Escribano

The Christoffel function: Some applications and connections
Jean Bernard Laserre

Linear systems of moment differential equations
Alberto Lastra

Jacobi matrices on binary trees: multilevel interpolations and boundedness
Vladimir Lysov

Lax-type pairs in the theory of bivariate orthogonal polynomials
Teresa E. Pérez, Amílcar Branquinho, Ana Foulquié-Moreno, Miguel Piñar

A generalisation of the Hermite-Biehler theorem
Mikhail Tyaglov, Rostyslav Kozhan

On generating Sobolev orthogonal polynomials
Niel Van Buggenhout

A matrix approach to the linearization and connection coefficients of orthogonal polynomial sequences
Luis Verde-Star

Inverse Darboux transformations and Sobolev inner products

M.J. Cantero[1], F. Marcellán[2], L. Moral[1] L. Velázquez[1]

[1] Dpt. Applied Mathematics, University of Zaragoza/ EINA and IUMA, Spain
E-mail: mjcante@unizar.es lmoral@unizar.es velazque@unizar.es
[2] Dpt. Mathematics 2, University Carlos III, Madrid/ Spain
E-mail: franciscojose.marcellan@uc3m.es

Abstract

Darboux transformation can be considered as the result of factoring a self-adjoint operator as a product of two others, whose permutation gives a new self-adjoint operator. This transformation, initially presented as a powerful tool in the field of integrable systems, also has applications in various areas of mathematics and physics.

When this transformation is applied to Jacobi matrices, canonical representation of self-adjoint operators, Darboux transformation is equivalent to a Christoffel modification of the corresponding orthogonality measure, which multiplies it by a polynomial of degree one. The inverse Darboux transformation corresponds to the so-called Geronimus transformation, which divides the measure by a polynomial of degree one and adds a possible mass point at the zero of the polynomial.

ln the unitary case, Darboux transformation of the corresponding canonical representatives, the CMV matrices, requires a prior modification of these unitary matrices by a Laurent polynomial, in order to transform them into self-adjoint ones.The Darboux transformation generated in this way is also equivalent to a Christoffel modification of the measure. In the corresponding inverse problem some drawbacks arise. In fact, the inverse Darboux transformation leads to spurious solutions which are neither unitary nor band matrices.

In this talk we will expose the Darboux transformation for CMV matrices highlighting these drawbacks, such as the presence of spurious solutions. We show how the spurious solutions are associated to certain Sobolev inner products.

The first part of this work was obtained in collaboration with Professor Francisco Marcellán.

Acknowledgements: Work (partially) supported by FEDER/Ministerio de Ciencia e Innovación-Agencia Estatal de Investigación of Spain, grant PID2021-122154NB-I00, and the Madrid Government (Comunidad de Madrid-Spain) under the Multiannual Agreement with UC3M in the line of Excellence of University Professors, grant EPUC3M23 in the context of the V PRICIT (Regional

Program of Research and Technological Innovation and by FEDER/Ministerio de Ciencia e Innovación-Agencia Estatal de Investigación of Spain, grant PID2021-124472NB-I00 and ERDF, 'Una manera de hacer Europa,' the project UAL18-FQM-B025-A (UAL/CECEU/FEDER) and the projects E26-17 and E48-20R from Diputación General de Aragón (Spain).

References

[1] M.J. Cantero, F.Marcellán, L. Moral, L. Velázquez, *Darboux transformations for orthogonal polynomials on the real line and on the unit circle*. Orthogonal polynomials: current trends and applications, 53-75 SEMA SIMAI Springer Ser., 22, 2021.

[2] M.J. Cantero, F.Marcellán, L. Moral, L. Velázquez, Darboux transformations for CMV matrices.*Advances in Mathematics.* 298:122-206 (2016)

[3] M.J. Cantero, L. Moral, L. Velázquez, Formas sesquilineales y productos tipo Sobolev discretos, *Preprint.*

Time-and-band limiting for exceptional orthogonal polynomials

Mirta M. Castro Smirnova[1]

[1] *Departamento de Matemática Aplicada II, Universidad de Sevilla, Spain*
E-mail: mirta@us.es

Abstract

In this talk we consider examples of exceptional orthogonal polynomials in connection with the problem of time-and band-limiting.

For a given family of orthogonal polynomials one considers the global operator defined by a full symmetric matrix whose entries are given by the truncated inner products or an operator defined by an integral kernel. The problem is to search for a local operator given by a narrow banded matrix or a differential operator (respectively), with simple spectrum, commuting with the global one. The existence of a commuting local operator is very useful to compute numerically the eigenfunctions of the given global operator.

This question is motivated by the work of Claude Shannon and a series of papers by D. Slepian, H. Landau and H. Pollak at Bell Labs in the 1960's.

This is a joint work with F. Alberto Grünbaum, University of California, Berkeley.

References

[1] M. Castro and F.A. Grünbaum, A new property of exceptional orthogonal polynomials, arXiv:2210.13928.

Discrete Darboux Transformations Leading to Nonstandard Orthogonality

Maxim Derevyagin

University of Connecticut, USA
E-mail: derevyagin.m@gmail.com

Abstract In this talk, we are going to quickly review the basics of discrete Darboux transformations for orthogonal polynomials. Then we will show how such transformations lead to various orthogonalities such as Sobolev and exceptional along with how they generate rational orthogonal functions.

Acknowledgements: Work partially supported by NSF DMS grant 2008844

References

[1] M. Derevyagin, J.C. García-Ardila, F. Marcellaán, Multiple Geronimus transformations. *Linear Algebra Appl.* 454: 158–183 (2014).

[2] R. Bailey, M. Derevyagin, Complex Jacobi matrices generated by Darboux transformations, arXiv:2107.09824

[3] R. Bailey, M. Derevyagin, DEK-Type exceptional orthogonal polynomials and a modification of the Christoffel formula, arXiv:2110.03038

Eigenvalues of infinite Hermitian matrices and Sobolev orthogonal polynomials

Carmen Escribano[1], *Raquel Gonzalo*[2],

[1] *Departamento de Matemática Aplicada a las Tecnologías de la Información y las Comunicaciones, Escuela Técnica Superior de Ingenieros Informáticos/Center for Computational Simulation, Universidad Politécnica de Madrid, Spain.*
E-mail: cescribano@fi.upm.es
[2] *Departamento de Matemática Aplicada a las Tecnologías de la Información y las Comunicaciones, Escuela Técnica Superior de Ingenieros Informáticos*
E-mail: rngonzalo@fi.upm.es

Abstract

Matrix algebra tools like generalized eigenvalues of Hermitian positive definite matrices have been used in [1] and [2] to obtain information on approximation of polynomials in spaces of square integrable functions $L^2(\mu)$ and also on the support of μ, for a certain measure μ supported on the complex plane.
Our aim in this work is to study Sobolev orthogonal polynomials with the approach of matrix analysis via the associated moment matrix. In particular we study the behaviour of eigenvalues of Hermitian positive definite matrices associated with inner Sobolev products with respect a set of measures in relation with the problem of the location of zeros of Sobolev orthogonal polynomials providing results in this context. As a consequence of our results we extend the characterization for the boundedness of the multiplication operator, and consequently the boundedness of the set of zeros of Sobolev polynomials given in [3].

Acknowledgements: Work partially supported by grant MadQuantum-CM supported by MCIN with funding from European Union NextGenerationEU (PRTR-C17.I1) and funding from the Comunidad de Madrid.

References

[1] C. Escribano, R. Gonzalo, E. Torrano. Small eigenvalues of large Hermitian moment matrices. *J. Math. Anal. Appl.*, 374: 470–480 (2011).

[2] C. Escribano, R. Gonzalo, E. Torrano. Smallest and largest generalized eigenvalues of large moment matrices and some applications. *J. Math. Anal. Appl.*, 521 (2023).

[3] J.M. Rodríguez. A simple characterization of weighted Sobolev spaces with bounded multiplication operator. *J. Approx. Theory*, 153(1):53–72 (2008).

Spectral theory for bounded banded matrices with positive bidiagonal factorization and mixed multiple orthogonal polynomials

Amílcar Branquinho[1], *Ana Foulquié-Moreno*[2], *Manuel Mañas*[3]

[1] *Departmento de Matemática, Universidade de Coimbra, Portugal*
E-mail: ajplb@mat.uc.pt
[2] *Departamento de Matemática,Universidade de Aveiro, Portugal*
E-mail: foulquie@ua.pt
[3] *Departamento de Física Teórica,Universidad Complutense de Madrid, España*
E-mail: manuel.manas@ucm.es

Abstract

Spectral and factorization properties of oscillatory matrices leads to a spectral Favard theorem for bounded banded matrices, that admit a positive bidiagonal factorization, in terms of sequences of mixed multiple orthogonal polynomials with respect to a set positive Lebesgue-Stieltjes measures. Also, a mixed multiple Gauss quadrature is proven and corresponding degrees of precision are found.

Acknowledgements: Work supported by CMUC, Centro de Matemática da Universidade de Coimbra and by FCT, UID/MAT/00324/2020, also by CIDMA, Center for Research and Development in Mathematics and Applications da Universidade de Aveiro and by FCT, UIDB/MAT/UID/04106/2020, and by Agencia Estatal de Investigación Española, under projects PGC2018-096504-B-C33, Ortogonalidad y Aproximación: Teoría y Aplicaciones en Física Matemática, and PID2021-122154NB-I00, Ortogonalidad y Aproximación con Aplicaciones en Machine Learning y Teoría de la Probabilidad

References

[1] *Spectral theory for bounded banded matrices with positive bidiagonal factorization and mixed multiple orthogonal polynomials*, arXiv:2212.10235

BERNSTEIN-SZEGŐ MEASURES IN THE PLANE

JEFFREY GERONIMO

Abstract. We consider a class of Bernstein-Szegő measures on R^2 which is a natural extension of the one-dimensional class of Bernstein-Szegő measures. The spectral properties of these measures will be discussed and conditions involving finitely many moments which completely characterize this class will be given. This work was in done in collaboration with P. Iliev.

A matrix approach to bounded point evaluation and zeros of Sobolev orthogonal polynomials

Carmen Escribano [1,2], *Raquel Gonzalo*[1],

[1] *Departamento de Matemática Aplicada a las Tecnologías de la Información y las Comunicaciones, Escuela Técnica Superior de Ingenieros Informáticos, Universidad Politécnica de Madrid, Spain.*
E-mail: cescribano@fi.upm.es; rngonzalo@fi.upm.es; emilio@fi.upm.es
[2] *Center for Computational Simulation, Universidad Politécnica de Madrid,Spain.*

Abstract

Our aim in this work is to study the problem of the location of zeros of Sobolev polynomials associated to compactly supported measures in the complex plane with the approach of matrix analysis via the associated moment matrix as in [2]. In particular, we obtain some results concerning boundedness of the multiplication operator in Sobolev spaces, and consequently of the set of zeros, using the matrix approach to bounded point evaluation of a measure introduced in [1]. The notion of bounded point evaluation is closely related to polynomial approximation [3], [1]. We also introduce and study certain polynomial inequalities involving the derivates of the polynomials in order to obtain examples of sets of measures for which the zeros of the associated Sobolev inner products are uniformly bounded.

References

[1] C. Escribano, R. Gonzalo, Torrano. *A characterization of polynomial density on curves via matrix algebra* Mathematics , **7(12)** (2019) pp: 1-12

[2] C. Escribano, R. Gonzalo, Torrano. Smallest and largest eigenvalues of large moment matrice and some applications. *J. Math. Anal. Appl.*, 521(2): (2023).

[3] Thomnson, J.E.,*Appoximation in the mean by polynomials* Ann. Mat., **133**, (1991), pp:477-507

The Christoffel function: Some applications and connections

J.B. Lasserre[1]

[1] *LAAS-CNRS & Institute of Mathematics, University of Toulouse, France*
E-mail: lasserre@laas.fr

Abstract

We introduce the Christoffel function (CF), a well-known tool in theory of approximation and orthogonal polynomials. We will briefly describe how the CF turns out to also provide a quite easy-to-use tool to help solve interesting problems in data analysis [1, 2] (e.g., outlier detection, support inference, density approximation, classification). We will also discuss some of its links (in the author's opinion some surprising) with seemingly unrelated topics, like e.g. certificates of positivity in real algebraic geometry, equilibrium measure of compact sets, polynomial Pell's equation, and duality in polynomial optimization [3, 4].

Acknowledgements: Work partially supported by the french ANR agency under ANR-NuSCAP-20-CE48-0014, as well as by AI Interdisciplinary Institute ANITI funding, through the French "Investing for the Future PIA3" program under the Grant agreement n°ANR-19-PI3A-0004. This research is also part of the programme DesCartes and is supported by the National Research Foundation, Prime Minister's Office, Singapore under its Campus for Research Excellence and Technological Enterprise (CREATE) programme.

References

[1] J.B. Lasserre, E. Pauwels, M. Putinar. *The Christoffel-Darboux Kernel for Data Analysis*, Cambridge University Press, Cambridge, UK, 2022

[2] J.B. Lasserre. On the Christoffel function and classification in data analysis. *Comptes Rendus Mathématique*, 360: 919–928 (2022). Lasserre J.B. (2022)

[3] J.B. Lasserre. A disintegration of the Christoffel function. *Comptes Rendus Mathématique*, 360: 1071–1079 (2022).

[4] J.B. Lasserre. Pell's equation, sum-of-squares and equilibrium measures on a compact set. *Comptes Rendus Mathématique*, 361: (2023). To appear.

Linear systems of moment differential equations

Alberto Lastra[1]

[1] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: alberto.lastra@uah.es

Abstract

The general solution to a linear system of moment differential equations

$$\partial_m y = Ay$$

is constructed via kernel functions for generalized summability. A stands for a matrix with complex elements. The operator ∂_m is a generalization of the classical derivative which depends on a fixed sequence m of positive real numbers.

Several applications related to different sequences m will be considered in the talk.

Also, the asymptotic growth of the solutions at infinity is described in terms of the associated moment sequence in some cases and from different points of view.

Acknowledgements: W ork partially supported by project PID2019-105621GB-I00 of Ministerio de Ciencia e Innovación, Spain, by Dirección General de Investigación e Innovación, Consejería de Educación e Investigación of the Comunidad de Madrid (Spain) and Universidad de Alcalá, under grant CM/JIN/2021-014, Proyectos de I+D para Jóvenes Investigadores de la Universidad de Alcalá 2021, and the Ministerio de Ciencia e Innovación-Agencia Estatal de Investigación MCIN/AEI/10.13039/501100011033 and the European Union "NextGeneration EU"/ PRTR, under grant TED2021-129813A-I00.

References

[1] A. Lastra.Entire Solutions of Linear Systems of Moment Differential Equations and Related Asymptotic Growth at Infinity. *Differ. Equ. Dyn. Syst.* (2022).

Jacobi matrices on binary trees: multilevel interpolations and boundedness

Vladimir Lysov [1]

[1] *Keldysh Institute of Applied Mathematics, Russia*
E-mail: v.g.lysov@gmail.com

Abstract

We consider tridiagonal Jacobi matrices (or so-called discrete Schrödinger operators) on graphs. One of the methods to implement such operators on homogeneous trees is based on the Hermite–Padé interpolation problems for perfect systems, see [1]. The Jacobi matrices are determined by the nearest-neighbor recurrence coefficients for the multiple orthogonal polynomials. For one important class of perfect systems, the so-called Nikishin systems, it is known [2] that such a construction leads to unbounded operators. We pose another interpolation problem for the Nikishin systems. Its solutions also satisfy the nearest-neighbor recurrent relations, but the corresponding coefficients and the Jacobi matrix turn out to be bounded, see [3].

Acknowledgements: Work supported by Moscow Center of Fundamental and Applied Mathematics, Agreement no. 075-15-2022-283 with the Ministry of Science and Higher Education of the Russian Federation.

References

[1] A. I. Aptekarev, S. A. Denisov, M. L. Yattselev. Self-adjoint Jacobi matrices on trees and multiple orthogonal polynomials. *Trans. Amer. Math. Soc.*, 373(2): 875–917 (2020).

[2] S. A. Denisov, M. L. Yattselev. Spectral theory of Jacobi matrices on trees whose coefficients are generated by multiple orthogonality. *Adv. Math.*, 396(4): 108114 (2021).

[3] A. I. Aptekarev, V. G. Lysov. Multilevel interpolation for Nikishin systems and boundedness of Jacobi matrices on binary trees. *Russian Math. Surveys*, 76(4): 726–728 (2021).

Lax-type pairs in the theory of bivariate orthogonal polynomials

Teresa E. Pérez

IMAG & Dpto. de Matemática Aplicada, Universidad de Granada (Spain)
E-mail: tperez@ugr.es

Abstract

The so-called bivariate orthogonal polynomial systems, that is, sequences of bivariate polynomials written as vector polynomials of increasing size orthogonal with respect to a weight function, satisfy two three term relations with matrix coefficients. In this work, introducing a time-dependent parameter, we solve a Lax-type pair system for the coefficients of the three term relations. We also deduce several characterizations relating the Lax-type pair, the shape of the weight, Stieltjes function, moments, a differential equation for the weight, and the bidimensional Toda-type systems.

This is a join work with A. Branquinho, A. Foulquié-Moreno and M. A. Piñar.

Acknowledgements: Work partially supported by FEDER/Junta de Andalucía under grant A-FQM-246-UGR20, and IMAG-María de Maeztu grant CEX2020-001105-M.

References

[1] G. Ariznabarreta, M. Mañas, *Multivariate Toda hierarchies and biorthogonal polynomials*, Phys. D 434 (2022), Paper No. 133202, 17 pp.

[2] C. F. Bracciali, T. E. Pérez, *Bivariate orthogonal polynomials, 2D Toda lattices and Lax-type pairs*, Appl. Math. Comput. 309 (2017), 142-155.

[3] C. F. Dunkl, Y. Xu, *Orthogonal Polynomials of Several Variables*, 2nd edition, Encyclopedia of Mathematics and its Applications, vol. **155**, Cambridge Univ. Press, 2014.

A generalisation of the Hermite–Biehler theorem

Rostyslav Kozhan[1], *Mikhail Tyaglov*[2]

[1] *Department of Mathematics, Uppsala University , Sweden*
E-mail: kozhan@math.uu.se
[2] *School of mathematical sciences, Shanghai Jiao Tong University, P.R. China*
E-mail: tyaglov@sjtu.edu.cn

Abstract

The classical Hermite–Biehler theorem describes possible zero sets of complex linear combinations of two real polynomials whose zeros strictly interlace. We provide the full characterization of zero sets for the case when this interlacing is broken at exactly one location.

Using this we solve the direct and inverse spectral problem for rank-one multiplicative and two-rank additive perturbations of finite Jacobi matrices.

Acknowledgements: The work of M. Tyaglov was partially supported by National Natural Science Foundation of China under grant no. 11871336.

On generating Sobolev orthogonal polynomials

Niel Van Buggenhout

Department of Numerical Mathematics, Charles University, Czech Republic
E-mail: buggenhout@karlin.mff.cuni.cz

Abstract

Polynomials orthogonal with respect to an inner product on the real line satisfy a three term recurrence relation. The recurrence coefficients can be represented by a tridiagonal matrix. For generating nonclassical orthogonal polynomials the method of choice is based on performing unitary similarity transformations on a diagonal matrix, whose entries are the nodes of a Gaussian quadrature rule for the inner product. The connection between this method and orthogonal polynomials can be described using Krylov subspaces. Consider a Krylov subspace generated by the diagonal matrix of the nodes and a starting vector containing the weights of the quadrature rule. Then the tridiagonal matrix is the orthogonal projection of the diagonal matrix onto this subspace. In this presentation we will explore the case where the Krylov subspace is generated by a Jordan matrix. The projection of this Jordan matrix onto the Krylov subspace results in a Hessenberg matrix that contains the recurrence coefficients for polynomials orthogonal with respect to a Sobolev inner product. This is an inner product that involves the polynomials themselves and the derivates of these polynomials. Based on this connection we propose two new algebraic methods for the numerical generation of Sobolev orthogonal polynomials and compare them to existing methods.

Acknowledgements: This work was supported by Charles University Research program No. PRIMUS/21/SCI/009.

A matrix approach to the linearization and connection coefficients of orthogonal polynomial sequences

Luis Verde-Star

Departamento de Matemáticas
Universidad Autónoma Metropolitana, Unidad Iztapalapa, Ciudad de México.
e-mail: verde@xanum.uam.mx

The three-term recurrence relation satisfied by a sequence of orthogonal polynomials

$$p_n(x) = \sum_{k=0}^{n} c_{n,k} x^k$$

can be expressed as the matrix equation $LC = CX$, where L is the infinite tridiagonal Jacobi matrix whose entries are the recurrence coefficients, $C = [c_{n,k}]$, and X is the right shift matrix that represents the operator of multiplication by x. Since C is invertible we have $C^{-1}LC = X$ and therefore $C^{-1}u(L)C = u(X)$ for every polynomial $u(x)$. We use this matrix equation to obtain simple algebraic formulas for the linearization coefficients $d(n, m, k)$ that satisfy

$$p_n(x)p_m(x) = \sum_{k=0}^{n+m} d(n, m, k)p_k(x),$$

and also for the corresponding coefficients for the linearization formula where the sequence p_k in the right-hand side is replaced by another sequence $v_k(x)$ of orthogonal polynomials. We also obtain the connection coefficients for the pair of sequences $\{p_n\}$ and $\{v_n\}$ and recurrence relations for the linearization coefficients. The linearization and connection coefficients are expressed in terms of the coefficients of the three-term recurrence relations. The proofs are algebraic and do not use the orthogonality of the polynomial sequences.

MSC17. Pattern restricted inverse eigenvalue problems

Jephian C.-H. Lin, Polona Oblak, Helena Smigoc

Orthogonal Realizations of Random Sign Patterns
Bryan Curtis, Zachary Brennan, Chris Cox, Enrique Gomez-Leos, Kimberly Hadaway, Leslie Hogben, Conor Leslie, Thompson Conor

Minimum rank bounds for cobipartite graphs and zero-nonzero patterns
Louis Deaett, Derek Young

The difficulty of minimum rank 3
H. Tracy Hall, Kevin Grace, Alathea Kevin, Rachel Lawrence

The number of distinct eigenvalues of joins of graphs
Mark Kempton, Aida Abiad, Shaun Fallat, Rupert Levene, Polona Oblak, Helena Smigoc, Michael Tait, Kevin Vander Meulen

Generic realisability and applications
Rupert Levene, Polona Oblak, Helena Šmigoc

Similarity via transversal intersection of manifolds
Zhongshan Li

The bifurcation lemma for strong properties in the inverse eigenvalue problem of a graph
Jephian C.-H. Lin, Shaun M. Fallat, Tracy Hall, Bryan L. Shader

Zq-Forcing Game for Some Families of Graphs

Shahla Nasserasr, Shaun Fallat, Neha Joshi, Roghayeh Maleki, Karen Meagher, Seyed Ahmad Mojallal, Mahsa N. Shirazi, Andriaherimanana Sarobidy Razafimahatratra, Brett Stevens

The liberation set of a graph
Polona Oblak, Polona Oblak

Changes in vertex status after removal of another vertex
Carlos Saiago

On the number of distinct eigenvalues allowed by a sign pattern
Kevin Vander Meulen, Jane Breen, Carraugh Brouwer, Minerva Catral, Michael Cavers, Pauline van den Driessche

Orthogonal Realizations of Random Sign Patterns

Z. Brennan[2], *C. Cox*[2], *Bryan A. Curtis*[1], *E. Gomez-Leos*[2], *K. Hadaway*[2], *L. Hogben*[2], *C. Thompson*[2]

[1] *Department of Mathematics, Iowa State University, USA*
E-mail: bcurtis1@iastate.edu
[2] *Department of Mathematics, Iowa State University, USA*

Abstract

A *sign pattern* is an array with entries in $\{+, -, 0\}$. A matrix Q is *row orthogonal* if $QQ^T = I$. The Strong Inner Product Property (SIPP), introduced in [B.A. Curtis and B.L. Shader, Sign patterns of orthogonal matrices and the strong inner product property, *Linear Algebra Appl.* 592: 228–259, 2020], is an important tool when determining whether a sign pattern allows row orthogonality because it guarantees there is a nearby matrix with the same property, allowing zero entries to be perturbed to nonzero entries, while preserving the sign of every nonzero entry. This paper uses the SIPP to initiate the study of conditions under which random sign patterns allow row orthogonality with high probability.

Acknowledgements: Work partially supported by NSF grant 18399.

References

[1] N. Alon and J.H. Spencer. The probabilistic method. John Wiley & Sons, 2016.

[2] R.F. Bailey and R. Craigen. On orthogonal matrices with zero diagonal. *Electron. J. Linear Algebra*, 35:307–318, 2019.

[3] J. Bourgain, V.H. Vu and P.M. Wood. On the singularity probability of discrete random matrices, *J.Functional Analysis* 258 (2010), no. 2, 559–603. MR2557947

[4] H. Cohn, A. Kumar and G. Minton. Optimal simplices and codes in projective spaces. *Geometry & Topology*, 20:1289–1357, 2016.

[5] B.A. Curtis. Sign Patterns of Row Orthogonal Matrices. PhD thesis, 2020.

[6] B.A. Curtis and B.L. Shader. Sign patterns of orthogonal matrices and the strong inner product property. *Linear Algebra and its Applications*, 592: 228–259, 2020.

[7] M. Fiedler, "Problem 12," in *Proceedings: Theory of Graphs and Its Application.* Prague: Publishing House of the Czechoslovakia Academy of Sciences, 1964, p. 160.

[8] L. Hogben, J.C.-H. Lin, B.L. Shader. *Inverse Problems and Zero Forcing for Graphs.* American Mathematical Society (Mathematical Surveys and Monographs, 260), Providence, RI, 2022.

[9] H. van der Holst, L. Lovász, and A. Schrijver. The Colin de Verdière graph parameter. In *Graph Theory and Combinatorial Biology* (L. Lovász, A. Gyárfás, G. Katona, A. Recski, and L. Székely, editors), János Bolyai Mathematical Society, Budapest, 1999, pp. 29–85.

[10] C.R. Johnson and C. Waters. Sign patterns occurring in orthogonal matrices. *Linear and Multilinear Algebra*, 44: 287–299, 1998.

[11] H. Tijms. *Probability: A lively introduction.* Cambridge University Press, 2018.

[12] K. Tikhomirov. Singularity of random Bernoulli matrices. *Ann. of Math.* (2) 191 (2) 593 - 634, 2018.

[13] D. Williams. *Probability with martingales.* Cambridge University Press, 1991.

Minimum rank bounds for cobipartite graphs and zero-nonzero patterns

Louis Deaett[1], *Derek Young*[2]

[1] *Department of Mathematics and Statistics, Quinnipiac University, USA*
E-mail: louis.deaett@quinnipiac.edu
[2] *Department of Mathematics and Statistics, Mount Holyoke College, USA*
E-mail: dyoung@mtholyoke.edu

Abstract

A *zero-nonzero pattern matrix* is a combinatorial object specifying exactly where the zero and nonzero entries occur in some matrix. What can such a description tell us about the rank of the matrix?

A related problem begins with a graph G on n vertices, and seeks the smallest rank of a symmetric $n \times n$ matrix whose nonzero entries off the diagonal occur exactly according to the edges of G.

Informally, we may refer to the former problem as the "minimum rank problem for zero-nonzero matrix patterns" and the latter problem as the "minimum rank problem for simple graphs" (which is equivalent to the maximum eigenvalue multiplicity problem for simple graphs). Foundational results on the former problem were given in [1], which also established connections between the two problems. In particular, the minimum rank of a matrix that is cobipartite (i.e., its complement is bipartite) was shown to be given by the minimum rank of the zero-nonzero pattern describing the pattern of edges between the two cocliques into which the vertex set is partitioned.

Our present work is focused on connecting combinatorial bounds in the setting of one problem to corresponding bounds in the other, specifically in the case of cobipartite graphs. For these graphs, we may consider various zero forcing parameters that are used to bound the minimum rank, while for the corresponding zero-nonzero patterns we have the lower bound of the triangle number. We present some results that relate these bounds, and explore how this can shed light on the question of which patterns and graphs actually achieve equality between these bounds and the minimum rank itself.

References

[1] F. Barioli, S. M. Fallat, H. T. Hall, D. Hershkowitz, L. Hogben, H. van der Holst, B. Shader. On the minimum rank of not necessarily symmetric matrices: A preliminary study. *Electron. J. Linear Algebra.*, 18: 126–145 (2009).

The difficulty of minimum rank 3

Kevin Grace[1], *H. Tracy Hall*[2], *Alathea Jensen*[3], *Rachel Lawrence*[4]

[1] *Vanderbilt University* *email:* kevin.m.grace@vanderbilt.edu
[2] *Hall Labs, LLC* *email:* h.tracy@gmail.com
[3] *Susquehanna University* *email:* alatheajensen@gmail.com
[4] *University of California Berkeley* *email:* rslawrence95@gmail.com

Abstract

The problem of minimum rank for a graph G is an easier subcase of the Inverse Eigenvalue Problem for G. It asks for the smallest possible rank of a real symmetric matrix whose pattern of off-diagonal nonzero entries is exactly specified by G. The minimum rank of any induced subgraph gives a lower bound, and to rule out rank 0, rank 1, or rank 2, a finite number of obstructions suffices. We show that for rank 3 no such finite family exists, and furthermore that given any system S of polynomial equations in multiple unknowns with integer coefficients, a graph G_S can be produced in polynomial time such that G_S has minimum rank 3 if and only if the equations in S can be solved simultaneously over the real numbers. As a result, the problem of minimum rank 3 is complete for the complexity class $\exists\mathbb{R}$, the existential theory of the reals.

Acknowledgements: This project was a part of the American Mathematical Society's 2021 Mathematics Research Community *Finding Needles in Haystacks: Approaches to Inverse Problems using Combinatorics and Linear Algebra*

The number of distinct eigenvalues of joins of graphs

Aida Abiad[1], *Shaun M. Fallat*[2], *Mark Kempton*[3], *Rupert H. Levene*[4], *Polona Oblak*[5], *Helena Šmigoc*[6], *Michael Tait*[7], *Kevin Vander Meulen*[8]

[1] *Department of Mathematics and Computer Science, Eindhoven University of Technology, The Netherlands*
E-mail: a.abiad.monge@tue.nl
[2] *Department of Mathematics and Statistics, University of Regina, Canada*
E-mail: shaun.fallat@uregina.ca
[3] *Department of Mathematics, Brigham Young University, United States*
E-mail: mkempton@mathematics.byu.edu
[4] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: rupert.levene@ucd.ie
[5] *Faculty of Computer and Information Science, University of Ljubljana, Slovenia*
E-mail: polona.oblak@fri.uni-lj.si
[6] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: helena.smigoc@ucd.ie
[7] *Department of Mathematics and Statistics, Villanova University, United States*
E-mail: michael.tait@villanova.edu
[8] *Department of Mathematics, Redeemer University, Canada*
E-mail: kvanderm@redeemer.ca

Abstract

The inverse eigenvalue problem for graphs asks what spectra can be obtained by a matrix whose zero/nonzero pattern is specified by a given graph. While very challenging in general, an important subquestion is to determine, for a graph G, the parameter $q(G)$, which is the minimum number of *distinct* eigenvalues that can be achieved by a matrix whose zero/nonzero pattern is given by G. We will discuss results relating to how bordering a matrix with new rows and columns can affect the number of distinct eigenvalues. We will further discuss how these techniques give results relating to determining $q(G)$ when G is the join of two graphs. This work was done in conjuction with the AIM research group on $q(G)$ for joins.

Generic realisability and applications

Rupert Levene[1], *Polona Oblak*[2], *Helena Šmigoc*[3]

[1] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: rupert.levene@ucd.ie
[2] *Faculty of Computer and Information Science, University of Ljubljana, Slovenia*
E-mail: polona.oblak@fri.uni-lj.si
[3] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: helena.smigoc@ucd.ie

Abstract

We introduce a notion of "generic realisability" for a graph, and explain why complete graphs and paths have this property. Applications are given to the problem of computing the minimum number of distinct eigenvalues of a graph.

References

[1] R. H. Levene, P. Oblak, H. Šmigoc. Paths are generically realisable. arXiv preprint 2103.04587 (2021).

[2] R. H. Levene, P. Oblak, H. Šmigoc. Orthogonal symmetric matrices and joins of graphs. *Linear Algebra and its Applications*, 652:213–238 (2022).

Similarity via transversal intersection of manifolds

Marina Arav[1], *Frank J. Hall* [1], *Hein van der Holst*, *Zhongshan Li*[1], *Aram Mathivanan* [1], *Jiamin Pan* [1], *Hanfei Xu* [1], *and Zheng Yang* [1]

[1] *Department of Mathematics and Statistics, Georgia State University, USA*
Presenter E-mail: zli@gsu.edu

Abstract

Let A be an $n \times n$ real matrix. As shown in the recent paper "The bifurcation lemma for strong properties in the inverse eigenvalue problem of a graph", Linear Algebra Appl. 648 (2022), 70–87, by S.M. Fallat, H.T. Hall, J.C.-H. Lin, and B.L. Shader, if the manifolds $\mathcal{M}_A = \{G^{-1}AG | G \in \mathrm{GL}(n, \mathbb{R})\}$ and $Q(\mathrm{sgn}(A))$ (consisting of all real matrices having the same sign pattern as A), both considered as embedded submanifolds of $\mathbb{R}^{n \times n}$, intersect transversally at A, then every superpattern of $\mathrm{sgn}(A)$ also allows a matrix similar to A. Let $X = [x_{ij}]$ be a generic matrix of order n whose entries are independent variables. In this paper, this similarity-transversality property is characterized in a direct and convenient way by the full row rank property of the Jacobian matrix of the entries of $AX - XA$ at the zero entry positions of A with respect to the nondiagonal entries of X. This new approach makes it possible to take better advantage of the combinatorial structure of the matrix A, and provides theoretical foundation for constructing matrices similar to a given matrix while the entries have certain desired signs. In particular, several important classes of zero-nonzero patterns and sign patterns that require or allow this transversality property are identified. Examples illustrating many possible applications (such as diagonalizability, number of distinct eigenvalues, nilpotence, idempotence, semi-stability, eigenvalues and their algebraic and geometric multiplicities, Jordan canonical form, minimal polynomial, and rank) are provided.

The bifurcation lemma for strong properties in the inverse eigenvalue problem of a graph

Shaun M. Fallat[1], *H. Tracy Hall*[2], *Jephian C.-H. Lin*[3], *Bryan L. Shader*[4]

[1] *Department of Mathematics and Statistics, University of Regina, Canada*
E-mail: shaun.fallat@uregina.ca
[2] *Hall Labs, LLC, USA*
E-mail: h.tracy@gmail.com
[3] *Department of Applied Mathematics, National Sun Yat-sen University, Taiwan*
E-mail: jephianlin@gmail.com
[3] *Department of Mathematics, University of Wyoming, USA*
E-mail: bshader@uwyo.edu

Abstract

Let G be a simple graph on n vertices. Let $\mathcal{S}(G)$ be the set of $n \times n$ real symmetric matrices whose i,j-entry, $i \neq j$, is nonzero whenever $\{i,j\}$ is an edge of G; the diagonal entries can be any real numbers. Suppose $A \in \mathcal{S}(G)$ has the strong spectral property (SSP) and H is a supergraph of G with $V(G) = V(H)$. It is known that there is a matrix $A' \in \mathcal{S}(H)$ such that $\text{spec}(A') = \text{spec}(A)$. This means the pattern of A can be perturbed while preserving the spectrum. In contrast, we will introduce the bifurcation lemma — if a matrix A has the SSP, then for any small perturbation Λ of $\text{spec}(A)$, there is a matrix $A' \in \mathcal{S}(G)$ with $\text{spec}(A') = \Lambda$. Many applications of the bifurcation lemma to the inverse eigenvalue problem and the sign pattern problem will be demonstrated.

Z_q-Forcing Game for Some Families of Graphs

Shaun Fallat[1], *Neha Joshi*[2], *Roghayeh Maleki*[3], *Karen Meagher*[4], *Seyed Ahmad Mojallal*[5], *Shahla Nasserasr* [6], *Mahsa N. Shirazi*[7], *Andriaherimanana Sarobidy Razafimahatratra*[8], *Brett Stevens*[9]

[1,2,4,5] *Department of Mathematics and Statistics, University of Regina, Canada*
E-mail: shaun.fallat@uregina.ca, njp008@uregina.ca, Karen.Meagher@uregina.ca, ahmad mojalal@yahoo.com
[3,8] *University of Primorska, Glagoljaška, Slovenia*
E-mail: r.maleki910@gmail.com, sarobidy@phystech.edu
[6] *School of Mathematical Sciences, Rochester Institute of Technology, USA*
E-mail: shahla@mail.rit.edu
[7] *Department of Mathematics, University of Manitoba, Canada*
E-mail: mahsa.nasrollahi@gmail.com
[9] *School of the Mathematics and Statistics, Carleton University, Canada*
E-mail: brett@math.carleton.ca

Abstract

Zero forcing game starts with an initial set of blue vertices of a graph, and the goal is to color all vertices blue under some color change rule at minimum cost. The original zero forcing game is a one-player game that gives a combinatorial upper bound for the maximum nullity of a family of matrices associated with the graph. A modified zero forcing game requires two players and gives a combinatorial upper bound, $Z_q(G)$, for the maximum nullity of the matrices with q negative eigenvalues. In this talk, we present a formula for Z_q of connected threshold graphs as well as some other families of graphs.

The liberation set of a graph

Jephian C.-H. Lin[1], *Polona Oblak*[2], *Helena Šmigoc*[3]

[1] *Department of Applied Mathematics, National Sun Yat-sen University, Kaohsiung 80424, Taiwan*
E-mail: jephianlin@gmail.com
[2] *Faculty of Computer and Information Science, University of Ljubljana, Večna pot 113, SI-1000 Ljubljana, Slovenia*
E-mail: polona.oblak@fri.uni-lj.si
[3] *School of Mathematics and Statistics, University College Dublin, Belfield, Dublin 4, Ireland*
E-mail: helena.smigoc@ucd.ie

Abstract

The Inverse Eigenvalue Problem (IEP-G) for a graph G is a problem of determining all possible multisets of eigenvalues of symmetric matrices, whose pattern is constrained by graph G. This question is in general very difficult to answer. The introduction of the strong spectral properties (SSP) in [2] has made a powerful impact on the IEP-G and related problems. The matrices with the SSP can be perturbed while preserving the eigenvalues and still controlling the pattern. The motivation for our work comes from the Matrix Liberation Lemma introduced in [3], which extends the similar conclusions for matrices that do not have the SSP.

In this talk we introduce the liberation set of a matrix to present an equivalent result to the Matrix Liberation Lemma, which is easier and less technical to apply. We define the liberation set of a graph G and present the application to the disjoint unions of graphs. In particular, in [1] a huge advance towards resolving the IEP-G for graphs on six vertices was made, with only spectral arbitrariness of few multiplicity lists left to be resolved, and we will present, how our methods resolve some of those open cases.

Acknowledgements: Work partially supported by the Young Scholar Fellowship Program grant no. NSTC-110-2628-M-110-003 from the National Science and Technology Council of Taiwan (Jephian C.-H. Lin) and Slovenian Research Agency research core funding no. P1-0222 and project no. J1-3004 (Polona Oblak).

References

[1] John Ahn, Christine Alar, Beth Bjorkman, Steve Butler, Joshua Carlson, Audrey Goodnight, Haley Knox, Casandra Monroe, and Michael C. Wigal. *Ordered multiplicity inverse eigenvalue problem for graphs on six vertices.* Electron. J. Linear Algebra, 37:316–358, 2021.

[2] Wayne Barrett, Shaun Fallat, H. Tracy Hall, Leslie Hogben, Jephian C.-H. Lin, and Bryan L. Shader. *Generalizations of the strong Arnold property and the minimum number of distinct eigenvalues of a graph.* Electron. J. Combin., 24(2):Paper No. 2.40, 28, 2017.

[3] Wayne Barrett, Steve Butler, Shaun M. Fallat, H. Tracy Hall, Leslie Hogben, Jephian C.-H. Lin, Bryan L. Shader, and Michael Young. *The inverse eigenvalue problem of a graph: Multiplicities and minors.* J. Combin. Theory Ser. B, 142:276–306, 2020.

Changes in vertex status after removal of another vertex

Carlos Saiago[1]

[1] *Center for Mathematics and Applications (NOVA Math), FCT NOVA and Department of Mathematics, Universidade NOVA de Lisboa, Portugal*
E-mail: cls@fct.unl.pt

Abstract

We will discuss the change in vertex status -as *Parter*, *neutral* or *downer*- (relative to an undirected graph G, a matrix A whose graph is G and a designated eigenvalue λ of A) after removal of another vertex of given status. By the *status* of a vertex v of G, we refer to the change in multiplicity of λ when v is removed from G to leave the principal submatrix $A(v)$ of A. Along the way we present a special type of neutral vertices that play an important role on explaining the differences between the case of real symmetric matrices and the case of real combinatorially symmetric matrices.

Acknowledgements: This is a joint work with Charles Johnson (College of William and Mary, Williamsburg, USA) and Kenji Toyonaga (National Institute of Technology, Kitakyushu, Japan). This work is funded by national funds through the FCT - Fundação para a Ciência e a Tecnologia, I.P., under the scope of the projects UIDB/00297/2020 and UIDP/00297/2020 (Center for Mathematics and Applications).

References

[1] C.R. Johnson, C.M. Saiago, and K. Toyonaga. Change in vertex status after removal of another vertex in the general setting. *Linear Algebra Appl.* 612:128–145 (2021).

[2] C.R. Johnson, C.M. Saiago, and K. Toyonaga. Classification of vertices and edges with respect to the geometric multiplicity of an eigenvalue in a matrix, with a given graph, over a field. *Linear Multilinear Algebra* 66(11):2168–2182 (2018).

[3] C.R. Johnson, A. Leal-Duarte, C.M. Saiago. The change in eigenvalue multiplicity associated with perturbation of a diagonal entry. *Linear Multilinear Algebra* 60(5):525–532 (2012).

On the number of distinct eigenvalues allowed by a sign pattern

Jane Breen[1], *Carraugh Brouwer*[2], *Minerva Catral*[3], *Michael Cavers*[4], *Pauline van den Driessche*[5], *Kevin Vander Meulen*[6]

[1] *Faculty of Science, Ontario Tech University, Canada.* jane.breen@ontariotechu.ca
[2] *Department of Chemistry, University of British Columbia, Canada.* carraugh@chem.ubc.ca
[3] *Department of Mathematics, Xavier University, USA.* catralm@xavier.edu
[4] *Department of Computer and Mathematical Sciences, University of Toronto Scarborough, Canada.* michael.cavers@utoronto.ca
[5] *Department of Mathematics, Univeristy of Victoria, Canada.* vandendr@uvic.ca
[6] *Department of Mathematics, Redeemer University, Canada.* kvanderm@redeemer.ca

Abstract

We explore the problem of determining the number of distinct eigenvalues allowed by a matrix sign pattern. While this inverse eigenvalue problem is analogous to a problem of determining the number of distinct eigenvalue allowed by a graph, there are significant differences. The graph problem restricts its attention to symmetric matrices and has freedom on the main diagonal of the matrix. The sign pattern problem involves fixed signs and/or zeros on the main diagonal and does not restrict attention to symmetric matrices. As such, while the graphs that allow for exactly one eigenvalue is limited to empty graphs, the class of sign patterns that allow exactly one eigenvalue is nontrivial, including spectrally arbitrary patterns and the wider class of potentially nilpotent sign patterns. We describe some digraph characteristics that bound the number of eigenvalues allowed by a sign pattern, and observe some Jacobian conditions that preserve properties of eigenvalues for superpatterns.

Acknowledgements: Research supported in part by NSERC Discovery Grants RGPIN-2021-03775 (JB), RGPIN-2016-03677 (PvD), RGPIN-2016-03867 (KVM), an NSERC USRA (CB) and the American Institute of Mathematics.

References

[1] J. Breen, C. Brouwer, M. Catral, M. Cavers, P. van den Driessche, and K. N. Vander Meulen. Minimum number of distinct eigenvalues allowed by a sign pattern. *Linear Algebra Appl.* 654: 311–338 (2022).

[2] M. Cavers and S. Fallat. Allow problems concerning spectral properties of patterns. *Elect. J. Linear Alg.* 23: 731–754 (2012).

[3] Z. Li and L. Harris. Sign patterns that require all distinct eigenvalues. *JP J. Algebra Number Theory Appl.* 2: 161–179 (2002).

MSC18. Riordan arrays and related topics

Ana Luzón, Manuel A. Morón

Properties of Riordan quotients
Paul Barry

Natural partial orderings and associated Riordan poset matrices
Gi-Sang Cheon, Hong Joon Choi, Bryan Curtis, Kuk-Won Kwon

From Alexandroff spaces to Riordan matrices
Pedro J. Chocano, Ana Luzón, Manuel A. Morón, Manuel A., Luis Felipe Prieto-Martínez

Professor
Tian-Xiao He

Exponential Riordan matrices and decomposition of Hankel matrices
Emanuele Munarini

The binary Pascal matrix and associated algebras
Nikolaos Pantelidis, Gi-Sang Cheon, Hong Joon Choi

Abstract cell complexes and Riordan matrices
Luis Felipe Prieto-Martínez, Pedro J. Chocano, Ana Luzón, Manuel A. Morón

Combinatorial statistics on Catalan words avoiding consecutive patterns
Jose Ramirez, Jean-Luc Baril

Total positivity of Riordan arrays
Roksana Słowik

Riordan Group Involutions
Louis Shapiro, Alexander Burstein

Minho Song, Jihyeug Jang, Louis Shapiro
Combinatorics on the negative part of Riordan matrices

Some properties of polynomial sequences associated with generalized Ri- ordan matrices
Luis Verde-Star

Properties of Riordan quotients

Paul Barry

Department of Computing and Mathematics, South East Technological University, Ireland.
E-mail: pbarry@wit.ie

Abstract

We investigate the qoutient matrix of two (ordinary) Riordan arrays (g_1, f_1) and (g_2, f_2). This is defined to be the lower-triangular matrix whose generating function is equal to the quotient of the generating functions of the two defining matrices. We detail such features as the row sums and column sums of these matrices, along with a characterization of their products. We show that these quotients are closely associated with almost-Riordan arrays of first order. In particular, each almost-Riordan array is shown to be the quotient of a Riordan array and a (possibly stretched) Riordan array. The coefficient array of the Chebyshev polynomials of the first kind occurs as a special example.

References

Y. Alp and E. Gokcen Kocer, Sequence characterization of almost-Riordan arrays, *Linear Algebra Appl.*, **664** (2023), 1–23.

P. Barry, On the group of almost-Riordan arrays, https://arxiv.org/abs/1606.05077.

P. Barry and N. Pantelidis, On pseudo-involutions, involutions and quasi-involutions in the group of almost Riordan arrays, *J. Algebraic Comb.*, (2020), 10.1007/s10801-020-00993-w.

R. Słowik, More about involutions in the group of almost-Riordan arrays, *Linear Algebra Appl.*, **624** (2021), 247–258.

Natural partial orderings and associated Riordan poset matrices

Gi-Sang Cheon[1], *Hong Joon Choi*[2], *Bryan Curtis*[3], *Kuk-Won Kwon*[4]

[1,4] *Department of Mathematics, Sungkyunkwan University, Suwon 16419, S. Korea*
E-mail: gscheon@skku.edu, tomy1995@skku.edu
[2] *Applied Algebra and Optimization Research Center, Sungkyunkwan University, Suwon 16419, S. Korea E-mail:* h.j.choi@skku.edu
[3] *Department of Mathematics, Iowa State University, Ames, IA 50011, USA*
E-mail: bcurtis1@iastate.edu

Abstract

A partial ordering $\preceq$ on a finite or infinite set X of integers $\{0, 1, 2, \ldots\}$ is *natural* if $x \preceq y$ implies $x \leq y$. Every partial ordering of a finite (possibly infinite) set is isomorphic to a natural partial ordering. Moreover, all natural partial orderings on the n-set $[n]$ are in bijection with $n \times n$ binary lower triangular matrices with ones on the main diagonal that contain no $\left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$ submatrix whose upper right entry 1 is on the main diagonal. These lower (possibly infinite) triangular matrices are called *poset matrices*. A natural partial ordering associated to a poset matrix B is called *Riordan* if B is a binary Riordan matrix over the field $\mathbb{Z}_2 = \{0, 1\}$. In this talk, we discuss about possible $(0, 1)$-patterns to be infinite Riordan poset matrices. In addition, we investgate avoiding posets for natural partial ordering to be a Riordan poset.

Acknowledgements: This work was supported by Science Research Center Program through the National Research Foundation of Korea (NRF) Grant funded by the Korean Government (MSIP) (NRF- 2016R1A5A1008055).

References

[1] G. Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25, American Mathematical Society, Providence, RI (1940).

[2] G.-S. Cheon, B. Curtis, G. Kwon and A. Mwafise, Riordan posets and associated incidence matrices, *Linear Algebra and its Applications*, 632: 308–331 (2022).

[3] G.-S. Cheon, J.-H. Jung, S. Kitaev and S. Mojallal, Riordan graphs I: Structural properties, *Linear Algebra and its Applications*, 579: 89–135 (2019).

[4] R. A. Dean and Gordon Keller, Natural Partial Orders, *Canadian Journal of Mathematics*, 20: 535–554 (1968).

From Alexandroff spaces to Riordan matrices

P. J. Chocano[1], *A. Luzón* [2], *M. A. Morón*[3], [4] *L. F. Prieto-Martínez*

[1] *Departamento de Matemática Aplicada, Ciencia e Ingeniería de los Materiales y Tecnología Electrónica, Universidad Rey Juan Carlos, Spain*
E-mail: pedro.chocano@urjc.es
[2] *Departamento de Matemática Aplicada, Universidad Politécnica de Madrid, Spain*
E-mail: anamaria.luzon@upm.es
[3] *Departamento de Álgebra, Geometría y Topología, Universidad Complutense de Madrid, Spain*
E-mail: mamoron@mat.ucm.es
[4] *Departamento de Matemática Aplicada, Universidad Politécnica de Madrid, Spain*
E-mail: luisfelipe.prieto@upm.es

Abstract

In this talk, we recall basic concepts of the theory of Alexandroff spaces (or partially ordered sets) and see how these spaces are related to simplicial complexes. Later, we introduce the non-hausdorff join. This construction allows us to construct some Riordan matrices, using m,q-cones, from partially ordered sets. Thus, for each pair of natural numbers m and q there exists an Alexandroff space realizing the previous Riordan matrix. We study the homotopy type of these spaces and consider some invariants for them.

Acknowledgements: Work partially supported by Ministerio de Ciencia e Innovación (Spain), PID2021-126124NB-I00

Vertical Recurrence Relation of Riordan Arrays, Quasi-Riordan Group and its Subgroups and Extended Subgroups

Tian-Xiao He[1]

[1] *Department of Mathematics/Illinois Wesleyan University, USA*
E-mail:the@iwu.edu

Abstract

It is known that the entries of a Riordan array satisfy horizontal recurrence relations represented by the A- and Z-sequences. Recently, we studied a vertical recurrence relation approach to Riordan arrays. This vertical recurrence approach gives a way to represent the entries of a Riordan array (g, f) in terms of a recurrence linear combination of the coefficients of g. This vertical recurrence relation can be represented by matrices. The set of all those matrices forms a group, named the quasi-Riordan group. This talk gives the normal subgroup of the quasi-Riordan group and studies the relationship between the quasi-Riordan and the Riordan group. By means of the study, several extended subgroups of the quasi-Riordan group and the Riordan group are obtained. The corresponding horizontal recurrence relation and vertical recurrence relation are given. The A-sequences and the vertical recurrence relations of the Riordan subgroups and those of the corresponding extensions are also given. Finally, we will show that every Riordan array is the half Riordan array of a unique Riordan array, and the vertical recurrence relation of the column entries of the half Riordan array is equivalent to the horizontal recurrence relation of the original Riordan array's row entries.

Exponential Riordan matrices and decomposition of Hankel matrices

Emanuele Munarini[1]

[1] *Dipartimento di Matematica, Politecnico di Milano, Italy*
E-mail: emanuele.munarini@polimi.it

Abstract

In this talk, we will characterize the infinite matrices $A = [a_{i,j}]_{i,j\geq 0}$ admitting an LDU-decomposition RDS^T, where D is a diagonal matrix and R and S are exponential Riordan matrices [4]. Similarly, we characterize the infinite matrices $A = [a_{i,j}]_{i,j\geq 0}$ admitting an LTU-decomposition RTS^T, where T is a tridiagonal matrix and R and S are exponential Riordan matrices. These characterizations are given in terms of the double exponential generating series of the matrix A and the exponential generating series of the other involved matrices.

When the matrix A admits a decomposition of this kind, these characterizations provide a relatively simple algebraic method to obtain explicitly such a decomposition. In particular, this method works when we consider the Hankel matrices generated by certain classical Appell and Sheffer sequences [3].

We illustrate this method presenting in detail some of the results obtained [2] for the Hankel matrices generated by the Appell sequence given by the generalized rencontres polynomials [1]. Furthermore, we give a characterization of these polynomials in terms of the determinants of the associated partial Hankel matrices. Finally, as a byproduct, we give a similar characterization of the generalized derangement numbers.

References

[1] S. Capparelli, M. M. Ferrari, E. Munarini, N. Zagaglia Salvi, A Generalization of the "*Problème des Rencontres*", *J. Integer Seq.* **21**: Article 18.2.8 (2018).

[2] M. M. Ferrari, E. Munarini, Decomposition of some Hankel matrices generated by the generalized rencontres polynomials, *Linear Algebra Appl.* **567**: 180-201 (2019).

[3] S. Roman, *The umbral calculus*, Academic Press, New York, 1984.

[4] L. Shapiro, R. Sprugnoli, P. Barry, G.-S. Cheon, T.-X. He, D. Merlini, W. Wang, *The Riordan group and applications*, Springer, Cham, 2022.

The binary Pascal matrix and associated algebras

Gi-Sang Cheon [1], *Hong Joon Choi* [2], *Nikolaos Pantelidis* [3]

[1] *Applied Algebra and Optimization Research Center, Sungkyunkwan University, Republic of Korea*
E-mail: gscheon@skku.edu
[2] *Applied Algebra and Optimization Research Center, Sungkyunkwan University, Republic of Korea*
E-mail: h.j.choi@skku.edu
[3] *Department of Computing Science and Mathematics, Dundalk Institute of Technology, Ireland*
E-mail: Nikolaos.Pantelidis@dkit.ie

Abstract

The concept of Riordan posets has been recently introduced by Cheon et al. [1]. The study of these objects has given a new dimension on the general research of Riordan arrays and related topics. In this talk, we discuss the algebras that defined by an important type of Riordan posets, such as the Pascal poset.

Presenting the Pascal group $\mathcal{P}_n$ and the incidence algebra *I(P)*, we analyse their structural and combinatorial properties through the number of the generators and the subgroups of $\mathcal{P}_n$, and we link it with other known groups, such as the Heisenberg group. We show that $\mathcal{P}_n$ is in fact a matrix Lie group and we present the Pascal Lie algebra. Finally, we discuss open problems and possible directions of our research.

References

[1] Gi-Sang Cheon, Bryan A. Curtis, Gukwon Kwon, and Arnauld Mesinga Mwafise. Riordan posets and associated incidence matrices. *Linear Algebra and its Applications* 632: 308-331 (2022).

[2] Gi-Sang Cheon, Hong Joon Choi, Bryan A. Curtis and Gukwon Kwon. Notes on Riordan Poset Matrices. unpublished work.

Abstract cell complexes and Riordan matrices

P. J. Chocano[1], *A. Luzón*[2], *M. A. Morón*[3], *L. F. Prieto-Martínez*[4]

[1] *Departamento de Matemática Aplicada, Ciencia e Ingeniería de los Materiales y Tecnología Electrónica, Universidad Rey Juan Carlos, Spain*
E-mail: pedro.chocano@urjc.es
[2] *Departamento de Matemática Aplicada, Universidad Politécnica de Madrid, Spain*
E-mail: anamaria.luzon@upm.es
[3] *Departamento de Álgebra, Geometría y Topología, Universidad Complutense de Madrid, Spain*
E-mail: mamoron@mat.ucm.es
[4] *Departamento de Matemática Aplicada, Universidad Politécnica de Madrid, Spain*
E-mail: luisfelipe.prieto@upm.es

Abstract

Abstract (or geometric) cell complexes are objects of great interest in Mathematics, with special mention to Image Analysis, that generalize *simplicial complexes* and *cubical complexes*. There are several approaches to this concept (see [1]), but all of them follow a similar idea: *cell complexes* are obtained from elementary pieces, called *cells*, that usually are abstract (or geometric) polytopes.

Their generality makes much more difficult the study of their combinatorial properties. So we will focus in these cell complexes such that their cells of the same dimension are (combinatorially) isomorphic.

In this talk: (i) we show that, for some interesting families of cells, if we write the number of j-faces of each i-cell in a matrix $[m_{ij}]_{0\leq i,j<\infty}$, we obtain a Riordan matrix of the type $(\frac{1}{(1-ax)^n}, \frac{x}{1-ax})$, for $a, n \in \mathbb{Z}$ (the case $a = 1, n = 2$ corresponds to simplices and has already been studied in depth, as the reader may see, for example, in [2]) and (ii) we study some consequences of the previous fact, such as the existence of linear forms for the set of face vectors which are invariants with respect to some topological properties.

Acknowledgements: Work partially supported by Ministerio de Ciencia e Innovación (Spain), PID2021-126124NB-I00.

References

[1] R. Klette. Cell complexes through time. Vision Geometry IX. Vol. 4117, SPIE (2000).

[2] A. Luzón, M. A. Morón, L. F. Prieto-Martínez. Pascal's Triangle, Stirling's Numbers and the Euler Characteristic. Chapter of the book *A mathematical Tribute to Professor José María Montesinos Amibilia* (ed. M. Castrillón, E. Martín Peinador and J. M. Sanjurjo). Departamento de Geometría y Topología UCM (2016).

Combinatorial statistics on Catalan words avoiding consecutive patterns

José L. Ramírez [1]

[1] *Departamento de Matemáticas, Universidad Nacional de Colombia, Colombia*
E-mail: jlramirezr@unal.edu.co

Abstract

A Catalan word $w = w_1 w_2 \cdots w_n$ is one over the set of positive integers satisfying $w_1 = 1$ and $1 \leq w_i \leq w_{i-1} + 1$ for $i = 2, \ldots, n$. Catalan words of length n are enumerated by the Catalan number $C_n = \frac{1}{n+1}\binom{2n}{n}$. We compute the distribution of the descent statistic and the last symbol on the Catalan words avoiding a consecutive pattern of length at most three. We use Riordan arrays and the symbolic method to characterize the associated counting sequences. Based on joint work with J.-L. Baril, D. Colmenares, D. Silva, L.M. Simbaqueba, and D. Toquica.

References

[1] J.-L. Baril, S. Kirgizov, and V. Vajnovszki, Descent distribution on Catalan words avoiding a pattern of length at most three, Discrete Math. **341** (2018), 2608–2615.

[2] J.-L. Baril, C. Khalil, and V. Vajnovszki, Catalan words avoiding pairs of length three patterns, Discret. Math. Theor. Comput. Sci. **22**(2) (2021), # 5.

[3] J.-L. Baril, D. Colmenares, J. L. Ramírez, E.D. Silva, L.M. Simbaqueba, and D. Toquica. Consecutive pattern-avoidance in Catalan words according to the last symbol. Submitted (2023).

[4] D. Callan, T. Mansour, and J. L. Ramírez, Statistics on bargraphs of Catalan words, J. Autom. Lang. Comb. **26** (2021), 177–196.

[5] S. Corteel, M.A. Martinez, C. Savage, and M. Weselcouch, Patterns in inversion sequences I, Discrete Math. Theor. Comput. Sci. **18**(2) (2016).

[6] T. Mansour, J. L. Ramírez, and D. A. Toquica, Counting lattice points on bargraphs of Catalan words, Math. Comput. Sci. **15** (2021), 701–713.

[7] J. L. Ramírez and A. Rojas-Osorio, Consecutive patterns in Catalan words and the descent distribution. Bol. Soc. Mat. Mex. Accepted.

[8] R. Stanley, *Catalan Numbers*, Cambridge University Press, Cambridge, 2015.

Total positivity of Riordan arrays

Roksana Słowik[1]

[1] *Faculty of Applied Mathematics, Silesian University of Technology, Poland*
E-mail: roksana.slowik@gmail.com

Abstract

We are going to present the results the results on the total positivity of Riordan arrays. We will start with recalling some sufficient conditions for a Riordan array to be totally positive [3, 4, 5]. Then, we will discuss why those conditions are not necessary [1, 2].

References

[1] R. Słowik, Some (counter)examples on totally positive Riordan arrays. *Linear Algebra Appl.*, 594: 117–123 (2020).

[2] R. Słowik, Corrigendum to: 'Some (counter)examples on totally positive Riordan arrays'. *Linear Algebra Appl.*, 619: 338-339 (2021).

[3] X. Chen, H. Liang, Y. Wang, Total positivity of Riordan arrays. *Eur. J. Comb.*, 46: 68–74 (2015).

[4] X. Chen, Y. Wang, Notes on the total positivity of Riordan arrays. *Linear Algebra Appl.*, 569: 156–161 (2019).

[5] J. Mao, L. Mu, Y. Wang, Yet another criterion for the total positivity of Riordan arrays. *Linear Algebra Appl.*, 634: 106–111 (2022).

abstract-ILAS Madrid 2023

Involutions and pseudo-involutions in the Riordan group often arise in enumeration and have been an active area of research recently. We review an efficient method of finding the pseudo-involution companion for many generating functions and some easy ways to create new pseudo-involutions. Then we discuss a simpler approach to finding all g for a self-involutary f Then in the other direction we can in many cases start with g, find f and then factor so that $f = h\widehat{h}$ where $\widehat{h}(z) = -\overline{h}(-z)$.

Combinatorics on the negative part of Riordan matrices

Jihyeug Jang[1], *Louis W. Shapiro*[2], *Minho Song*[3]

[1] *Mathematics, Sungkyunkwan University, South Korea*
E-mail: aabb4242@gmail.com
[2] *Mathematics, Howard University, USA*
E-mail: lou.shapiro@gmail.com
[3] *Applied Algebra and Optimization Research Center, Sungkyunkwan University, South Korea*
E-mail: smh3227@skku.edu

Abstract

If a sequence indexed by nonnegative integers satisfies a linear recurrence without constant terms, one can extend the indices of the sequence to negative integers using the recurrence. Such a combinatorial result about the 'negative terms' of a sequence is called a combinatorial reciprocity theorem. In this talk, we introduce Riordan matrices and establish their combinatorial reciprocities considering negatively indexed columns of Riordan matrices. Actual examples will be presented, which are related to generalized Catalan numbers, generalized Schröder numbers, etc. This is joint work with Jihyeug Jang and Louis W. Shapiro.

Acknowledgements:
This work was (partially) supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2022R1C1C2009025).

References

[1] R. L. Graham, D. E. Knuth, O. Patashnik, and S. Liu. *Concrete mathematics: a foundation for computer science*, volume 3. American Institute of Physics, 1989.

[2] T.-X. He and L. W. Shapiro. Fuss-Catalan matrices, their weighted sums, and stabilizer subgroups of the Riordan group. *Linear Algebra Appl.*, 532:25–42, 2017.

[3] A. Luzón, D. Merlini, M. A. Morón, and R. Sprugnoli. Complementary Riordan arrays. *Discrete Applied Mathematics*, 172:75–87, 2014.

Luis Verde-Star
Department of Mathematics
Universidad Autónoma Metropolitana, Ciudad de Mexico.

Some properties of polynomial sequences associated with generalized Riordan matrices

We use the theory of polynomial sequences of interpolatory type to obtain properties of the polynomial sequences associated with the rows of Riordan matrices and exponential Riordan matrices. We describe the connection of such polynomial sequences with polynomial interpolation formulas. We also find linearization and connection coefficients, recurrence relations, and generating functions. The representation of composition operators as pseudo-exponential functions of matrices and the similarity of infinite Hessenberg matrices are other useful tools to study all kinds of infinite matrices that we will discuss.

The theory of polynomial sequences of interpolatory type was introduced around 1992 and is a form of umbral calculus, closely related with composition and multiplication operators, that can be used to obtain easily numerous results about Riordan matrices and their generalizations.

MSC19. Totally positive matrices

Ana Marco, José-Javier Martínez, Raquel Viaña

Properties of Riordan quotients
Paul Barry

Natural partial orderings and associated Riordan poset matrices
Gi-Sang Cheon, Hong Joon Choi, Bryan Curtis, Kuk-Won Kwon

From Alexandroff spaces to Riordan matrices
Pedro J. Chocano, Ana Luzón, Manuel A. Morón, Manuel A., Luis Felipe Prieto-Martínez

Professor
Tian-Xiao He

Exponential Riordan matrices and decomposition of Hankel matrices
Emanuele Munarini

The binary Pascal matrix and associated algebras
Nikolaos Pantelidis, Gi-Sang Cheon, Hong Joon Choi

Abstract cell complexes and Riordan matrices
Luis Felipe Prieto-Martínez, Pedro J. Chocano, Ana Luzón, Manuel A. Morón

Combinatorial statistics on Catalan words avoiding consecutive patterns
Jose Ramirez, Jean-Luc Baril

Total positivity of Riordan arrays
Roksana Słowik

Riordan Group Involutions
Louis Shapiro, Alexander Burstein

Combinatorics on the negative part of Riordan matrices
Minho Song, Jihyeug Jang, Louis Shapiro

Some properties of polynomial sequences associated with generalized Ri- ordan matrices
Luis Verde-Star

Accurate eigenvalues of some generalized sign regular matrices via relatively robust representations

Rong Huang[1]

[1] *School of Mathematics and Computational Science, Hunan University of Science and Technology, Xiangtan 411201, Hunan, China.*
E-mail: rongh98@aliyun.com

Abstract In this talk, we consider how to accurately solve the nonsymmetric eigenvalue problem for a class of generalized sign regular matrices including extremely ill-conditioned quasi-Cauchy and quasi-Vandermonde matrices. The problem of performing accurate computations with structured matrices is very much a representation problem. We first develop a relatively robust representation (RRR) for this class of matrices by introducing a free parameter, which exceeds an essential threshold, into an indefinite factorization. We then design a new $O(n^3)$ algorithm to compute all the eigenvalues of such matrices with high relative accuracy, as warranted by the RRR. Error analysis and numerical experiments are performed to illustrate the high relative accuracy.

Acknowledgements: Research supported by the National Natural Science Foundation of China (Grant No. 11871020), the Natural Science Foundation for Distinguished Young Scholars of Hunan Province (Grant No. 2017JJ1025) and the Research Foundation of Education Bureau of Hunan Province (Grant No. 18A198).

References

[1] A.S. Alfa, J. Xue and Q. Ye, *Accurate computation of the smallest eigenvalue of a diagonally dominant M-matrix*, Math. Comp., 71 (2002), pp. 217-236.

[2] J. Demmel, I. Dumitriu, O. Holtz and P. Koev, *Accurate and efficient expression evaluation and linear algebra*, Acta Numerica, 17 (2008), pp. 87-145.

[3] J. Demmel, M. Gu, S. Eisenstat, I. Slapničar, K. Veselić and Z. Drmač, *Computing the singular value decomposition with high relative accuracy*, Linear Algebra Appl., 299 (1999), pp. 21-80.

[4] P. Koev, *Accurate eigenvalues and SVDs of totally nonnegative matrices*, SIAM J. Matrix Anal. Appl., 27 (2005), pp. 1-23.

[5] P. Koev, *Accurate computations with totally nonnegative matrices*, SIAM J. Matrix Anal. Appl., 29 (2007), pp. 731-751.

[6] P. Koev, http://math.mit.edu/ plamen/software/TNTool.html.

[7] P. Koev, *Accurate eigenvalues and exact zero Jordan blocks of totally nonnegative matrices*, Numer. Math., 141 (2019), pp. 693-713.

[8] P. Koev, http://math.mit.edu/ plamen/software/TNTool.html

[9] D.S. Watkins, *Product eigenvalue problems*, SIAM Rev., 47 (2005), pp. 3-40.

Bidiagonal decompositions of singular sign regular matrices of signature $(1, \ldots, 1, -1)$

Plamen Koev[1]

[1] *Department of Mathematics, San Jose State University, San Jose, CA 95192, U.S.A.*
E-mail: plamen.koev@sjsu.edu

Abstract

The bidiagonal decompositions of the totally nonnegative and sign regular matrices have become an instrumental tool to parameterize, study the properties of, and perform accurate computation with these matrices.

Recently, Rong Huang [1] demonstrated how to parameterize the class of nonsingular sign regular matrices with signature $(1, \ldots, 1, -1)$ as a product of (anti) bidiagonals and also, how to perform computations with them to high relative accuracy.

In this talk we show how to generalize the above results to the singular case and to incorporate the singularity seamlessly into their existing bidiagonal decompositions and algorithms for accurate computations.

These singular sign regular matrices are also totally nonnegative, a class which has been well studied and understood [2] and we will also show how the corresponding bidiagonal decompositions are intricately related.

Acknowledgements: This work is partially supported by the Woodward Fund of the Mathematics Department of San Jose State University.

References

[1] Rong Huang, *Accurate Eigenvalues of Some Generalized Sign Regular Matrices via Relatively Robust Representations*, Journal of Scientific Computing 82, 2020.

[2] Plamen Koev, *Accurate eigenvalues and exact zero Jordan blocks of totally nonnegative matrices*, Numerische Mathematik, 141, 693-713, 2019.

On the total positivity of Gram matrices of polynomial bases

E. Mainar[1], J. M. Peña[1], B. Rubio[1]

[1] Departamento de Matemática Aplicada, Universidad de Zaragoza/IUMA, España
E-mail: esmemain@unizar.es

Abstract

Hilbert matrices $H_n = (1/(i-j-1))_{1\leq i,j\leq n+1}$ are Hankel matrices corresponding to the moment sequence

$$s_n = \frac{1}{n+1} = \int_0^1 x^n\, dx$$

(cf. [4]). These matrices are strictly totally positive and can be seen as grammian matrices of monomial polynomial bases with respect to the usual inner product

$$\langle f, g\rangle := \int_0^1 f(x)g(x)\, dx.$$

Bernstein mass matrices are grammian matrices of Bernstein bases with respect to the above mentioned inner product (see [1], [5]). In [7], it is shown that these matrices are also strictly totally positive. Bidiagonal factorizations to Hilbert and Bernstein mass matrices are obtained in [6, 8] and [7], respectively, providing matrix computations to high relative accuracy.

Bernstein bases are the polynomial bases most used in computer aided geometric design due to their optimal shape preserving and stability properties (see [2]). This good behaviour is related to the fact that Bernstein bases are normalized B-bases (cf. [2, 3]) and then, any totally positive polynomial basis on a compact interval can be written in terms of a Bersntein basis and a nonsigular totally positive matrix.

Taking into account the previous facts, the total positivity of grammian matrices of totally positive polynomial basis with respect to several inner products will be analyzed. Furthermore, conditions to guarantee computations to high relative accuracy with those matrices will be obtained. The numerical experimentation will confirm the accuracy of the proposed procedures.

Acknowledgements: Work partially supported by the Spanish research grant PGC2018-096321-B-I00 (MCIU/AEI) and by Gobierno de Aragón (E41_20R)

References

[1] L. Allen, R. C. Kirby, Structured Inversion of the Bernstein Mass Matrix. *SIAM J. Matrix Anal. Appl.*, 41, 413–431 (2020).

[2] J.M. Carnicer, J.M. Peña, Shape preserving representations and optimality of the Bernstein basis. *Adv. Comput. Math.*, 1: 173–196 (1993).

[3] Carnicer, J.M., Peña, J.M., Totally positive bases for shape preserving curve design and optimality of B-splines. *Comput. Aided Geom. Des.*, 11: 633–654 (1994).

[4] D. Hilbert, Ein Beitrag zur Theorie des Legendreschen Polynoms. *Acta Math.*, 18 (1894).

[5] R. C. Kirby. Fast simplicial finite element algorithms using Bernstein polynomials. *Numer. Math.*, 117 (4): 631–652 (2011).

[6] P. Koev, Accurate computations with totally nonnegative matrices. *SIAM J. Matrix Anal. Appl.*, 29: 731–751 (2007).

[7] E. Mainar, J.M. Peña, B.Rubio, Total positivity and accurate computations with Gram matrices of Bernstein bases. *Numer. Algor.*, 91: 841–859 (2022).

[8] E. Mainar, J.M. Peña, B.Rubio.Accurate computations with matrices related to bases $\{t^i e^{\lambda t}\}$. *Adv. Comput. Math.*, 48, 38 (2022).

Bidiagonal decomposition of rectangular totally positive Lagrange-Vandermonde matrices and applications

Ana Marco[1], *José-Javier Martínez*[2], *Raquel Viaña*[3]

[1] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: ana.marco@uah.es
[2] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: jjavier.martinez@uah.es
[3] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: raquel.viana@uah.es

Abstract

In this work we present an accurate and fast algorithm to compute the bidiagonal decomposition of rectangular totally positive Lagrange-Vandermonde matrices, which are the generalization of rectangular Vandermonde matrices arising when taking a Lagrange-type basis for the space of the algebraic polynomials of degree less than or equal to n, instead of the monomial basis.

The algorithm is based on results on total positivity and Neville elimination, and the explicit expressions obtained for the determinants involved in the process make it both fast and accurate.

Our algorithm has been applied to compute with high relative accuracy the singular values and the Moore-Penrose inverse of rectangular Lagrange-Vandermonde matrices, using also some of the algorithms of Koev [1, 2]. The results of the numerical experiments, which illustrate the good behavior of the proposed algorithm, are also included.

Acknowledgements: T he authors are members of the Research Group ASYNACS (Ref. CT-CE2019/683) of Universidad de Alcalá.

References

[1] P. Koev, Accurate eigenvalues and SVDs of totally nonnegative matrices, *SIAM Journal on Matrix Analysis and Applications*, 21: 1–23 (2005) .

[2] P. Koev. Accurate computations with totally nonnegative matrices. *SIAM Journal on Matrix Analysis and Applications*, 29: 731–751 (2007).

Linear Algebra in Approximation Theory: a new hope

Ana Marco[1], *José-Javier Martínez*[2], *Raquel Viaña*[3]

[1] *Dpt. de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: ana.marco@uah.es
[2] *Dpt. de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: jjavier.martinez@uah.es
[3] *Dpt. de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: raquel.viana@uah.es

Abstract

In [5] the Lagrange polynomial basis is considered for polynomial interpolation, and its good properties are remarked [2]. On the other hand, in [1] algorithms related to totally positive matrices are presented. However, these approaches (Lagrange basis and totally positive matrices) are only used in the square case, i.e. in interpolation problems.

We propose to combine the Lagrange basis and totally positive matrices in the rectangular case, i.e. to solve least squares problems, in the line of work presented for the Bernstein basis in [4]. The aim of our work is to take advantage of existing algorithms for totally positive matrices [3], in particular for computing the QR factorization when solving the normal equations in least squares problems.

References

[1] A. Björck *Numerical Methods in Matrix Computations*. Springer, 2015.

[2] N. J. Higham. The numerical stability of barycentric Lagrange interpolation. *IMA J. Numer. Anal.*, 24: 547–556 (2004).

[3] P. Koev. Accurate computations with totally nonnegative matrices. *SIAM J. Matrix Anal. Appl.*, 29: 731–751 (2007).

[4] A. Marco, J.J. Martínez. Polynomial least squares fitting in the Bernstein basis. *Linear Algebra and its Applications*, 433(7): 1254–1264 (2010).

[5] L. N. Trefethen *Approximation Theory and Approximation Practice*. SIAM, 2013.

Tropical totally positive matrices

S.Gaubert[1], *A. Niv*[2]

[1] *INRIA Saclay–Île-de-France, France and CMAP, École polytechnique, CNRS, France*
E-mail: Stephane.Gaubert@inria.fr
[2] *Science faculty, Kibbutzim College of Education, Israel*
E-mail: adi.niv@smkb.ac.il

Abstract

We investigate the tropical analogues of totally positive and totally nonnegative matrices. These arise when considering the images by the nonarchimedean valuation of the corresponding classes of matrices over a real nonarchimedean valued field, like the field of real Puiseux series. We show that the nonarchimedean valuation sends the totally positive matrices precisely to the Monge matrices. This leads to explicit polyhedral representations of the tropical analogues of totally positive and totally nonnegative matrices. We also show that tropical totally nonnegative matrices with a finite permanent can be factorized in terms of elementary matrices. We finally determine the eigenvalues of tropical totally nonnegative matrices, and relate them with the eigenvalues of totally nonnegative matrices over nonarchimedean fields.

Acknowledgements: The first author has been partially supported (at the time) by the Gaspard Monge Program (PGMO) of FMJH and EDF, by a public grant as part of the Investissement d'avenir project, reference ANR-11-LABX-0056-LMH, LabEx LMH, and by the MALTHY Project of the ANR Program.

The second author was sported (at the time) by the French Chateaubriand grant and INRIA postdoctoral fellowship.

References

[1] P. Butkovic. Max-algebra: the linear algebra of combinatorics? *Linear Algebra Appl.*, 367:313–335 (2003).

[2] M. Fiedler. Subtotally positive and Monge matrices. *Linear Algebra Appl.*, 413:177–188 (2006).

[3] S. M. Fallat and C. R. Johnson *Totally Nonnegative Matrices*. Princeton Series in Applied Mathematics, Princeton university press, 2011.

[4] S. Fomin and A. Zelevinsky. Total positivity: Tests and parametrizations. *The Mathematical Intelligencer*, 22(1):23–33 (2000).

[5] A. Niv. Factorization of tropical matrices. *Algebra Appl.*, 13(1):1350066 (2014).

[6] S. Gaubert and P. Butkovic. Sign-nonsingular matrices and matrices with unbalanced determinant in symmetrised semirings. *Linear Algebra Appl.*, 301(1-3):195–201 (1999).

[7] S. Gaubert and A. Niv. Tropical totally positive matrices. *Journal of Algebra*, 515:522–544 (2018).

Some optimal properties related to Total Positivity

J.M. Peña[1]

[1] *Departamento de Matemática Aplicada/IUMA, Universidad de Zaragoza, Spain*
E-mail: jmpena@unizar.es

Abstract

Several optimal properties related to Total Positivity and totally positive matrices will be commented. In particular, we shall deal with optimal properties of the B-basis of a space of univariate functions U with a totally positive basis (see [2]). The B-basis of U generates all totally positive bases of U by means of totally positive matrices. A first example of B-basis is the Bernstein basis of polynomials, which has optimal shape preserving propeties for curve design (see [1]), property shared by the normalized B-bases [2]. Later, optimal stability properties of many B-bases was also shown (see [3]). Optimal properties of collocation matrices of B-bases can be found in [4] and these properties could be extended to multivariate spaces (cf. [5]). This talk will present some new contributions to this field.

Acknowledgements: Work (partially) supported by the Spanish research grant PGC2018-096321-B-I00 (MCIU/AEI) and by Gobierno de Aragón (E41 20R)

References

[1] J. M. Carnicer, J. M. Peña. Shape preserving representations and optimality of the Bernstein basis. *Advances in Computational Mathematics*, 1: 173–196 (1993).

[2] J. M. Carnicer, J. M. Peña. Totally positive bases for shape preserving curve design and optimality of B-splines. *Computer Aided Geometric Design*, 11: 633–654 (1994).

[3] J. M. Peña. On the optimal stability of bases of univariate functions. *Numerische Mathematik*, 91: 305–318 (2002).

[4] J. Delgado, J. M. Peña. Extremal and optimal properties of B-bases collocation matrices. *Numerische Mathematik*, 146: 105–118 (2020).

[5] J. Delgado, H. Orera, J. M. Peña. Optimal properties of tensor product of B-bases. *Applied Mathematics Letters*, 121: Paper 107473 (2021).

Accurate computations with rectangular totally positive collocation matrices of the Lupaş-type (p,q)-analogue of the Bernstein basis

Ana Marco[1], *José-Javier Martínez*[2], *Raquel Viaña*[3]

[1] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: ana.marco@uah.es
[2] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: jjavier.martinez@uah.es
[3] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: raquel.viana@uah.es

Abstract

An accurate and fast algorithm for computing the bidiagonal decomposition of rectangular totally positive collocation matrices of the Lupaş-type (p,q)-analogue of the Bernstein basis is presented. These matrices are a generalization of the Vandermonde matrices obtained when replacing the monomial basis by a generalization of the Bernstein basis used in the area of CAGD: Lupaş-type (p,q)-analogue of the Bernstein basis [2]. This bidiagonal decomposition, together with the algorithms in [1], allows us to solve in an accurate and efficient way several numerical linear algebra problems for these matrices.

Acknowledgements: T his research has been partially supported by Spanish Research Grant PGC2018-096321- B-I00 from the Spanish Ministerio de Ciencia, Innovación y Universidades. The authors are members of the Research Group ASYNACS (Ref. CT-CE2019/683) of Universidad de Alcalá.

References

[1] P. Koev. Accurate computations with totally nonnegative matrices. *SIAM Journal on Matrix Analysis and Applications*, 29: 731–751 (2007).

[2] A. Marco, J.J. Martínez, R. Viaña. Accurate computations with collocation matrices of the Lupaş-type (p,q)-analogue of the Bernstein basis, *Linear Algebra and its Applications*, 651: 312–331 (2022) .

MSC20. Euclidean Jordan algebras and related systems

Muddappa Gowda

Fan-Theobald-Von Neumann systems
Muddappa Gowda

Hadamard product and related inequalities in the Jordan spin algebra
Juyoung Jeong, Sangho Kum, Yongdo Lim

Jordan automorphisms and derivatives of symmetric cones
Michael Orlitzky

A Fiedler-type determinantal inequality in Euclidean Jordan algebras
David Sossa

On certain properties of the second order cone and some of its generalizations
Roman Sznajder

Fan-Theobald-von Neumann systems

Muddappa Gowda[1]

[1] *Department of Mathematics and Statistics, University of Maryland Baltimore County, USA*
E-mail: gowda@umbc.edu

Abstract

A Fan-Theobald-von Neumann system is a triple (V, W, λ), where V and W are real inner product spaces and $\lambda : V \to W$ is a norm-preserving map satisfying a Fan-Theobald-von Neumann type inequality together with a condition for equality. A simple example is $(\mathcal{H}^n, \mathcal{R}^n, \lambda)$, where $\mathcal{H}^n$ is the space of n by n complex Hermitian matrices and, for any $X \in \mathcal{H}^n$, $\lambda(X)$ is the vector of eigenvalues of X written in the decreasing order. Other examples include the space of n by n complex matrices (with λ denoting the singular value map), Euclidean Jordan algebras, systems induced by certain hyperbolic polynomials, and normal decomposition systems (Eaton triples). This talk is aimed at describing some examples, basic properties, and the concepts of commutativity, automorphisms, majorization, etc.

References

[1] M.S. Gowda. Optimizing certain combinations of spectral and linear/distance functions over spectral sets. *arXiv.org*, arXiv:1902.06640, (2019).

[2] M.S. Gowda, J. Juyoung. Commutativity, Majorization, and Reduction in Fan–Theobald–von Neumann Systems. *Results Math.*, 78:72, https://doi.org/10.1007/s00025-023-01845-2, (2023).

Hadamard product and related inequalities in the Jordan spin algebra

Sangho Kum[1], *Yongdo Lim*[2], *Juyoung Jeong*[3]

[1] *Department of Mathematics Education, Chungbuk National University, Republic of Korea*
E-mail: shkum@chungbuk.ac.kr
[2] *Department of Mathematics, Sungkyunkwan University, Republic of Korea*
E-mail: ylim@skku.edu
[3] *Applied Algebra and Optimization Research Center, Sungkyunkwan University, Republic of Korea*
E-mail: jjycjn@gmail.com, jjycjn@skku.edu

Abstract

Due to its simple yet elegant structure, the study of an entry-wise product of matrices, called the Hadamard product, has received extensive attention from researchers and has expanded to various disciplines, including Euclidean Jordan algebras. As an ongoing effort to extend this product to Euclidean Jordan algebras, in this article, we propose a Hadamard product in the setting of Jordan spin algebra, $\mathcal{L}^n$, under the scheme of the Peirce decomposition, and show that it preserves the diagonal structure of the elements in the algebra. It is shown that this new product corresponds to the standard Hadamard product of 2×2 symmetric matrices in the case of $\mathcal{L}^3$. Lastly, we prove that this novel product satisfies an analog of the Schur product theorem as well as the inequalities of Hadamard, Oppenheim, Fiedler, etc.

Acknowledgements: The presenting author is supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIT) No. 2021R1C1C2008350.

References

[1] Sangho Kum, Yongdo Lim, and Juyoung Jeong. *Hadamard product and related inequalities in the Jordan spin algebra*. Linear and Multilinear Algebra. Published online (2022). DOI: 10.1080/03081087.2022.2093319

Jordan automorphisms and derivatives of symmetric cones

Michael Orlitzky[1]

[1] *Baltimore, Maryland*
E-mail: michael@orlitzky.com

Abstract

Hyperbolicity cones, and in particular symmetric cones, are of great interest in optimization. Renegar showed that every hyperbolicity cone has a family of "derivatives" that approximate it [3], and Ito and Lourenço recently characterized the automorphisms of the derivatives of symmetric cones [2]. Symmetric cones famously correspond to Euclidean Jordan algebras [1], and we show that the automorphisms of a symmetric cone's derivatives are closely related to the automorphisms of its associated Jordan algebra.

References

[1] Jacques Faraut and Adam Korányi. *Analysis on Symmetric Cones*. Clarendon Press, Oxford, 1994. ISBN 9780198534778.

[2] Masaru Ito and Bruno F. Lourenço. *Automorphisms of rank-one generated hyperbolicity cones and their derivative relaxations*. August 2022. URL https://arxiv.org/abs/2207.11986.

[3] James Renegar. *Hyperbolic programs, and their derivative relaxations*. Foundations of Computational Mathematics volume, 6(1):59–79, 2006, doi:10.1007/s10208-004-0136-z.

A Fiedler-type determinantal inequality in Euclidean Jordan algebras

David Sossa[1]

[1] *Instituto de Ciencias de la Ingeniería, Universidad de O'Higgins, Chile*
E-mail: david.sossa@uoh.cl

Abstract

The Fiedler's determinantal inequality stated in [1] provides bounds for the determinant of the sum of two Hermitian matrices. In this talk, we present an extension of the Fiedler's result to Euclidean Jordan algebras. In our proof, we use the commutation principle for variational problems which were recently introduced in these algebras. As an application, we obtain some Minkowski-type determinantal inequalities in Euclidean Jordan algebras.

Acknowledgements: Work (partially) supported by FONDECYT (Chile) through grant number 11220268

References

[1] M. Fiedler. Bounds for the determinant of the sum of Hermitian matrices. *Proc. Am. Math. Soc.* 30 (1971), 27–31.

[2] D. Sossa. A Fiedler-type determinantal inequality in Euclidean Jordan algebras. Submitted, 2022.

Abstract

We present two generalizations of the classical second-order cone (the Lorentz cone), namely the Extended Second Order Cone (ESOC) and the Monotone Extended Second Order Cone (MESOC). We will discuss the irreducibility, the dual cones, Laupunov rank, and related issues pertaining to ESOC and MESOC cones. The results on MESOC were obtained in collaboration with Yingchao Gao and Sándor Zoltán Németh.

MSC21. Robust and efficient linear algebra computations at exascale

Roman Iakymchuk

Associate Professor
Roman Iakymchuk

Mixed-precision eigenvalue solver on GPUs
Toshiyuki Imamura, Takeshi Terao

GEMM-based numerical algorithm for accurate matrix multiplication
Ozaki Katsuhisa

Acceleration of iterative refinement for symmetric eigenvalue decomposition with clustered eigenvalues
Yuki Uchino, Katsuhisa Ozaki

Robust iterative solvers

Roman Iakymchuk[1]

[1] *Umeå University, Sweden*
E-mail: riakymch@cs.umu.se

Abstract

Parallel implementations of Krylov subspace methods often help to accelerate the procedure of finding an approximate solution of a linear system. However, such parallelization coupled with asynchronous and out-of-order execution often enlarge the non-associativity impact in floating-point operations. These problems are even amplified when communication-hiding pipelined algorithms are used to improve the parallelization of Krylov subspace methods. Introducing reproducibility in the implementations avoids these problems by getting more robust and correct solutions. We propose a general framework [1] for deriving robust (reproducible and accurate) variants of Krylov subspace methods. The proposed algorithmic strategies are reinforced by programmability suggestions to assure deterministic and accurate executions. The framework is illustrated on the preconditioned BiCGStab method and its pipelined modification, which in fact is a distinctive method from the Krylov subspace family.

Due to the energy consumption constraint for large-scale computing that encourages the revision of the architecture design, scientists also review the applications and the underlying algorithms organization. The main aim is to make computing sustainable and apply the lagom principle ("not too much, not too little, the right amount"), especially when it comes to working/ storage precision. Thus, I will introduce an approach to address the issue of sustainable, but still reliable, computations from the perspective of computer arithmetic tools [2] and will present some preliminary results.

Acknowledgements: This work was partially supported by the EU H2020 MSCA-IF Robust project (No. 842528); the EU H2020 CoE CEEC (No. 101093393 the French ANR InterFLOP project (No. ANR-20-CE46-0009).

References

[1] R. Iakymchuk and J. I. Aliaga *General framework for re-assuring numerical reliability in parallel Krylov solvers: A case of BiCGStab methods*. arXiv preprint arXiv:2302.04180, February 2023.

[2] Y. Chatelain , E. Petit, P. de Oliveira Castro, Gh. Lartigue, and D. Defour. *Automatic Exploration of Reduced Floating-Point Representations in Iterative Methods*. Euro-Par 2019: Parallel Processing, pp 481-494.

Mixed-precision eigenvalue solver on GPUs

Toshiyuki Imamura[1], *Takeshi Terao*[12]

[1] *Center for Computational Science, RIKEN, Japan*
E-mail: imamura.toshiyuki@riken.jp
[2] *Kyoshu, University, Japan*
E-mail: takeshi.terao@a.riken.jp

Abstract

Mixed-precision algorithm is a challenging and advanced approach in the recent numerical linear algebra. Focusing on the reduced precision such as fp16, bfloat16, or special reduced precision matrix multiplier accumulator, these are optimized for the computational speed and have a significant advantage on energy efficiency for domain sciences.

The reduced precision arithmetic works on small-sized problems or successive accuracy refinement by step-by-step precision up-casting. Batched-style eigensolver [1] with a reduced precision calculation is applied for the initial step of Ogita-Aishima's iterative refinement scheme [2]. It can accelerate the iterative refinement using a 'matrix-multiply arithmetic' or similar accumulators to accelerate major operations of matrix-matrix products. Mixing up with different data formats and actual numerical arithmetic can be controlled by the idea of our templated BLAS (tmBLAS) proposal [3]. For the larger dense matrix cases, some extensions to the subspace problem and more advanced implementation are done by Terao, Uchino, Ozaki, and so on (see the MS at ILAS23).

In the presentation of the Mini-symposium, we will demonstrate the computational behavior of the successively refined approach and preliminary performance on the latest GPU systems.

Acknowledgements: This work was supported by a COE research grant in computational science from the Hyogo prefecture and Kobe City through the Foundation for Computational Science, also partially by Grant-in-Aid for Scientific Research (B) No. 19H04127 from the JSPS.

References

[1] Y. Hasegawa, T. Imamura, T. Ina, N. Onodera, Y. Asahi and Y. Idomura. GPU Optimization of Lattice Boltzmann Method with Local Ensemble Transform Kalman Filter, Proc. ScalAH (2022), 10–17.

[2] T. Ogita and K. Aishima. Iterative refinement for symmetric eigenvalue decomposition, Jpn. J. Ind. Appl. Math., **35** (2018), 1007–1035.

[3] D. Mukunoki, A. Suzuki, and T. Imamura. Multiple and Mixed Precision BLAS with C++ Template, 5th R-CCS Intl. Sympo. Poster 14 (2023)

GEMM-based numerical algorithm for accurate matrix multiplication

Katsuhisa Ozaki[1]

[1] *Dpt. 1, Shibaura Institute of Technology, Japan*
E-mail: ozaki@sic.shibaura-it.ac.jp

Abstract

Floating-point numbers and their arithmetic, typically in accordance with IEEE 754 standard [1], are widely used in scientific computing owing to their speed. However, rounding errors can be problematic when performing computations with floating-point arithmetic. This presentation focuses on accurate numerical algorithms for matrix multiplication [2]. For floating-point matrices A and B, the original method [2] splits A and B into sum of p and q floating-point matrices such that

$$A = \sum_{i=1}^{p} A^{(i)}, \quad B = \sum_{j=1}^{q} B^{(j)}, \quad p, q \in \mathbb{N},$$

$$|a_{ij}^{(k)}| \geq |a_{ij}^{(k+1)}| \text{ for } |a_{ij}^{(k)}| \neq 0, \quad |b_{ij}^{(\ell)}| \geq |b_{ij}^{(\ell+1)}| \text{ for } |b_{ij}^{(\ell)}| \neq 0.$$

For $1 \leq i \leq p-1$ and $1 \leq j \leq q-1$, no rounding error occurs in the evaluation of $A^{(i)}B^{(j)}$. Let $p = q$, we have

$$AB = \sum_{i+j \leq p} A^{(i)}B^{(j)} + \sum_{i=1}^{p} A^{(i)} \left(\sum_{j=p-i+1}^{p} B^{(j)} \right), \tag{1}$$

and it involves $p(p-1)/2$ matrix multiplications.

Although [2], only discusses the case where $p = q$, we have recently discovered that efficient algorithms for accurate matrix multiplication exist even when $p \neq q$. In this study, we primarily develop accurate algorithms with 4, 5, 8, and 9 matrix multiplications that are not expressed in the form 1. We demonstrate that the computational cost and accuracy of the computed results are correlated with the number of matrix multiplications used in the algorithm.

Acknowledgements: This work was supported by JSPS KAKENHI Grant Number 20H04195.

References

[1] IEEE standard for floating-point arithmetic, IEEE Std 754-2019, 1–84, 2019.

[2] K. Ozaki, T. Ogita, S. Oishi, S. M. Rum. Error-Free Transformation of Matrix Multiplication by Using Fast Routines of Matrix Multiplication and its Applications, *Numer. Algorithms*, 59: 95–118 (2012).

Acceleration of iterative refinement for symmetric eigenvalue decomposition with clustered eigenvalues

Yuki Uchino[1], *Katsuhisa Ozaki*[2]

[1] *Graduate School of Engineering and Science, Shibaura Institute of Technology, Japan*
E-mail: nb22105@shibaura-it.ac.jp
[2] *Department of Mathematical Sciences, Shibaura Institute of Technology, Japan*
E-mail: ozaki@sic.shibaura-it.ac.jp

Abstract

This study investigates iterative refinement algorithms for the eigenvalue decomposition of a real symmetric matrix $A \in \mathbb{R}^{n \times n}$:

$$A = XDX^T,$$

where the ith columns of the orthgonal matrix $X \in \mathbb{R}^{n \times n}$ are the eigenvectors and the ith diagonal elements of the diagonal matrix $D \in \mathbb{R}^n$ are the eigenvalues for $i = 1, \ldots, n$. For the non-clustered eigenvalues, Ogita and Aishima proposed the algorithm in the same manner as Newton's method [1]. Building on their work, we presented two mixed-precision algorithms: one involves $4n^3 + \mathcal{O}(n^2)$ to $6n^3 + \mathcal{O}(n^2)$ highly accurate operations per iteration, and the other involves $2n^3 + \mathcal{O}(n^2)$ highly accurate operations per iteration [2].

For the clustered eigenvalues, Ogita and Aishima proposed the algorithm extending their algorithm without clustered eigenvalues [3]. In this study, we propose an alternative algorithm based on our algorithm without clustered eigenvalues. Our proposed method offers advantages in terms of computational speed comparable with Ogita and Aishima's algorithm, as demonstrated through numerical experiments.

Acknowledgements: This work was supported by a Grant-in-Aid for JSPS Fellows No. 22J20869.

References

[1] T. Ogita, K. Aishima: Iterative refinement for symmetric eigenvalue decomposition, *Jpn. J. Ind. Appl. Math.*, 35:3 (2018), 1007–1035.

[2] Y. Uchino, K. Ozaki, T. Ogita: Acceleration of iterative refinement for symmetric eigenvalue decomposition (in Japanese), *IPSJ Transactions on Advanced Computing System*, 15:1 (2022), 1–12.

[3] T. Ogita, K. Aishima: Iterative refinement for symmetric eigenvalue decomposition II: clustered eigenvalues, *Jpn. J. Ind. Appl. Math.*, 36:2 (2019), 435–459.

MSC22. State-of-the-art in algorithms and applications

Sirani M. Perera, Natalia Bebiano

Solving an inverse eigenvalue problem using a divide-and-conquer method
Natalia Bebiano, Wei-Ru Xu

A Vandermonde Neural Operator: Extending the Fourier Neural Operator to Nonequispaced Distributions
Levi Lingsch, Mike Michelis, Sirani Perera

Minisymposia Organizer and Presenter
Sirani M. Perera

Computing Approximate Solutions of Ill-Conditioned Linear Systems in Low and Mixed Precision
James Nagy

Adapted AZNN Methods for Time-Varying\\ and Static Matrix Problems
Frank Uhlig

Updating a Sequence of Orthogonal Rational Functions
Raf Vandebril, Marc Van Barel, Niel Van Buggenhout

Structured Matrices Approach for Legendre Pairs
Ilias Kotsireas

Solving an inverse eigenvalue problem using a divide-and-conquer method

Natalia Bebiano[1], *Wei-Ru Xu*[2]

[1] *CMUC, Department of Mathematics, University of Coimbra, P 3001-454 Coimbra, Portugal*
E-mail: bebiano@mat.uc.pt
[2] *School of Mathematical Sciences, Laurent Mathematics Center, Sichuan Normal University, Chengdu 610066, P. R. China*
E-mail: weiruxu@sicnu.edu.cn, weiruxu@foxmail.com (Wei-Ru Xu)

Abstract For the given signature operator $\mathcal{H} = I_r \oplus -I_{n-r}$, a pseudo-Jacobi matrix is a self-adjoint matrix relatively to a symmetric bilinear form $\langle \cdot, \cdot \rangle_{\mathcal{H}}$, and it is the counterpart of a classical Jacobi matrix to the indefinite scalar product space setting. In this talk, we consider recent inverse eigenvalue problems for this class of matrices. Namely, an $n \times n$ pseudo-Jacobi matrix is constructed from a prescribed n-tuple of distinct real numbers and a Jacobi matrix of order not less than $\lfloor \frac{n}{2} \rfloor$, such that its spectrum is this tuple and the given Jacobi matrix is its trailing principal submatrix. A divide-and-conquer scheme is used to solve this problem, and a necessary and sufficient condition under which the problem is solvable is presented. A numerical algorithm is designed to solve this pseudo-Jacobi matrix inverse eigenvalue problem according to the obtained results. Illustrative numerical examples are given to test the reconstructive algorithm.

Acknowledgements: Work (partially) supported by the Centre for Mathematics of University of Coimbra (funded by the Portuguese Government through FCT/MEC and by European RDF through Partership Agreement PT2020).

References

[1] W.-R. Xu, Y. Gong, N. Bebiano, G.-L. Chen, How to choose the signature operator such that the periodic pseudo-Jacobi inverse eigenvalue problem is solvable?, Appl. Math. Lett. 126 (2022) 107803, 8pp.

A Vandermonde Neural Operator: Extending the Fourier Neural Operator to Nonequispaced Distributions

Levi Lingsch[1], *Mike Michelis*[2], *Sirani Perera*[3]

[1] *Dpt. Mathematics, ETH Zurich, Switzerland*
E-mail: levi.lingsch@sam.math.ethz.ch
[2] *Dpt. Mechanical and Process Engineering, ETH Zurich, Switzerland*
E-mail: mike.michelis@ai.ethz.ch
[3] *Dpt. Mathematics, Embry-Riddle Aeronautical University, Florida, USA*
E-mail: Sirani.Perera@erau.edu

Abstract

The Fourier neural operator has shown impressive capabilities to learn parametric partial differential equations. It relies on the fast Fourier transform, which imposes a strict requirement for equispaced data. In this work, we propose a neural operator learning method utilizing a Vandermonde structured matrix to act as a surrogate for the forward and backward of the discrete Fourier transform on nonequispaced data while maintaining quasi-linear operational complexity. Our results show that the proposed Vandermonde neural operator surpasses the Fourier neural operator in speed, while maintaining its accuracy. The simple structure makes this method ideal for applying neural operator learning to the nonequispaced lattice, as well as real-world data collected on a global scale.

A Low-complexity Algorithm in Navigating Unmanned Aerial Systems

Sirani M. Perera [1], *Rodman J. Myers*[2], *Killian Sullivan*[3], *Kyle Byassee*[4], *Houbing Song*[5], *Arjuna Madanayake*[6]

[1] *Department of Mathematics, Embry-Riddle Aeronautical University, USA*
E-mail: pereras2@erau.edu
[2] *Department of Computer Science, Florida Southern College, USA*
E-mail: rmyers3@mocs.flsouthern.edu
[3] *Department of Mathematics, Florida Southern College, USA*
E-mail: ksullivan6@mocs.flsouthern.edu
[4] *Department of Engineering, University of Tennessee at Martin, USA*
E-mail: kylhbyas@ut.utm.edu
[5] *Department of Information Systems, University of Maryland Baltimore County, USA*
E-mail: songh4@erau.edu
[6] *Department of Electrical & Computer Engineering, Florida International University, USA*
E-mail: amadanay@fiu.edu

Abstract

The advent of 5G/6G cellular networking has greatly enhanced the capacity of beamforming in Unmanned Aerial Systems (UAS) networking, enabling significant advancements in industrial and residential applications.

In this talk, we explore the integration of beamforming and sensor arrays to develop a mathematical model for routing a collection of Unmanned Autonomous Aerial Systems (UAAS), or drone swarms, without the need for a base station communication. Our proposed SWARM routing algorithm is based on structured matrices, multi-beam beamforming, and sensor array concepts. The precision and reliability of the model are discussed, and the bit-error rate is analyzed based on the number of elements in the sensor arrays and the beamformed output of the swarm members. This is to ensure secure, decentralized networking for the UAAS. Lastly, we introduce a low-cost routing algorithm to navigate the UAAS collection.

Acknowledgements: This research was partially funded by the National Science Foundation with the Award Number 2150213.

Computing Approximate Solutions of Ill-Conditioned Linear Systems in Low and Mixed Precision

James Nagy[1]

[1] *Department of Mathematics, Emory University, USA*
E-mail: jnagy@emory.edu

Abstract

In this talk we consider approaches to compute approximate solutions of large scale, severely ill-conditioned linear systems that arise in many important applications, such as machine learning and image reconstruction. We focus on iterative methods, and consider implications of using low and mixed numerical precisions in the computations. Special considerations, which normally do not arise when solving well-conditioned problems, such as incorporating regularization into the developed methods, need to be considered. We consider operator approximation, iterative refinement exploiting mixed precision formats to ensure sufficient accuracy, and preconditioning for Krylov subspace iterative methods.

Acknowledgements: Work (partially) supported by the US National Science Foundation grants DMS-2038118 and DMS-2208294.

Adapted AZNN Methods for Time-Varying and Static Matrix Problems

Frank Uhlig
Department of Mathematics and Statistics,
Auburn University, Auburn, AL 36849-5310
(uhligfd@auburn.edu)

Abstract

We present adapted Zhang Neural Networks (AZNN) in which the parameter settings for the exponential decay constant η and the length of the start-up phase of basic ZNN are adapted to the problem at hand. Specifically we study experiments with AZNN for time-varying square matrix factorizations as a product of two time-varying symmetric matrices and for the time-varying matrix square root problem.

Differing from generally used small η values and minimal length start-up phases in ZNN, we adapt the basic ZNN method to work with large or even gigantic η settings and arbitrary length start-up phases using the Euler low accuracy and unstable finite difference formula. These adaptations improve the speed of AZNN's convergence and lower its solution error bounds for our chosen problems significantly to near machine constant levels.

Parameter-varying AZNN also allows us to find full rank symmetrizers of static matrices reliably, such as for the Kahan and Frank matrices, for matrices with highly ill-conditioned eigenvalues and for matrices with complicated Jordan structures of dimensions from $n = 2$ on up where standard eigendata based symmetrizer algorithms generally have failed. AZNN helps us to find full rank static matrix symmetrizers that have never been successfully computed before.

Updating a Sequence of Orthogonal Rational Functions

Marc Van Barel[1], *Niel Van Buggenhout*[2], *Raf Vandebril*[3]

[1] *Dept. Computer Science, KU Leuven, Belgium*
E-mail: marc.vanbarel@kuleuven.be
[2] *Dept. Numerical Mathematics, Charles University, Czech Republic*
E-mail: niel.vanbuggenhout@karlin.mff.cuni.cz
[3] *Dept. Computer Science, KU Leuven, Belgium*
E-mail: raf.vandebril@kuleuven.be

Abstract

Orthogonal polynomials are an important tool to approximate functions. Orthogonal rational functions provide a powerful alternative if the function of interest is not well approximated by polynomials. Polynomials orthogonal with respect to certain discrete inner products can be constructed by applying the Lanczos or Arnoldi iteration to appropriately chosen diagonal matrix and vector. This can be viewed as a matrix version of the Stieltjes procedure. The generated nested orthonormal basis can be interpreted as a sequence of orthogonal polynomials. The corresponding Hessenberg matrix, containing the recurrence coefficients, also represents the sequence of orthogonal polynomials.

Alternatively, this Hessenberg matrix can be generated by an updating procedure. The goal of this procedure is to enforce Hessenberg structure onto a matrix which shares its eigenvalues with the given diagonal matrix and the first entries of its eigenvectors must correspond to the elements of the given vector. Plane rotations are used to introduce the elements of the given vector one by one and to enforce Hessenberg structure. The updating procedure is stable thanks to the use of unitary similarity transformations. In this talk rational generalizations of the Lanczos and Arnoldi iterations are discussed. These iterations generate nested orthonormal bases which can be interpreted as a sequence of orthogonal rational functions with prescribed poles. A matrix pencil of Hessenberg structure underlies these iterations. We show that this Hessenberg pencil can also be used to represent the orthogonal rational function sequence and we propose an updating procedure for this case. The proposed procedure applies unitary similarity transformations and its numerical stability is illustrated.

Acknowledgements: The research was partially supported by the Research Council KU Leuven (Belgium), project C16/21/002 (Manifactor: Factor Analysis for Maps into Manifolds) and by the Fund for Scientific Research – Flanders (Belgium), projects G0A9923N (Low rank tensor approximation techniques for up- and downdating of massive online time series clustering) and G0B0123N (Short recurrence relations for rational Krylov and orthogonal rational functions inspired by modified moments).

References

[1] M. Van Barel, N. Van Buggenhout, R. Vandebril. Algorithms for Modifying Recurrence Relations of Orthogonal Polynomial and Rational Functions when Changing the Discrete Inner Product. *arXiv: 2302.00355*, 2023.

Structured Matrices Approach for Legendre Pairs

Ilias Kotsireas[1]

[1] *Wilfird Laurier University, Waterloo, ON, Canada*
E-mail: ikotsire@wlu.ca

Legendre Pairs are combinatorial objects with a rich 20+ years history. Their main application is that they furnish a structured form of the Hadamard conjecture and they have been studied by several authors. We bring into bear a number of Linear Algebraic concepts and tools, in the context of Legendre Pairs. This allows us to improve existing computational schemes to search efficiently for Legendre Pairs. Preliminary implementations indicate that using custom-tailored FFT-type algorithms for computing the DFT of odd-length input vectors clearly out-performs previous techniques used for this purpose.

Joint work with Dr. Shirani M. Perera, ERAU, Florida, United States.

MSC23. Tensors and quantum information

Shmuel Friedland, Michal Eckstein, Chi-Kwong Li

Tensors and quantum information
Mohsen Aliabadi, Shmuel Friedland

Quantum Wasserstein energy distance
Rafał Bistroń, Tomasz Miller, Michał Eckstein, Shmuel Friedland, Karol Życzkowski

Apolarity for border rank
Jarosław Buczyński, Weronika Buczyńska

Measurement sharpness and incompatibility as quantum resources
Francesco Buscemi

Quantum Monge-Kantorovich problem and transport distance between density matrices
Michal Eckstein, Sam Cole, Shmuel Friedland, Karol Zyczkowski

Tensor optimal transport, distance between sets of measures and tensor scaling
Shmuel Friedland

Turing-model complexity estimates in geometric programming and application to the computation of the spectral radius of nonnegative tensors
Stephane Gaubert, Shmuel Friedland

Inevitability of knowing less than nothing
Gilad Gour, Mark Wilde, Sam Brandsen, Isabelle Jianing Geng

Hyperdeterminant, Fermionic Fock space and Entanglement
Frederic Holweck, Luke Oeding

A new distance between pure qudits
Tomasz Miller, Rafał Bistroń, Michał Eckstein, Shmuel Friedland, Karol Życzkowski

Operator Schmidt decomposition of attenings of a 4-tensor: A solution of the quantum version of the Euler's problem of 36 officers
Karol Zyczkowski

On the complexity of finding tensor ranks

Shmuel Friedland[1], *Mohsen Aliabadi*[2]

[1] *Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, United States*
E-mail: friedlan@uic.edu
[2] *Department of Mathematics, University of California, San Diego, United States*
E-mail: maliabadisr@ucsd.edu

Abstract

The purpose of this talk is to give a linear algebra algorithm to find out if a rank of a given tensor over a field F is at most k over the algebraic closure of F, where k is a given positive integer. We estimate the arithmetic complexity of our algorithm. This is based on a joint work with Shmuel Friedland [1].

References

[1] M. Aliabadi, S. Friedland. *On the complexity of finding tensor ranks.* Commun. Appl. Math. Comput. 3 (2021), no. 2, 281–289.

Quantum Wasserstein energy distance

Rafał Bistroń [1], Tomasz Miller [2], Shmuel Friedland[3], Michał Eckstein [1], Karol Życzkowski [1,4]

[1] Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, ul. Łojasiewicza 11, 30-348, Kraków, Poland
E-mail: rafal.bistron@uj.edu.pl , michal.eckstein@uj.edu.pl
[2] Copernicus Center for Interdisciplinary Studies, Jagiellonian University, Szczepańska 1/5, 31-011, Kraków, Poland
E-mail: tomasz.miller@uj.edu.pl
[3] Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, Chicago, Illinois, 60607-7045, USA
E-mail: friedlan@uic.edu
[4] Center for Theoretical Physics, Polish Academy of Science, Warsaw, Poland
E-mail: karol.zyczkowski@uj.edu.pl

Abstract

Recently, the theory of quantum optimal transport has been developed in connection with quantum information processing, see [1, 2, 3, 4, 5, 6], and their references. This area of research aims to identify a suitable analogue of the optimal transport problem or the Wasserstein distance [7] for density matrices instead of probability distributions. Some authors have already look for the applications of the quantum Wasserstein distance, for example by constructing quantum Wasserstein Generative Adversarial Networks [8].

Currently, the most promising approach [5, 6, 8] involves identifying quantum couplings ρ^{AB} of size $N^2 \times N^2$ for two $N \times N$ density matrices ρ^A and ρ^B such that $\mathrm{Tr}_A \rho^{AB} = \rho^B$ and $\mathrm{Tr}_B \rho^{AB} = \rho^A$. The set of all such couplings is denoted as $\Gamma^Q(\rho^A, \rho^B)$. The objective of the quantum optimal transport problem is to minimize the *quantum transport cost*,

$$T^Q(\rho^A, \rho^B) := \min_{\rho^{AB} \in \Gamma^Q(\rho^A, \rho^B)} \left(\mathrm{Tr}\, C^Q \rho^{AB} \right), \tag{1}$$

for a suitable *quantum cost matrix* C^Q. Analogically to the quantum transport cost one can define [5, 6] the *quantum Wasserstein (semi) distance of order two* as:

$$W^Q(\rho^A, \rho^B) := \sqrt{\min_{\rho^{AB} \in \Gamma^Q(\rho^A, \rho^B)} \left(\mathrm{Tr}\, {C^Q}^2 \rho^{AB} \right)}. \tag{2}$$

In [5] it was shown that W^Q is a semi-distance if and only if the quantum cost matrix has the following form:

$$C^Q = \sum_{i<j} E_{ij} |\psi_-(ij)\rangle\langle\psi_-(ij)| \,,$$

where $|\psi_-(ij)\rangle = \frac{1}{\sqrt{2}}(|ij\rangle - |ji\rangle)$ and the coefficients $E_{ij} > 0$ are called the cost coefficients.

In my talk I will focus on the quantum Wasserstein distance for pure states with cost coefficients $E_{ij} = |E_i - E_j|$ encoding differences of energy levels in some physical system, which we name *quantum Wasserstein energy distance* W_H. Note that for the pure states there exist only one coupling

$$\Gamma^Q(\rho^A \otimes \rho^B) = \{\rho^A \otimes \rho^B\} \ .$$

Hence if $\rho^A = |\psi\rangle\langle\psi|$, $\rho^B = |\phi\rangle\langle\phi|$, the formula for Wasserstein energy distance simplifies to:

$$W_H(|\psi\rangle, |\phi\rangle) = \sqrt{\sum_{i<j}(E_i - E_j)^2 |\psi_i\phi_j - \phi_i\psi_j|^2}$$

By choosing these specific cost coefficients, we not only incorporate the "system structure" into the system Hilbert space but also establish the triangle inequality for W_H, regardless of the dimension of Hilbert space or involved Hamiltonian. Hence, if the spectrum of the used Hamiltonian is non degenerate then W_H is a true distance.

Moreover, I will present a tight upper and lower bound of the quantum Wasserstein energy distance $W_H(|\psi\rangle, |\phi\rangle)$ using Hamiltonian expectation values and variances. Finally, we construct a quantum speed limit using the quantum Wasserstein energy distance corresponding to evolution in the interaction picture.

Acknowledgements: Work supported by Foundation for Polish Science under the Team-Net Project No. POIR.04.04.00-00-17C1/18-00, and the Quant Era II grant project (Grant Agreement no. 731473 and 101017733)

References

[1] J. Agredo and F. Fagnola, On quantum versions of the classical Wasserstein distance, *Stochastics* **89**:6-7 910 (2017).

[2] Y. Chen, W. Gangbo, T. T. Georgiou, and A. Tannenbaum, On the matrix Monge-Kantorovich problem, *Eur. J. Appl. Math.* **31**, 574 (2020).

[3] G. De Palma and D. Trevisan, Quantum optimal transport with quantum channels, *Ann. Henri Poincaré* **22**, 3199–3234 (2021).

[4] E. A. Carlen and J. Maas, Non-commutative calculus, optimal transport and functional inequalities in dissipative quantum systems, *J. Stat. Phys.* **178**, 319 (2020).

[5] S. Friedland, M. Eckstein, S. Cole, and K. Życzkowski Quantum Monge–Kantorovich problem and transport distance between density matrices, *Phys. Rev. Lett.* **129** 110403 (2022).

[6] R. Bistro n, M. Eckstein and K. Życzkowski Monotonicity of a quantum 2-Wasserstein distance, *J. Phys. A* **56** 095301 (2023)

[7] C. Villani, *Optimal transport, Old and new*, Grundlehren der Mathematischen Wissenschaften, 338, Springer-Verlag, Berlin, 2009.

[8] S. Chakrabarti, Y. Huang, T. Li, S. Feizi, and X. Wu 2019 Quantum Wasserstein Generative Adversarial Networks, in *Advances in Neural Information Processing Systems*, ID:3674

Apolarity for border rank

Weronika Buczyńska[1], *Jarosław Buczyński*[2]

[1] *Faculty of Mathematics, Computer Science and Mechanics of University of Warsaw, Poland*
E-mail: wkrych@mimuw.edu.pl
[1] *Institute of Mathematics of Polish Academy of Sciences, Poland*
E-mail: jabu@mimuw.edu.pl

Abstract

We introduce an elementary method to study the border rank of polynomials and tensors, analogous to the apolarity lemma. This can be used to describe the border rank of all cases uniformly, including those very special ones that resisted a systematic approach. We also define a border rank version of the variety of sums of powers and analyse its usefulness in studying tensors and polynomials with large symmetries. In particular, it can be applied to provide lower bounds for the border rank of some interesting tensors, such as the matrix multiplication tensor.

Acknowledgements: The authors are supported by two Polish National Science Centre projects: 2013/08/A/ST1/00804 and 2017/26/E/ST1/00231.

References

[1] W. Buczyńska, J. Buczyński. Apolarity, border rank and multigraded Hilbert scheme. *Duke Math. J.*, 170: 3659–3702 (2021).

Measurement sharpness and incompatibility as quantum resources

Francesco Buscemi[1]

[1] *Dpt. Mathematical Informatics/Nagoya University, Japan*
E-mail: buscemi@i.nagoya-u.ac.jp

Abstract

In this talk I will discuss two aspects of quantum measurements, namely sharpness and incompatibility, from a resource-theoretic perspective. Our construction, based on various matrix preorders and Blackwell's statistical comparison theorem, settles a debate in the literature and fills some gaps in the mathematical and conceptual foundations of quantum theory.

Acknowledgements: Work (partially) supported by the MEXT Quantum Leap Flagship Program (MEXT QLEAP) Grant No. JPMXS0120319794; the MEXT-JSPS Grant-in-Aid for Transformative Research Areas (A) "Extreme Universe" No. 21H05183, and from JSPS KAKENHI, Grants No. 20K03746 and No. 23XXXXXXX.

References

[1] F.B., E. Chitambar, W. Zhou, "A complete resource theory of quantumincompatibility as quantum programmability". Link: https://arxiv.org/abs/1908.11274

[2] F.B., K. Kobayashi, S. Minagawa, A. Tosini, P. Perinotti, "Unifying different notions of quantum incompatibility into a strict hierarchy of resource theories of communication". Link: https://arxiv.org/abs/2211.09226

[3] F.B., K. Kobayashi, S. Minagawa, "A complete and operational resource theory of measurement sharpness". Link: https://arxiv.org/abs/2303.07737

Quantum Monge–Kantorovich problem and transport distance between density matrices

Sam Cole [1], *Michał Eckstein* [2], *Shmuel Friedland* [3], *Karol Życzkowski* [2,4]

[1] *NielsenIQ, Chicago, Illinois, 60606-6910, USA*
E-mail: samuel.cole@nielseniq.com
[2] *Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, ul. Łojasiewicza 11, 30-348, Kraków, Poland*
E-mail: michal.eckstein@uj.edu.pl
[3] *Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, Chicago, Illinois, 60607-7045, USA*
E-mail: friedlan@uic.edu
[4] *Center for Theoretical Physics, Polish Academy of Science, Warsaw, Poland*
E-mail: karol.zyczkowski@uj.edu.pl

Abstract

The optimal transport problem, established by Monge and refined by Kantorovich and Wasserstein, is modern domain of mathematics [1] with ubiquitous applications in physics, statistics and computer science. Recently, there is a growing interest in a "quantum" version of the optimal transport, in which the probability distributions are substituted by density matrices (see e.g. [2, 3, 4, 5, 6, 7, 8] and references therein).

The central idea is as follows: Let ρ^A and ρ^B be two $N \times N$ density matrices, and consider a density matrix ρ^{AB} of size N^2 such that both partial traces agree, $\mathrm{Tr}_A \rho^{AB} = \rho^B$ and $\mathrm{Tr}_B \rho^{AB} = \rho^A$. Such matrices, forming a set $\Gamma^Q(\rho^A, \rho^B)$, play the role of *quantum transport plans*. The *quantum optimal transport* problem seeks the minimal *quantum transport cost*,

$$T^Q(\rho^A, \rho^B) := \min_{\rho^{AB} \in \Gamma^Q(\rho^A, \rho^B)} \big(\mathrm{Tr}\, C\rho^{AB} \big), \tag{1}$$

for a suitable *quantum cost matrix* C^Q.

In the talk I will provide an overview of the quantum optimal transport problem basing on our recent works [8] and [9]. Therein we showed that the quantum transport cost has some exceptional properties if the quantum cost matrix is chosen to be the projector onto the antisymmetric subspace,

$$C^Q := \tfrac{1}{2}(\mathrm{id} - S) \tag{2}$$

where S is the SWAP operator $S(|x\rangle \otimes |y\rangle) = |y\rangle \otimes |x\rangle$.

Our key result is that the root of the quantum transport cost, $\sqrt{T^Q}$, with the cost matrix C^Q, is a unitarily invariant distance on the space of 2×2 density matrices. For $N \geq 3$, and more general quantum cost matrices, we proved that $\sqrt{T^Q}$ is a unitarily invariant semidistance, and we conjecture that it does satisfy the triangle inequality in general.

For $N = 2$ the solution to (1) is equivalent to solving a 6th order polynomial equation with roots on the unit circle. It leads to the conclusion that the analogue of the p-Wasserstein distance, $(T^Q)^{1/p}$, is indeed a distance for $p \geq 2$, but the triangle inequality fails for $p \in [1, 2)$. Moreover, closed formulas are available in some specific cases: if the density matrices ρ^A and ρ^B are (i) isospectral, or (ii) commute, or (iii) either of them is pure. Furthermore, the quantum transport cost T^Q is bounded from below by the rescaled Bures distance and from above by the rescaled root infidelity.

Finally, I will briefly mention the possible applications of the quantum optimal transport in quantum information processing.

Acknowledgements: Work supported by Foundation for Polish Science under the Team-Net Project No. POIR.04.04.00-00-17C1/18-00.

References

[1] C. Villani, *Optimal transport, Old and new*, Grundlehren der Mathematischen Wissenschaften, 338, Springer-Verlag, Berlin, 2009.

[2] J. Agredo and F. Fagnola, On quantum versions of the classical Wasserstein distance, *Stochastics* **89**:6-7 910 (2017).

[3] F. Golse and T. Paul, Wave packets and the quadratic Monge-Kantorovich distance in quantum mechanics, *Comptes Rendus Math.* **356**, 177 (2018).

[4] S. Friedland, Tensor optimal transport, distance between sets of measures and tensor scaling, preprint arXiv:2005.00945.

[5] Y. Chen, W. Gangbo, T. T. Georgiou, and A. Tannenbaum, On the matrix Monge-Kantorovich problem, *Eur. J. Appl. Math.* **31**, 574 (2020).

[6] G. De Palma and D. Trevisan, Quantum optimal transport with quantum channels, *Ann. Henri Poincaré* **22**, 3199–3234 (2021).

[7] E. A. Carlen and J. Maas, Non-commutative calculus, optimal transport and functional inequalities in dissipative quantum systems, *J. Stat. Phys.* **178**, 319 (2020).

[8] S. Friedland, S. Cole, M. Eckstein, and K. Życzkowski, Quantum optimal transport, *arXiv:2105.06922* (2021).

[9] S. Friedland, M. Eckstein, S. Cole, and K. Życzkowski Quantum Monge–Kantorovich problem and transport distance between density matrices, *Phys. Rev. Lett.* **129** 110403 (2022).

Tensor optimal transport, distance between sets of measures and tensor scaling

Shmuel Friedland

Department of Mathematics and Computer Science, University of Illinois at Chicago, Chicago, Illinois, 60607-7045, USA
E-mail: friedlan@uic.edu

Abstract

We discuss the optimal transport problem for $d > 2$ discrete measures. This is a linear programming problem on d-tensors. It gives a way to compute a "distance" between two sets of discrete measures. We introduce an entropic regularization term, which gives rise to a scaling of tensors. We give a variation of the celebrated Sinkhorn scaling algorithm. We show that this algorithm can be viewed as a partial minimization algorithm of a strictly convex function. Under appropriate conditions the rate of convergence is geometric, and we estimate the rate.

Acknowledgements: Work (partially) supported by the Simons Collaboration Grant for Mathematicians.

References

[1] S. Friedland, Tensor optimal transport, distance between sets of measures and tensor scaling, arXiv:2005.00945, 2020.

[2] T. Lin, N. Ho, M. Cuturi, and M.I. Jordan, On the Complexity of Approximating Multimarginal Optimal Transport, arXiv:1910.00152.

Turing-model complexity estimates in geometric programming and application to the computation of the spectral radius of nonnegative tensors

Shmuel Friedland[1], *Stéphane Gaubert*[2]

[1] *Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, Chicago, Illinois, USA*
E-mail: A, friedlan@uic.edu
[2] *INRIA and Centre de Mathématiques Appliquées (CMAP), Ecole polytechnique, IP Paris, UMR 7641 CNRS, 91128 Palaiseau, France*
E-mail: Stephane.Gaubert@inria.fr

Abstract

We show that a class of geometric programs, consisting in minimizing the maximum of finitely many log-Laplace transforms of nonnegative measures with finite support, can be solved in polynomial time in the Turing model of computation. To do so, we establish explicit bit-size estimates for near minimizers. Then, by exploiting variational formulations of the spectral radius of non-negative tensors in terms of geometric programs [2], we deduce that the spectral radius of a nonnegative tensor can be approximated within ε error in polynomial time, and that the maximum of a nonnegative homogeneous d-form in the unit ball with respect to d-Hölder norm can also be approximated in polynomial time. Relations of these results with entropy games [1], as well as alternative approaches to compute the spectral radius, will also be discussed. This talk is based on [3].

Acknowledgements: The work of the first author was partially supported by the Simons Collaboration Grant for Mathematicians. The work of the two authors was partially supported by the FAACTS (France and Chicago Collaborating in the Sciences) program.

References

[1] M. Akian, S. Gaubert, J. Grand-Clément, and J. Guillaud, The operator approach to entropy games, *Theor. Comp. Sys.* 63(5), 1089-1130, July 2019.

[2] S. Friedland and S. Gaubert, Inequalities for the spectral radius and spectral norms of nonnegative tensors, Vietnam Journal of Mathematics, Vietnam J. Math. 48 (2020), no. 4, special issue in the honor of Volker Mehrmann, 893-928.

[3] S. Friedland and S. Gaubert, Bit-complexity estimates in geometric programming, and application to the polynomial-time computation of the spectral radius of nonnegative tensors, arXIv:2301.10637, 2023.

Inevitability of knowing less than nothing

Gilad Gour[1], Mark Wilde[2], Sam Brandsen[3], Isabelle Jianing Geng[1]

[1] Department of Mathematics and Statistics, University of Calgary, Canada
E-mail: giladgour@gmail.com
[2] School of Electrical and Computer Engineering, Cornell University, USA
E-mail: mwilde@gmail.com
[3] Department of Physics, Duke University, USA
E-mail: sam.brandsen@duke.edu

Abstract

A colloquial interpretation of entropy is that it is the knowledge gained upon learning the outcome of a random experiment. Conditional entropy is then interpreted as the knowledge gained upon learning the outcome of one random experiment after learning the outcome of another, possibly statistically dependent, random experiment. In the classical world, entropy and conditional entropy take only non-negative values, consistent with the intuition that one has regarding the aforementioned interpretations. However, for certain entangled states, one obtains negative values when evaluating commonly accepted and information-theoretically justified formulas for the quantum conditional entropy, leading to the confounding conclusion that one can know less than nothing in the quantum world. In this talk I will introduce a physically motivated framework for defining quantum conditional entropy, based on two simple postulates inspired by the second law of thermodynamics (non-decrease of entropy) and extensivity of entropy, and I argue that all plausible definitions of quantum conditional entropy should respect these two postulates. I will then prove that all plausible quantum conditional entropies take on negative values for certain entangled states, so that it is inevitable that one can know less than nothing in the quantum world. All of my arguments are based on constructions of physical processes that respect the first postulate, the one inspired by the second law of thermodynamics. My talk is based on a joint work with Mark Wilde, Sarah Brandsen, and Isabelle Jianing Geng, and is available in [1].

Acknowledgements: Work (partially) supported by the Natural Sciences and Engineering Research Council of Canada (NSERC), the National Science Foundation (NSF) under Grant Nos. 1908730 and 1910571, and NSF under Grant No. 1907615.

References

[1] Gilad Gour, Mark M. Wilde, Sam Brandsen, and Isabelle Jianing Geng *Inevitability of knowing less than nothing*. eprint: arxiv.org/abs/2208.14424 (2022)

Hyperdeterminant, Fermionic Fock space and Entanglement

Frédéric Holweck

March 13, 2023

Abstract

In the past twenty years, classical algebraic invariants like Cayley generalization of the 2×2 determinant, i.e. the well known Cayley $2 \times 2 \times 2$-Hyperdeterminant, have been investigated in the quantum information litterature to help classify, measure and distinguish classes of entanglement for multipartite pure quantum states. In this talk, I'll explain how projective duality can help finding such algebraic invariants. In particular, I'll show how one obtained with Luke Oeding a polynomial invariant that generalize the $2 \times 2 \times 2 \times 2$ Hyperdeterminant and what is its interpretation in quantum information theory.

A new distance between pure qudits

Rafał Bistroń [1], *Michał Eckstein* [1], *Shmuel Friedland*[2], *Tomasz Miller* [3], *Karol Życzkowski* [1,4]

[1] *Faculty of Physics, Astronomy and Applied Computer Science, Jagiellonian University, Łojasiewicza 11, 30-348, Kraków, Poland*
E-mail: rafal.bistron@uj.edu.pl
E-mail: michal.eckstein@uj.edu.pl
[2] *Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, Chicago, Illinois, 60607-7045, USA*
E-mail: friedlan@uic.edu
[3] *Copernicus Center for Interdisciplinary Studies, Jagiellonian University, Szczepańska 1/5, 31-011, Kraków, Poland*
E-mail: tomasz.miller@uj.edu.pl
[4] *Center for Theoretical Physics, Polish Academy of Science, Warsaw, Poland*
E-mail: karol.zyczkowski@uj.edu.pl

Abstract

Recently, in the context of developing a "quantum" extension of the optimal transport theory of Monge, Kantorovich and Wasserstein, in [1] and [2] the following conjecture was posed.

Let $E = [E_{ij}]$ be any N-by-N distance matrix. On an N-dimensional Hilbert space $\mathcal{H}$ with some fixed orthonormal basis $\{|i\rangle\}$, define the *quantum cost matrix* $C_E \in \mathcal{B}(\mathcal{H} \otimes \mathcal{H})$ via

$$C_E := \tfrac{1}{2} \sum_{i<j} E_{ij}(|ij\rangle - |ji\rangle)(\langle ij| - \langle ji|)$$

and consider the *transport cost* between two state vectors $|\psi\rangle, |\phi\rangle \in \mathcal{H}$, $\|\psi\| = \|\phi\| = 1$ given by

$$T_E(|\psi\rangle, |\phi\rangle) := \langle \psi \otimes \phi | C_E | \psi \otimes \phi \rangle$$

In [2] it was proven that $d_E := \sqrt{T_E}$ is a semidistance, and it was conjectured to be an actual distance, i.e. to satisfy the triangle inequality

$$d_E(|\psi_1\rangle, |\psi_2\rangle) \leq d_E(|\psi_1\rangle, |\psi_3\rangle) + d_E(|\psi_3\rangle, |\psi_2\rangle)$$

for all normalized state vectors $|\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle \in \mathcal{H}$.

In my talk I will present some partial results of our joint work in progress on this conjecture. More concretely, I will discuss the case when E is a *Euclidean* distance matrix and the case when E_{ij}'s satisfy the stronger version of the triangle inequality: $E_{ij} \leq E_{kl} + E_{mn}$ for any distinct $\{i, j\}, \{k, l\}, \{m, n\}$ (where of course $i \neq j$, $k \neq l$ and $m \neq n$).

Acknowledgements: Work supported by Foundation for Polish Science under the Team-Net Project No. POIR.04.04.00-00-17C1/18-00.

References

[1] S. Friedland, S. Cole, M. Eckstein, and K. Życzkowski, Quantum optimal transport, *arXiv:2105.06922* (2021).

[2] S. Friedland, M. Eckstein, S. Cole, and K. Życzkowski Quantum Monge–Kantorovich problem and transport distance between density matrices, *Phys. Rev. Lett.* **129** 110403 (2022).

Operator Schmidt decomposition of flattenings of a 4-tensor: A solution of the quantum version of the Euler's problem of 36 officers

Karol Życzkowski[1] [a,b]

[a] *Jagiellonian University, ul. Łojasiewicza 11, 30-348, Kraków, Poland*
[b] *Center for Theoretical Physics, Polish Academy of Science, Warsaw, Poland*
E-mail: karol.zyczkowski@uj.edu.pl

Abstract

Any pure state of four-partite quantum system $ABCD$, written $|\psi\rangle \in \mathcal{H}_d^{\otimes 4}$, can be represented in a product basis by a 4–tensor, $|\psi\rangle = \sum_{ijk\ell=1}^{d} T_{ijk\ell}|i,j,k,\ell\rangle$. Three flattenings of the tensor produce matrices X, Y, Z of order d^2, which correspond to three different partition of the system: $AB|CD$, $AC|BD$ and $AD|BC$. The state $|\psi\rangle$ is *absolutely maximally entangled* (AME), if it is maximally entangled with respect to all three partitions, which is the case if X, Y, Z are unitary, so all components of their Schmidt vectors are equal. This is equivalent to the condition that the matrix X is 2-unitary, which means that its partial transpose, $Y = X^{\Gamma}$, and reshuffling, $Z = X^{R}$, are also unitary.

Negative solution to the famous problem of 36 officers of Euler implies that there are no two orthogonal Latin squares of order six. We show that the problem has a solution, provided the ranks and units of the officers are allowed to be entangled, and construct [1] orthogonal quantum Latin squares of order $d = 6$. The solution can be represented by a 2-unitary matrix $X = U_{36} \in U(36)$ and implies existence of an AME state of four subsystems with six levels each and a pure non-additive quhex quantum error detection code, which allows one to encode a 6-level state into a triple of such states [2].

References

[1] S.A Rather, A.Burchardt, W. Bruzda, G. Rajchel-Mieldzioć, A. Lakshminarayan and K. Życzkowski, Thirty-six entangled officers of Euler, *Phys. Rev. Lett.* **128**, 080507 (2022).

[2] D. Garisto, Eulers 243-Year-Old Impossible Puzzle Gets a Quantum Solution, Quanta Magazine, Jan. 10, 2022; https://www.quantamagazine.org/

[1] joint work with W. Bruzda, A. Burchardt, A. Lakshminarayan, G. Rajchel-Mieldzioć and S.A. Rather

MSC24. Representations of groups and algebras and related topics

Samuel Lopes, Ana Paula Santana

Shintani descent for supercharacters of finite algebra groups
Carlos André, Ana Branco-Correia, João Dias

Carnot graded Li algebras and chain ideal lattices
Pilar Benito, Jorge Roldán-López

Exact Borel subalgebras of stratified algebras
Teresa Conde, Julian Külshammer

Blocks of endomorphism algebras via quasi-hereditary algebras
Stephen Donkin

Representation theory of quantum algebras at roots of unity through linear algebra techniques
Stéphane Launois

Jordan type Artinian Gorenstein algebras and related invariants
Pedro Macias Marques, Anthony Iarrobino, Johanna Steinmeyer

The graphs of reduced words of a permutation
Ricardo Mamede, Gonçalo Gutierres, José Luis Santos

U(h)-free modules and weight representations
Eduardo Mendonça

Compression functor and Auslander-Reiten sequences
Maria José Souto Salorio

Shintani descent for supercharacters of finite algebra groups

Carlos A. M. André[1], Ana L. Branco-Correia[2], João Dias[3]

[1] Departamento de Matemática, Universidade de Lisboa & CEAFEL(FCUL), Portugal
E-mail: caandre@ciencias.ulisboa.pt
[2] Departamento de Ciências Exatas e Engenharias, Academia Militar & CEAFEL (FCUL), Portugal
E-mail: alcorreia@ciencias.ulisboa.pt
[3] Departamento de Matemática, Universidade de Lisboa & CEAFEL (FCUL), Portugal
E-mail: joaodias104@gmail.com

Abstract

Let $A(q)$ be a finite-dimensional nilpotent algebra over a finite field $\mathbb{F}_q$ with q elements, and let $G(q) = 1+A(q)$. On the other hand, let $\mathbb{K}$ denote the algebraic closure of $\mathbb{F}_q$, and let $A = A(q) \otimes_{\mathbb{F}_q} \mathbb{K}$. Then, $G = 1 + A$ is an algebraic group over $\mathbb{K}$ equipped with an $\mathbb{F}_q$-rational structure given by the usual Frobenius map $F\colon G \to G$, and $G(q)$ can be regarded as the fixed point subgroup G^F. For every $n \in \mathbb{N}$, the nth power $F^n\colon G \to G$ is also a Frobenius map, and G^{F^n} identifies naturally with $G(q^n) = 1 + A(q^n)$ where $A(q^n) = A(q) \otimes_{\mathbb{F}_q} \mathbb{F}_{q^n}$. The Frobenius map restricts to a group automorphism $F\colon G(q^n) \to G(q^n)$, and hence it acts on the set of irreducible characters of $G(q^n)$. Shintani descent provides a method to compare F-invariant irreducible characters of $G(q^n)$ and irreducible characters of $G(q)$. It also provides a uniform way of studying supercharacters of $G(q^n)$ for $n \in \mathbb{N}$. These groups form an inductive system with respect to the inclusion maps $G(q^m) \hookrightarrow G(q^n)$ whenever $m \mid n$, and this fact allows us to study all supercharacter theories simultaneously, to establish connections between them, and to relate them to the algebraic group $G = \bigcup_{n\in\mathbb{N}} G(q^n)$. Indeed, Shintani descent permits the definition of a certain *"superdual algebra"* which encodes information about the supercharacters of $G(q^n)$ for $n \in \mathbb{N}$.

Acknowledgements: Work partially supported by the Portuguese Science Foundation (FCT) through the Strategic Projects UID/MAT/04721/2013 and UIDB/04721/2020.

References

[1] P. Diaconis, I.M. Isaacs. Supercharacters and superclasses for algebra groups. *Trans. Amer. Math. Soc.*, 360, 5: 2359–2392 (2008).

[2] G. Lusztig. Character sheaves and generalizations. In *The unity of mathematics*. vol. 244 of *Progr. Math.*. Birkhäuser Boston, Boston, MA, 2006.

[3] T. Shoji. Shintani descent for algebraic groups over a finite field I. *J. Algebra*. 145, 2: 468–524 (1992).

Carnot graded Lie algebras and chain ideal lattices

Pilar Benito[1], *Jorge Roldán-López*[1],

[1] *Departamento de Matemáticas y Computación , University of La Rioja, Spain*
E-mail: pilar.benito@unirioja.es; jorge.roldanl@unirioja.es

Abstract

Over algebraically closed fields of characteristic zero, Heissenberg type algebras, filiforms and finite-dimensional thin algebras are the only possible varieties of algebras which appear as nilradicals of solvable Lie algebras in which the lattice of ideals is a n-string [1]. All this algebras admit a positive natural grading of Carnot type [3]. This graded pattern also follows for non-solvable Lie algebras. Along this talk we will present some parametric families of Carnot graded Lie algebras (firstly introduced as quasi-cyclic by Leger in [4]) that appear as nilradical of mixed Lie algebras with chain ideal lattices. This talk is based on the preprint [2].

Acknowledgements: The authors have been supported by research grant MTM2017-83506-C2-1-P of 'Ministerio de Economía, Industria y Competitividad, Gobierno de España' (Spain) until 2022 and by grant PID2021-123461NB-C21, funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe" since then. J. Roldán-López was also supported by a predoctoral research grant FPI-2018 of 'Universidad de La Rioja'.

References

[1] P. Benito. Lie algebras in which the lattice formed by the ideals is a chain. *Communications in Algebra* , 20 (1): 93-108 (1992).

[2] P. Benito, J. Roldán-López. Lie structures and chain ideals lattices. https://doi.org/10.48550/arXiv.2212.01466

[3] Y. Cornulier. Gradings on Lie algebras, systolic growth, and cohopfian properties of nilpotent groups. *Bulletin de la Société Mathématique de France*, 144(4): 693–744 (2016).

[4] G. F. Leger. Derivations of Lie algebras iii. *Duke Mathematical Journal*, 30 (4): 637–645 (1963).

Exact Borel subalgebras of stratified algebras

Teresa Conde[1], *Julian Külshammer*[2]

[1] *Institute of Algebra and Number Theory, University of Stuttgart, Germany*
E-mail: tconde@mathematik.uni-stuttgart.de
[2] *Department of Mathematics, Uppsala University, Sweden*
E-mail: julian.kuelshammer@math.uu.se

Abstract

Standardly stratified algebras are an axiomatic abstraction of concepts and phenomena in Lie theory. The class of standardly stratified algebras includes all quasi-hereditary algebras, so in particular, blocks of the Bernstein–Gelfand–Gelfand category $\mathcal{O}$ of a complex semisimple Lie algebra, Schur algebras and all algebras of global dimension at most 2. Exact Borel subalgebras of standardly stratified algebras are the counterpart of Borel subalgebras of complex semisimple Lie algebras.

Not every standardly stratified algebra A has an exact Borel subalgebra. However, a theorem by Koenig, Külshammer and Ovsienko ([5]), generalised by Bautista, Pérez and Salmerón ([1]) and by Goto ([4]), states that there always exists a standardly stratified algebra Morita equivalent to A that has an exact Borel subalgebra. Such results establish the existence of exact Borel subalgebras "up to equivalence", but determining these subalgebras and their embeddings into stratified algebras is a complex problem.

The aim of this talk is twofold. First, we shall see that exact Borel subalgebras and standardly stratified algebras containing them are unique in precise circumstances ([6, 2]), and I will provide techniques to deduce information about these algebras ([2]). Secondly, I will explain how the results presented are compatible with the recursive nature of standardly stratified algebras ([3]).

Acknowledgements: Work supported by the Deutsche Forschungsgemeinschaft through the grant KO 1281/18 1+2.

References

[1] R. Bautista, E. Pérez, L. Salmerón, *Homological systems and bocses*, Journal of Algebra **617** (2023), 192–274.

[2] T. Conde, *All quasihereditary algebras with a regular exact Borel subalgebra*, Advances in Mathematics **384** (2021), p. 45.

[3] T. Conde, J. Külshammer, *Exact Borel subalgebras, good quotients and good subalgebras*, in preparation.

[4] Y. Goto, *Generalizations of the relationship between quasi-hereditary algebras and directed bocses*, arXiv e-print 2202.00385 (2022), p. 21.

[5] S. Koenig, J. Külshammer, S. Ovsienko, *Quasi-hereditary algebras, exact Borel subalgebras, A_∞-categories and boxes*, Advances in Mathematics **262** (2014), 546–592.

[6] J. Külshammer, V. Miemietz, *Uniqueness of exact Borel subalgebras and bocses*, arXiv e-print 2109.03586 (2021), p. 191.

Blocks of endomorphism algebras via quasi-hereditary algebras

Stephen Donkin

Department of Mathematics, University of York , UK
E-mail: stephen.donkin@york.ac.uk

Abstract

Let S be a finite dimensional algebra over a field k and let e be a non-zero idempotent in S. Then $S_e = eSe$ has the structure of an algebra and the block structures of S and of S_e are related. If $L(\lambda)$, $\lambda \in \Lambda$, is a labelling of the simple S-modules then $eL(\lambda)$, $\lambda \in \Lambda_e$ is a labelling of the simple S_e-modules, where $\Lambda_e = \{\lambda \in \Lambda \,|\, eL(\lambda) \neq 0\}$ (see [3]). Viewing a block of S as a subset of Λ and a block of S_e as a subset of Λ_e we are interested in situations in which for each block $\mathcal{B}$ of S the intersection $\mathcal{B} \bigcap \Lambda_e$ is a block of S_e (or empty). The classical case is that in which S is a Schur algebra $S(n,r)$, with $n \geq r$, and S_e is the group algebra of the symmetric group of degree r. In this case the blocks (of S and of S_e) are described by the so-called Nakayama Conjecture (long since proved). We consider more generally situations in which this is true with S a quasi-hereditary endomorphism algebra of a module X (over some finite dimensional k-algebra) and S_e is the endomorphism algebra of a summand of X.

References

[1] Stephen Donkin *On the blocks of endomorphism algebras of certain quantised Young permutation modules* , preprint, (2023).

[2] Stephen Donkin *Distributions on quantum general linear groups and applications*, preprint, (2023).

[3] J. A. Green, Polynomial Representations of GL_n. Second Edition with an Appendix on Schenstead Correspondence and Littelmann Paths by K. Erdmann, J. A. Green and M. Schocker, *Lecture Notes in Mathematics.*, 830, (2007).

Representation theory of quantum algebras at roots of unity through linear algebra techniques

Jason P. Bell[1], *Stéphane Launois*[2], *Samuel A. Lopes*[3], *Alexandra Rogers*[4]

[1] *Department of Pure Mathematics, University of Waterloo, Canada*
E-mail: jpbell@uwaterloo.ca
[2] *School of Mathematics, Statistics and Actuarial Science, University of Kent, UK*
E-mail: S.Launois@kent.ac.uk
[3] *CMUP, Universidade do Porto, Portugal*
E-mail: slopes@fc.up.pt
[4] *E-mail:* alex@hence.ai

Abstract

In this talk, I will discuss linear algebra techniques to study irreducible representations of various quantum algebras at roots of unity. Our main example will be quantum Schubert varieties in the quantum grassmannian. This is based on joint work with Jason Bell, Samuel Lopes and Alexandra Rogers.

Acknowledgements: Work (partially) supported by NSERC grant RGPIN-2022-02951 and EPSRC grant EP/R009279/1.

References

[1] J. P. Bell, S. Launois, A. Rogers. *PI Degree and Irreducible Representations of Quantum Determinantal Rings and their Associated Quantum Schubert Varieties*. arXiv:2212.03799

[2] S. Launois, S. A. Lopes, A. Rogers. *A Deleting Derivations Algorithm for Quantum Nilpotent Algebras at Roots of Unity*. In preparation.

Jordan type Artinian Gorenstein algebras and related invariants

Anthony Iarrobino[1], *Pedro Macias Marques*[2], *Johanna Steinmeyer*[3]

[1] *Mathematics Department, Northeastern University, USA*
E-mail: a.iarrobino@northeastern.edu
[2] *Departamento de Matemática, Universidade de Évora, Portugal*
E-mail: pmm@uevora.pt
[3] *Institut for Matematiske Fag, Københavns Universitet , Danmark*
E-mail: jks@math.ku.dk

Abstract

We study local Artinian Gorenstein (AG) algebras and consider the set of Jordan types of elements of the maximal ideal, i.e. the partition giving the Jordan blocks of the respective multiplication map.

In a joint work with Tony Iarrobino, we construct examples of families $\mathrm{Gor}(T)$ of local AG algebras with given Hilbert function T, and use obstructions that the symmetric decomposition of the associated graded algebra of an AG algebra A imposes on the Jordan type of A to study their irreducible components. Together with Johanna Steinmeyer, we explore possible generalisations of Jordan type and try to understand how they might apply to these topological questions.

Acknowledgements: Work partially supported by FCT – Fundação para a Ciência e a Tecnologia (UIDB/04674/2020)

The graphs of reduced words of a permutation

Gonçalo Gutierres[1], *Ricardo Mamede*[1]
José Luis Santos[1]

[1] *University of Coimbra, CMUC, Department of Mathematics*
E-mail: ggutc@mat.uc.pt, mamede@mat.uc.pt, zeluis@mat.uc.pt

Abstract

Any permutation w of the symmetric group can be generated by a product of adjacent transpositions. The products of minimal length for w are called reduced words, and we can consider the graph $G(w)$ whose vertices are reduced words for w and whose edges are braid relations. We establish a statistic on $R(w)$, inducing a rank poset structure on the graph $G(w)$. This statistic allows to prove a conjecture made by Reiner and Roichman on bounds for the diameter of $G(w)$, and to compute the diameter of $G(w)$ for certain permutations w. The diameter for the associated graphs $C(w)$ and $B(w)$, obtained from $G(w)$ by contracting the commutations moves, or the long braid moves, respectively, are also considered. We recover, as special cases, the diameter of the commutation graph for the longest element of the symmetric group and the characterization of fully commutative permutations obtained by S. Billey, W. Jockusch and R. Stanley.

Acknowledgements: This work was partially supported by the Centre for Mathematics of the University of Coimbra - UIDB/00324/2020, funded by the Portuguese Government through FCT/MCTES

References

[1] S. Billey, W. Jockusch, and R. P. Stanley (1993). Some combinatorial properties of Schubert polynomials, Journal of Algebraic Combinatorics, Vol. 2, 345–374.

[2] Gutierres, G., Mamede, R., Santos, J.L., 2020. Commutation classes of the reduced words for the longest element of Sn. The Electronic Journal of Combinatorics 27(2), P2.21.

[3] Gutierres, G., Mamede, R., Santos, J.L., 2022. Diameter of the commutation classes graph of a permutation. European Journal of Combinatorics 103, Paper No.103525.

[4] V. Reiner and Y. Roichman (2013). Diameter of graphs of reduced words and galleries, Trans. Amer. Math. Soc., 365(5), 2779–2802.

$\mathcal{U}(\mathfrak{h})$-free modules and weight representations

Eduardo Monteiro Mendonça[1]

[1] *Institute of Mathematics and Statistics, University of São Paulo, Brazil*
E-mail: edummend@ime.usp.br

Abstract

Lie algebras and their representations appear throughout multiple areas of mathematics, and the classification of simple modules is an important first step to understand the theory. Unfortunately, a complete classification of simple modules for a Lie algebra $\mathfrak{g}$ seems beyond reach: it is complete only for the Lie algebra $\mathfrak{sl}(2)$ (due the work of R.Block [1]). However, some classes of simple $\mathfrak{g}$-modules are well understood. For example, in the case of simple complex finite-dimensional Lie algebras, irreducible representations with finite-dimensional weight spaces were classified due to the efforts of S.Fernando [2] and the O. Mathieu [3].

Classes of non-weight modules have also been studied, such as Whittaker modules and Gelfand-Zetlin modules. Recently such class has drawn attention in the community: the category of $\mathfrak{h}$-free modules, i.e, of $\mathfrak{g}$-modules on which the Cartan subalgebra $\mathfrak{h}$ acts freely. Still, the complete classification of $\mathfrak{h}$-free modules of finite-rank seems to be a hard project and the only known case is when the rank equals one ([4] and [5]).

Surprisingly, in spite of being of a completely opposite nature, weight modules and $\mathfrak{h}$-free modules carry interesting connections. These connections are mostly obtained due to the *weighting functor* $\mathcal{W}$. As the name suggests, the functor $\mathcal{W}$ assigns to a $\mathfrak{h}$-free module M a weight module $\mathcal{W}(M)$. And in fact the assignment is stronger: $\mathcal{W}(M)$ is a *coherent family* - a very large weight module whose support coincides with whole $\mathfrak{h}^*$. Coherent families played a crucial role in Mathieu's breakthrough paper [3], where he obtains an algebraic construction, geometric realizations and a complete classification of the semissimple irreducible ones. Therefore such relations hint that a deeper study of coherent families and the weighting functor can bring important results for both classes of representations.

Based on the PhD project of the speaker, this talk intends to illustrate interesting properties of the weighing functor and its applications. In fact, by constructing and studying the left derived functors of $\mathcal{W}$, we are able to show that any simple infinite-dimensional $\mathfrak{g}$-module M that is $\mathcal{U}(\mathfrak{h})$-finitely generated is $\mathcal{U}(\mathfrak{h})$-locally free in all maximal ideals of $\mathcal{U}(\mathfrak{h})$, except for a finite set. Moreover, we can see that $\mathcal{W}(M)$ is very close to being a coherent family, which inspires us to define a generalization of coherent families that appears to have very similar properties to the original ones.

Acknowledgements: Work supported by São Paulo Research Foundation (FAPESP), grants #2020/14313-4 and #2022/05915-6

References

[1] Richard E. Block. The irreducible representations of the Lie algebra $sl(2)$ and of the Weyl algebra. *Adv. in Math.*, 39(1):69–110, 1981.

[2] S. L. Fernando. Lie algebra modules with finite-dimensional weight spaces. I. *Trans. Amer. Math. Soc.*, 322(2):757–781, 1990.

[3] Olivier Mathieu. Classification of irreducible weight modules. *Ann. Inst. Fourier (Grenoble)*, 50(2):537–592, 2000.

[4] Jonathan Nilsson. Simple $sl(n+1)$-module structures on $U(h)$. *Journal of Algebra*, 424:294–329, 2015.

[5] Jonathan Nilsson. $U(h)$-free modules and coherent families. *Journal of Pure and Applied Algebra*, 220(4):1475–1488, 2016.

Compression of bounded complexes and Auslander-Reiten sequences

Claudia Chaio[1], *Alfredo G. Chaio*[2], *Isabel Pratti*[3], *María José Souto Salorio*[4]

[1 2 3] *Universidad Nacional de Mar del Plata, Argentina*
[4] *Facultade de Informática, Universidade da Coruña, España*
E-mail: maria.souto.salorio@udc.es

Let A be a finite dimensional algebra with finite global dimension. In [6], we found a link between the category of periodic complexes and the Happel's root category using the compression functor. Related to these ideas, in [3] and [4] the authors described the almost split sequences for periodic complexes, in the case that A is a hereditary algebra. In this talk, we present some properties concerning the compression functor of bounded complexes over a more general context. In particular, we prove that it preserves indecomposable complexes and irreducible morphisms. The covering notion given in [1] and the complexes of fixed size studied in [2] will play a relevant role in our work.

Acknowledgements: Work (partially) supported by MICINN, PID2020-113230RB-C21

References

[1] R. Bautista, S. Liu. Covering theory for linear categories with application to derived categories. *Journal of Algebra* 406: 173–225 (2014).

[2] R. Bautista, M. J. Souto Salorio, R. Zuazua. Almost split sequences for complexes of fixed size. *Journal of Algebra,* 287: 140–168 (2005).

[3] Q. Chen, B. Deng. Cyclic complexes, Hall polynomials and simple Lie algebras. *Journal of Algebra.* 440: 1–32 (2015).

[4] Q. Chen. Almost split sequences of $m-$cyclic complexes. *Arch. Math.* 119: 569–581 (2022).

[5] D. Happel. *Triangulated categories in the representation theory of finite dimensional algebras.* London Math. Soc. Lecture Note Ser. 119, Cambridge, 1988.

[6] L. Peng, J. Xiao. Root Categories and Simple Lie Algebras. *Journal of Algebra.* 198: 19–56 (1997).

[7] T. Stai. The triangulated hull of periodic complexes. *Math. Res. Lett 25,* 1: 199–236 (2018).

MSC25. Solving matrix and tensor equations

Qingwen Wang, Yang Zhang,
Dragana Cvetkovic Ilic

Minusymposium
Dragana Cvetkovic Ilic

Galois group actions and rational solutions of p(X) = A.
Gerrit Goosen, Andre Ran, Gilbert Groenewald. Madelein van Straaten, Frieda Madelein, Dawie Janse van Rensburg

Automated proofs of operator statements
Clemens Hofstadler, Clemens Raab, Georg Regensburger

Trace Minimization Principles
Ren-Cang LI, Xin Liang

The η-(anti-)Hermitain solution to a constrained Sylvester-type matrix equation over the generalized commutative quaternions
Qing-Wen Wang, Xue-Ying Chen

Singular value decomposition of commutative quaternion tensors
Yang Zhang, Xin Liu Cui-E Yu

Completion of operator matrices with application to solving operator equations

Dragana Cvetković Ilić

Department of mathematics, College of Science, Kuwait University, Kuwait
E-mail: dragana.ilic@ku.edu.kw

Abstract

We will discuss certain problems on completions of different types of operator matrices to the classes of Fredholm, Weyl, Browder and closed range operators. Using these results we will give certain necessary and sufficient conditions for the existence of a solution of certain operator equations that belong to mentioned classes of operators. Also, we will discuss some reverse order law improvements and applications in completion problems. In this talk we will answer an open question of the existence of a positive solution of the operator equation $AXB = C$ without any additional range or regularity assumptions using two well-known results of Douglas and Zoltán. Also we will give a general form of a positive solution and consider some possible applications.

References

[1] D. Cvetković Ilić and J. Milenković, Improvements on the reverse order laws, Math. Nachr. (2023), 1-9. https://doi.org/10.1002/mana.202100661

[2] D. Cvetković Ilić, *Completion problems on operator matrices*, Amer. Math. Soc., Series: Mathematical Surveys and Monographs, Vol. 267, 2022.

Galois group actions and rational solutions of $p(X) = A$.

André Ran[1], *Dawie Janse van Rensburg*[3], *Frieda Theron*[2], *Gerrit Goosen*[3], *Gilbert Groenewald*[3], *Madelein van Straaten*[3]

[1] *Vrije Universiteit Amsterdam, Nederland*
E-mail: a.c.m.ran@vu.nl
[2] *Akademia, South Africa*
E-mail: FriedaT@akademia.ac.za
[3] *North-West University, South Africa*
E-mail: dawie.jansevanrensburg@nwu.ac.za gerrit.goosen@nwu.ac.za
gilbert.groenewald@nwu.ac.za madelein.vanstraaten@nwu.ac.za

Abstract

The solutions of the matrix polynomial equation $p(X) = A$ over the complex field are well understood. The case where the underlying field is the rationals was studied by Drazin [1], who showed how to obtain exact rational solutions by a certain linearization procedure. Robert Reams [2] investigated the special case where $p(x) = x^m$ and m is odd, showing that the existence of a solution is equivalent to a certain condition on the size of the Galois groups of $f(x)$ and $f(p(x))$, where $f(x)$ is the irreducible characteristic polynomial of A. In this talk we show how the action of the Galois group of $f(x)$ on the roots of $f(p(x))$ "generates" the solutions obtained Drazin.

References

[1] Drazin, M.P. *Exact rational solution of the matrix equation $A = p(X)$ by linearization.* Linear algebra and its applications, 426(2-3), pp.502-515., 2007.

[2] Reams, R. *A Galois approach to mth roots of matrices with rational entries.* Linear algebra and its applications, 258, pp.187-194., 1997.

Automated proofs of operator statements

Clemens Hofstadler[1], *Clemens G. Raab*[2], *Georg Regensburger*[1]

[1] *Institute of Mathematics, University of Kassel, Germany*
E-mail: {clemens.hofstadler,regensburger}@mathematik.uni-kassel.de
[2] *Institute for Algebra, Johannes Kepler University Linz, Austria*
E-mail: clemensr@algebra.uni-linz.ac.at

Abstract

Linear operators appear in several forms in many different settings all across mathematics. They can be ring elements (as in C^*-Algebras), but also (rectangular) matrices, or, more generally, vector space and module homomorphisms. In this talk, we present a recently developed framework for efficiently proving statements about linear operators by verifying ideal membership of noncommutative polynomials [2]. More precisely, any statement about operator identities that can be phrased within first-order logic can be treated. Our main result is a semi-decision procedure that allows to automatically proof operator statements based on a single computation with noncommutative polynomials. The resulting proof is valid in all of the settings mentioned above. We also illustrate the framework by concrete examples, including recent work [1], and show how computer algebra software can be used to automatise computations.

Acknowledgements: The first author was supported by the Austrian Science Fund (FWF) grant P 32301. The second author was supported by the Austrian Science Fund (FWF) grant P 31952.

References

[1] Dragana S. Cvetković-Ilić, Clemens Hofstadler, Jamal Hossein Poor, Jovana Milošević, Clemens G. Raab, and Georg Regensburger. *Algebraic proof methods for identities of matrices and operators: improvements of Hartwig's triple reverse order law*. Applied Mathematics and Computation 409 (2021): 126357.

[2] Clemens Hofstadler, Clemens G. Raab, and Georg Regensburger. *Universal truth of operator statements via ideal membership*. arXiv preprint, arXiv:2212.11662, 2023.

Trace Minimization Principles

Xin Liang[1], *Ren-Cang Li*[3]

[1] *Yau Mathematical Sciences Center, Tsinghua University, Beijing 100084, China*
E-mail: liangxinslm@tsinghua.edu.cn
[3] *Department of Mathematics, University of Texas at Arlington, Arlington, TX 76019, USA*
E-mail: rcli@uta.edu

Abstract

Various trace minimization principles have interplayed with numerical computations for the standard eigenvalue and generalized eigenvalue problems in general, as well as important applied eigenvalue problems including the linear response eigenvalue problem from electronic structure calculation and the symplectic eigenvalue problem of positive definite matrices that play important roles in classical Hamiltonian dynamics, quantum mechanics, and quantum information, among others. In this talk, we will review these trace minimization principles and their most recent extensions that we have been working on.

Acknowledgements: Work (partially) supported by NSF grants DMS-1719620 and DMS-2009689.

References

[1] Zhaojun Bai and Ren-Cang Li. Minimization principle for linear response eigenvalue problem, I: Theory. *SIAM J. Matrix Anal. Appl.*, 33(4):1075–1100, 2012.

[2] Xin Liang, Ren-Cang Li, and Zhaojun Bai. Trace minimization principles for positive semi-definite pencils. *Linear Algebra Appl.*, 438:3085–3106, 2013.

[3] Xin Liang and Ren-Cang Li. Extensions of Wielandt's min-max principles for positive semi-definite pencils. *Lin. Multilin. Alg.*, 62(8):1032–1048, 2014.

[4] Ren-Cang Li. Rayleigh quotient based optimization methods for eigenvalue problems. In Zhaojun Bai, Weiguo Gao, and Yangfeng Su, editors, *Matrix Functions and Matrix Equations*, volume 19 of *Series in Contemporary Applied Mathematics*, pages 76–108. World Scientific, Singapore, 2015.

[5] Xin Liang, Li Wang, Lei-Hong Zhang, and Ren-Cang Li. On generalizing trace minimization principles. *Linear Algebra Appl.*, 656:483–509, 2023.

[6] Xin Liang and Ren-Cang Li. On generalizing trace minimization principles,

The η-(anti-)Hermitain solution to a constrained Sylvester-type matrix equation over the generalized commutative quaternions

Qing-Wen Wang, Xue-Ying Chen

Department of Mathematics, Shanghai University, Shanghai, China

E-mail: wqw@t.shu.edu.cn

Abstract

In this talk, we consider the general η-(anti-)Hermitian solution to the constrained Sylvester-type matrix equation over the generalized commutative quaternions

$$s.t. \quad \begin{cases} \sum_{s=1}^{n} E_s X_s F_s = H, \\ \begin{cases} A_1X_1 = P_1, \\ X_1B_1 = Q_1, \\ A_2X_2 = P_2, \\ X_2B_2 = Q_2, \\ \cdots \\ A_nX_n = P_n, \\ X_nB_n = Q_n, \end{cases} \end{cases} \tag{1}$$

where $A_s, P_s, B_s, Q_s, E_s, F_s$ and H are given matrices and X_s are unknowns.

We present some practical necessary and sufficient conditions for the existence of an η-(anti-)Hermitian solution to (1). We also provide the general η-(anti-)Hermitian solution to the constrained matrix equation when it is solvable. Moreover, we present algorithms and numerical examples to illustrate the results of this talk.

Singular value decomposition of commutative quaternion tensors

Cui-E Yu[1], *Xin Liu*[1], *Yang Zhang*[2]

[1] *Faculty of Innovation Engineering, Macau University of Science and Technology, Macau, China*
E-mail: cyu@must.edu.mo; xiliu@must.edu.mo
[2] *Department of Mathematics, University of Manitoba, Canada*
E-mail: yang.zhang@umanitoba.ca

Abstract In this talk, we discuss the singular value decomposition of a commutative quaternion tensor under some product. As applications, we give the expression of the Moore-Penrose inverse of a commutative quaternion tensor as well as some properties. Finally. we investigate the general solutions to some commutative quaternion tensor equations when they are consistent and the least-square solutions.

MSC26. Bohemian matrices and related topics in matrix theory

Robert M. Corless, Leili Rafiee Sevyeri,
George Labahn, Mark Giesbrecht

Searching for Rigidity in Algebraic Starscapes
Gabriel Dorfsman-Hopkins, Shuchang Xu

Numerical Examples on Backward Stability of Algebraic Linearizations
Eunice Y.S. Chan, Robert M. Corless

Bohemian Doubly Companion Matrices
Robert M. Corless

On the the eigenvalues of (Bohemian) Q-matrices and P-matrices
Laureano Gonzalez-Vega

Eigenvectors of the block Kronecker formulation of Mandelbrot matrices
Piers Lawrence

On the orthogonal decomposition of real square matrices over the co-Latin and semi-magic symmetry classes
Matthew Lettington, Nicholas Higham, Karl Michael Schmidt

On eigenvalue gaps of integer matrices
James Pommersheim, Aaron Abrams, Zeph Landau, Nikhil Srivastava

Computing the maximum spread of a Bohemian symmetric matrix with entrees in [a, b]
J. Sendra, R. M. Corless, L. Gonzalez-Vega, J. R. Sendra Pons

Inner Bohemian Matrices: a particular case of Generalized Inverse Bohemian Matrices
J. Sendra, L. Gonzalez-Vega, J.R. Sendra Pons

Searching for Rigidity in Algebraic Starscapes

Gabriel Dorfsman-Hopkins[1], *Shuchang Xu*[2]

[1] *Dpt. Math, Statistic, and Computer Science, Saint Lawrence University, Canton, NY, United States*
E-mail: gdhopkins@stlawu.edu
[2] *University of California, Berkeley, United States*
E-mail: candyxu@berkeley.edu

Abstract

We study plots of algebraic integers in the complex plane, so-called algebraic starscapes. Algebraic starscapes have a rich and collaborative history, bringing together pure and computational mathematics with digital art. As we explore the mysterious patterns lying in the images, we find deep relationships between geometry and arithmetic. In this talk we will cover some history of the visual geometry of algebraic starscapes, and then explore the effect of creating starscapes that emphasize points with fewer than the expected number of symmetries, exhibiting previously hidden geometries. Finally, we will explain how the resulting imagery informs and inspires research questions in algebraic number theory.

Acknowledgements: This work was partially by NSF grant DMS-1439786, which funded the ICERM semester entitled Illustrating Mathematics where this project began. It was also supported by NSF grant DMS-1646385 while Dorfsman-Hopkins was part of the Research Training Group in arithmetic geometry at the University of California, Berkeley.

References

[1] Dorfsman-Hopkins, G., Xu, S. (2022). Searching for rigidity in algebraic starscapes. Journal of Mathematics and the Arts, 16(1–2), 57–74.

[2] Harriss, E., Stange, K., Trettel, S. (2022). Algebraic number starscapes. Experimental Mathematics, 31(4) 1098-1149

Numerical Examples on Backward Stability of Algebraic Linearizations

Eunice Y.S. Chan[1], *Robert M. Corless*[2]

[1] *School of Medicine, The Chinese University of Hong Kong, Shenzhen, P.R. China*
E-mail: eunicechan@cuhk.edu.cn
[2] *Department of Computer Science, Western University, London, ON, Canada, and Cheriton School of Computer Science, University of Waterloo, Waterloo, ON, Canada*
E-mail: rcorless@uwo.ca

Abstract

A "linearization" of a matrix polynomial $\boldsymbol{P}(x) = \sum_{k=0}^{n} \boldsymbol{C}_k x^k$ where each coefficient $\boldsymbol{C}_k$ is a square matrix of dimension m by m is a pair of matrices $\boldsymbol{A}$, $\boldsymbol{B}$ usually of larger dimension, typically $d = mn$, for which several properties hold including that $\det \boldsymbol{P}(x) = \det(x\boldsymbol{B} - \boldsymbol{A})$. This means that the (generalized) eigenvalues of the linearization give us the (polynomial) eigenvalues of the original matrix polynomial. Finding such polynomial eigenvalues is of interest in several applications.

Recently we have introduced a new kind of linearization of matrix polynomials, which we termed "algebraic linearizations" [1]. Our formulation is chiefly useful for matrix polynomials that are or can be recursively constructed from lower-degree matrix polynomials: $\boldsymbol{h}(x) = x\boldsymbol{a}(x) \cdot \boldsymbol{b}(x) + \boldsymbol{c}$. In several instances this results in linearizations with lower *height*, that is maximal absolute value of any entry in the matrices. For matrices over the integers, for instance, this has resulted in more numerically stable linearizations.

In this talk we report on some experimental results, and show that the usual notion of pseudospectra of matrix polynomials needs to be extended and that new theorems are needed.

This work is related to work done jointly with Laureano Gonzalez-Vega, J. Rafael Sendra and Juana Sendra.

Acknowledgements: We acknowledge the support of Western University, The National Science and Engineering Research Council of Canada, the University of Alcalá, the Ontario Research Centre of Computer Algebra, and the Rotman Institute of Philosophy. Part of this work was developed while RMC and EYSC was visiting the University of Alcalá, in the frame of the project Giner de los Rios and Mitacs Globalink Research Award, respectively.

References

[1] EYS Chan, RM Corless, L Gonzalez-Vega, JR Sendra, and J Sendra. Algebraic linearizations of matrix polynomials. *Linear Algebra and its Applications*, 563:373–399, 2019.

Bohemian Doubly Companion Matrices

Robert M. Corless[1]

[1] *Editor-in-Chief, Maple Transactions*
E-mail: rcorless@uwo.ca

Abstract

"Doubly companion" matrices were introduced in a 1999 paper by Butcher and Chartier in order to help analyze certain Runge–Kutta methods, and later General Linear Methods, for numerically solving ordinary differential equations. Doubly companion matrices are not studied outside of this application, so far as I know, so this talk will include a discussion of their elementary properties. For concreteness, here is a four-by-four doubly companion matrix:

$$\begin{bmatrix} -\beta_1 & -\beta_2 & -\beta_3 & -\alpha_4-\beta_4 \\ 1 & 0 & 0 & -\alpha_3 \\ 0 & 1 & 0 & -\alpha_2 \\ 0 & 0 & 1 & -\alpha_1 \end{bmatrix}.$$

It was already known from the 1999 paper that if $a(\lambda) = \alpha_4\lambda^4 + \alpha_3\lambda^3 + \alpha_2\lambda^2 + \alpha_1\lambda + 1$ and $b(\lambda) = \lambda^4\beta_4 + \lambda^3\beta_3 + \lambda^2\beta_2 + \lambda\beta_1 + 1$, and $c(\lambda) = a(\lambda)b(\lambda)$ truncated to remove terms of degree 5 or higher, then the characteristic polynomial of this example matrix is $\lambda^4 c(1/\lambda)$, the reversal of $c(\lambda)$. This construction works for general dimension.

In this talk we explore doubly companion Bohemian matrices for various populations for the coefficients α_k and β_k, and present several new puzzles.

Acknowledgements: Work partially supported by NSERC and by the Spanish MICINN. This is joint work with many people, including some present at this session.

References

[1] Neil J. Calkin, Eunice Y. S. Chan, and Robert M. Corless, *Computational Discovery on Jupyter*, online at https://computational-discovery-on-jupyter.github.io/Computational-Discovery-on-Jupyter/ and to be published in book form by SIAM. See in particular unit 5, on Bohemian Matrices.

On the the eigenvalues of (Bohemian) Q-matrices and P-matrices

Laureano Gonzalez-Vega[1]

[1] *Department of Quantitative Methods, CUNEF Universidad, Spain*
E-mail: laureano.gonzalez@cunef.edu

Abstract

P-matrices are matrices all of whose principal minors are positive. Q-matrices are matrices whose sums of principal minors of the same order are all positive. A matrix is said to be a PM-matrix (resp. QM-matrix) if all its powers are P-matrices (resp. Q-matrices). A matrix family is called Bohemian if its entries come from a fixed finite discrete (and hence bounded) set, called the population, usually of integers.

First, we will fully characterise Bohemian P-matrices with population $\{-1, 1\}$. For size n, there are $2^{\frac{n(n-1)}{2}}$ such matrices and we show how to construct a P-matrix of size $n+1$ starting from a Bohemian P-matrices of size n.

Second, the study of the eigenvalues of these matrices (and some of its powers) brings in many cases open questions. For example we do not know if the eigenvalues of a matrix A such that A and A^2 are P-matrices necessarily have positive real parts or if the eigenvalues of a PM-matrix are necessarily positive (see [2, 3]). In order to get a complete answer to these questions we will fully characterise the real QM-matrices up-to size 5 and we characterise those real matrices A, 4×4, such that A and A^2 are Q-matrices but not all eigenvalues of A have positive real parts.

Acknowledgements: Partially supported by the grant PID2020-113192GB-I00/AEI/10.13039/501100011033 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish State Research Agency (Ministerio de Ciencia e Innovación).

References

[1] R. Corless. What can we learn from Bohemian Matrices? *Maple Transactions*, 1: 7–7 (2021).

[2] D. Hershkowitz, C. R. Johnson. Spectra of matrices with P-matrix powers. *Linear Algebra and its Applications*, 80: 159–171 (1986).

[3] D. Hershkowitz, N. Keller. Positivity of principal minors, sign symmetry and stability. *Linear Algebra and its Applications*, 364: 105–124 (2003).

Eigenvectors of the block Kronecker formulation of Mandelbrot matrices

Piers W. Lawrence[1]

[1] *EigenPoly, Belgium*
E-mail: piers.lawrence@eigenpoly.com

Abstract

This work explores the structure of the eigenvectors of Mandelbrot matrices [1], a Bohemian family of matrices with binary entries. Their eigenvalues are related to the periodic points of the Mandelbrot set, and can be computed efficiently using Krylov subspace methods due to the availability and sparseness of the LU decomposition of the residual pencil. The family of matrices are constructed recursively, and can be related to and seen as block Kronecker linearizations [2] of the underlying scalar Mandelbrot polynomials. Thus, the dual minimal basis relations used to construct these linearizations can give significant insight into the structure of the eigenvectors and can potentially be exploited in computations.

References

[1] N. J. Calkin, E. Y. Chan, R. M. Corless, D. J. Jeffrey, and P. W. Lawrence. A Fractal Eigenvector. *The American Mathematical Monthly*, 129(6): 503–523 (2022).

[2] F. M. Dopico, P. W. Lawrence, J. Pérez, and P. Van Dooren. Block Kronecker linearizations of matrix polynomials and their backward errors. *Numerische Mathematik* 140(2): 373–426 (2018).

On the orthogonal decomposition of real square matrices over the co-Latin and semi-magic symmetry classes.

N. J. Higham[1], *M. C. Lettington*[2], *K. M. Schmidt*[3]

[1] *Dpt. 1, Manchester University, UK*
E-mail: nick.higham@manchester.ac.uk
[2] *Dpt. 2, Cardiff University, UK*
E-mail: LettingtonMC@cardiff.ac.uk
[3] *Dpt. 3, Cardiff University, UK*
E-mail: SchmidtKM@cardiff.ac.uk

Abstract

For real $n \times n$ matrices, we identify a co-Latin symmetry space, orthogonal to the space of all $n \times n$ real matrices with constant row and column sum (semi-magic), with respect to the Frobenius norm. The co-Latin and semi-magic symmetry spaces form a superalgebra, yielding a unique decomposition over these symmetry spaces for every real square matrix.

Explicit construction formula are established for the co-Latin matrices which have at most rank 2, and can be expressed as the sum of the outer products of two vector pairs. We outline how the assumed decompositions lead to the identification of a quadratic form obstruction to the classical problem of factorising an integer matrix M as $M = Z^T Z$, with Z an integer matrix.

References

[1] N. J. Higham, M. C. Lettington, *Optimizing and factorizing the Wilson matrix*, Amer. Math. Monthly, 129 (5): 454-465 (2022).

[2] N. J. Higham, M. C. Lettington, K. M. Schmidt, *Integer matrix factorisations, superalgebras and the quadratic form obstruction*, Linear Algebra Appl., 622: 250-267 (2021).

[3] S. L. Hill, M. C. Lettington, K. M. Schmidt, *On superalgebras of matrices with symmetry properties*, Linear Multilinear Algebra, 66: 1538-1563 (2018).

On Eigenvalue Gaps of Integer Matrices

Aaron Abrams[1], *Zeph Landau*[2], *Jamie Pommersheim*[3], *Nikhil Srivastava*[4]

[1] *Washington and Lee University, USA*
E-mail: abramsa@wlu.edu
[2] *UC Berkeley, USA*
E-mail: zlandau@berkeley.edu
[3] *Reed College, USA*
E-mail: jamie@reed.edu
[4] *UC Berkeley, USA*
E-mail: nikhil@math.berkeley.edu

Abstract

Given an $n \times n$ matrix with integer entries in the range $[-h, h]$, how close can two of its distinct eigenvalues be?

Previous authors ([3, 4]) achieved a minimum gap of $h^{-O(n)}$. In this work, we give an explicit construction of matrices with entries in $[0, h]$ with two eigenvalues separated by at most $h^{-n^2/16+o(n^2)}$. Up to a constant in the exponent, this agrees with the known lower bound of $\Omega((2\sqrt{n})^{-n^2} h^{-n^2})$ [1]. Bounds on the minimum gap are relevant to the worst case analysis of algorithms for diagonalization and computing canonical forms of integer matrices (e.g. [2]).

In addition to our explicit construction, we show there are many matrices with a slightly larger gap of roughly $h^{-n^2/32}$. We also construct 0-1 matrices which have two eigenvalues separated by at most $2^{-n^2/64+o(n^2)}$.

Acknowledgements: The fourth author was supported by NSF Grant CCF-2009011

References

[1] K. Mahler. An inequality for the discriminant of a polynomial. *Michigan Math. J.* 11:3 257–262 (1964).

[2] P. Dey, R. Kannan, N. Ryder, and N. Srivastava. Bit complexity of Jordan normal form and spectral factorization. *arXiv:2109.13956*, (2021)

[3] J. H. Wilkinson. *The Algebraic Eigenvalue Problem.* Oxford University Press, 1988.

[4] B. Parlett and T.-T. Lu. Minimal eigenvalue separation. *Techinical report, UC Berkeley Dept. Math.* (1992).

Computing the maximum spread of a Bohemian symmetric matrix with entrees in $[a, b]$

R. M. Corless[1], *L. Gonzalez-Vega*[2], *J.R. Sendra* [3], *J. Sendra*[2]

[1] *School of Mathematical and Statistical Sciences, Western University, Canada*
E-mail: rcorless@uwo.ca
[2] *Departamento de Métodos Cuantitativos, CUNEF Universidad, Spain*
E-mail: laureano.gonzalez@cunef.edu & juana.sendra@cunef.edu
[3] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: rafael.sendra@uah.es

Abstract

The spread of an square matrix with entries in $\mathbb{C}$ is defined as the maximum of the distances among its eigenvalues. Different upper and lower bounds for the spread can be found in the literature (see e.g. [3] or [4]).

Let $S_n[a, b]$ denote the set of all $n \times n$ symmetric matrices with entries in the real interval $[a, b]$ and let $S_n\{a, b\}$ be the subset of $S_n[a, b]$ of Bohemian matrices with population $\{a, b\}$ (i.e. the entries of such a matrix are equal to a or b).

In [2], the following conjecture is proposed: *for $A \in S_n[a, b]$, the maximum spread in this class is attained by some $A \in S_n\{a, b\}$ with rank equal to* 2. We will show how interpreting this problem geometrically, via polynomial resultants and symbolic computing, one can prove this conjecture for $S_n[0, 1]$ when $n \leq 6$ and for $S_n[a, 1]$ when $n \leq 5$ and $a \in [-1, 1)$ which covers the general case for $S_n[a, b]$ (see [2]). In this way we improve the results in [1] where the previous conjecture was proved only for $S_3[0, 1]$.

Acknowledgements: Authors are partially supported by the grant PID2020-113192GB-I00/AEI/10.13039/501100011033 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish State Research Agency (Ministerio de Ciencia e Innovación).

References

[1] I. Biborski. Note on the spread of real symmetric matrices with entries in fixed interval. *Linear Algebra and its Applications*, 632:246-257 (2022).

[2] S.M. Fallat, J.J. Xing. On the spread of certain normal matrices. *Linear Multilinear Algebra*, 60:1391-1407 (2012).

[3] C.R. Johnson, R. Kumar and H. Wolkowicz. Lower bounds for the spread of a matrix. *Linear Algebra and its Applications*, 71:161-173 (1985).

[4] L. Mirsky, The spread of matrix. *Mathematica*, 3:127-130 (1956) .

Inner Bohemian Matrices: a particular case of Generalized Inverse Bohemian Matrices

L. Gonzalez-Vega[1], *J.R. Sendra* [2], *J. Sendra*[1]

[1] *Departamento de Métodos Cuantitativos, CUNEF Universidad, Spain*
E-mail: laureano.gonzalez@cunef.edu & juana.sendra@cunef.edu
[2] *Departamento de Física y Matemáticas, Universidad de Alcalá, Spain*
E-mail: rafael.sendra@uah.es

Abstract

This talk is framed in the intersection area of Bohemian matrices and Generalized inverses. Or maybe we should say, that is an attempt of the Bohemian foray into the realm of generalized inverses. More precisely, for certain type of structured $\{0, 1, -1\}$–matrices, we present a complete description of the inner Bohemian inverses over any population containing the set $\{0, 1, -1\}$. In addition, when the population is exactly $\{0, 1, -1\}$ we provide explicit formulas for the number of inner Bohemian matrices of this type of matrices.

Acknowledgements: Authors are partially supported by the grant PID2020-113192GB-I00/AEI/10.13039/501100011033 (Mathematical Visualization: Foundations, Algorithms and Applications) from the Spanish State Research Agency (Ministerio de Ciencia e Innovación).

References

[1] A. Ben-Israel, T. N. E. Greville, *Generalized Inverses: Theory and Applications, Second edition*, Springer, New York, 2003.

[2] E.Y.S. Chan, R.M. Corless, L. Gonzalez-Vega, J.R. Sendra, J. Sendra, S. E. Thornton, *Upper Hessenberg and Toeplitz Bohemians*. Linear Algebra Appl. 601 (2020) 372–100.

[3] R.M. Corless, S.E. Thornton, *The Bohemian eigenvalue project*. ACM Commun. Comput. Algebra 50 (2016) 158–160.

[4] E.Y.S. Chan, R.M. Corless, L. Gonzalez-Vega, J.R. Sendra, J. Sendra. *Inner Bohemian Inverses* (submitted to Applied Mathematics and Computation).

[5] P. S. Stanimirović, R. Behera, J. K. Sahoo, D. Mosic, J. R. Sendra, J. Sendra, A. Lastra. *Computing tensor generalized inverses via specialization and rationalization.* Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas (2021) 115:116 pp.1–16.

MSI01. Combinatorial matrices

Richard A. Brualdi, Geir Dahl

On weight partitions of graphs and their applications
Aida Abiad, Sjanne Zeijlemaker

Combinatorics behind signed graphs
Milica Andelic

New results on graph partition and Fiedler theory
Enide Andrade, Geir Dahl

Approximate Graph Colouring and Crystals
Lorenzo Ciardo, Standa Živný

Integer eigenvalues of n-Queens graph
Inês Serôdio Costa, Domingos M. Cardoso, Rui Duarte

Alternating Sign Matrices and Generalizations
Geir Dahl, Richard Brualdi

Rigid spectra - a surprising consequence of invertible subtrees
Seth Meyer, Shaun Fallat, H. Tracy Hall, Rupert Levene, Shahla Nasserasr, Polona Oblak, Helena Smigoc

Multiplicative structures generated by alternating sign matrices
Rachel Quinlan, Cian O'Brien

On weight partitions of graphs and their applications

Aida Abiad[1,2,3], *Sjanne Zeijemaker*[1]

[1] *Dpt. of Mathematics and Computer Science, Eindhoven University of Technology, The Netherlands*
E-mail: a.abiad.monge@tue.nl
[2] *Dpt. of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University, Belgium*
[3] *Dpt. of Mathematics and Data Science of Vrije Universiteit Brussel, Belgium, Belgium*

Abstract

Weight-equitable partitions of graphs, which are a natural extension of the well-known equitable partitions, have been shown to be a powerful tool to weaken the regularity assumption in several classic eigenvalue bounds. Weight-equitable partitions assign to each vertex $u \in V$ a weight that equals the corresponding entry ν_u of the Perron eigenvector ν. In this talk we will present some new algebraic characterizations and two new applications: a condition under which Hoffman's ratio bound can be improved, and a new version of the Expander Mixing Lemma for general graphs.

Acknowledgements: Work (partially) supported by the Dutch Research Council through the grant VI.Vidi.213.085 and by the Research Foundation Flanders through the grant 1285921N.

Combinatorics behind signed graphs

Milica Anđelić[1]

[1] *Department of Mathematics, Kuwait University, Kuwait*
E-mail: milica.andelic@ku.edu.kw

Abstract

We investigate generalized inverses of several matrices associated to signed graphs and their combinatorial interpretations. In particular, we focus on trees and unbalanced unicyclic graphs.

This is a joint work with S. Mallik (Northern Arizona University, USA) and A. Alazemi (Kuwait University, Kuwait).

New results on graph partition and Fiedler theory

Enide Andrade[1], *Geir Dahl* [2]

[1] *Dept. of Mathematics, University of Aveiro, Portugal*
E-mail: enide@ua.pt
[2] *Dept. of Mathematics, University of Oslo, Norway*
E-mail: geird@math.uio.no

Abstract

In this talk we recall the spectral partitioning method based on a Fiedler vector ([1]), i.e., an eigenvector corresponding to the second smallest eigenvalue of the Laplacian matrix of a graph. This problem corresponds to the minimization of a quadratic form associated with this matrix, under a certain constraint. We introduce a similar problem using the ℓ_1-norm to measure distances and compare the optimal solutions for both problems.

Acknowledgements: Work (partially) supported by Portuguese funds through the CIDMA - Center for Research and Development in Mathematics and Applications, and the Portuguese Foundation for Science and Technology (FCT-Fundação para a Ciência e a Tecnologia), within projects UIDB/04106/2020 and UIDP/04106/2020.

References

[1] M. Fiedler. A property of eigenvectors of nonnegative symmetric matrices and its application to graph theory. *Czech. Math. J.*, 25: 619–633 (1975).

Approximate Graph Colouring and Crystals

Lorenzo Ciardo[1], Standa Živný[2]

[1] University of Oxford, UK
E-mail: lorenzo.ciardo@cs.ox.ac.uk
[2] University of Oxford, UK
E-mail: standa.zivny@cs.ox.ac.uk

Abstract

The *approximate graph colouring* problem (AGC) asks to find a d-colouring of a graph that is promised to be c-colourable, for $3 \leq c \leq d$. Though conjectured to be NP-hard in 1976 [3], the complexity of AGC is still unknown. We show that AGC is not solved by the strongest known algorithm for this type of computational problems, which is based on a blend of linear programming and Gaussian elimination. To this end, we build a class of highly symmetric *crystal* tensors whose structure is able to fool the algorithm. In order to design crystals, we use multilinear algebra. In particular, we provide a combinatorial characterisation for *realisable systems of tensors*; i.e., families of low-dimensional integral tensors that can be realised as the projections of a single high-dimensional tensor.

This talk is based on the two papers [1, 2].

References

[1] L. Ciardo, S. Živný. Approximate graph colouring and crystals. In *Proceedings of the Annual ACM-SIAM Symposium on Discrete Algorithms (SODA'23)*, 2256–2267 (2023).

[2] L. Ciardo, S. Živný. Approximate graph colouring and the hollow shadow. To appear in *Proceedings of the Annual ACM Symposium on Theory of Computing (STOC'23)*, (2023).

[3] M.R. Garey, D.S. Johnson. The complexity of near-optimal graph coloring. *Journal of the ACM (JACM)*, 23(1), 43–49 (1976).

Integer eigenvalues of n-Queens graph

Domingos M. Cardoso[1], *Inês Serôdio Costa*[2], *Rui Duarte*[3]

[1] *Department of Mathematics, University of Aveiro, Portugal*
E-mail: dcardoso@ua.pt
[2] *Department of Mathematics, University of Aveiro, Portugal*
E-mail: inesserodiocosta@ua.pt
[3] *Department of Mathematics, University of Aveiro, Portugal*
E-mail: rduarte@ua.pt

Abstract

The n-Queens graph, $\mathcal{Q}(n)$, is the graph obtained from a $n \times n$ chessboard where each of its n^2 squares is a vertex and two vertices are adjacent if and only if they are in the same row, column or diagonal.

Several combinatorial properties of n-Queens graph are known, such as its clique number, vertex independence number, chromatic number and domination number. However, as far as we know, there are no published results concerning its spectral properties.

In this talk, we present the integer spectrum of $\mathcal{Q}(n)$, for $n \geq 4$. We prove that -4 is the least eigenvalue of $\mathcal{Q}(n)$ with multiplicity $(n-3)^2$ and $n-4$ is also an eigenvalue of $\mathcal{Q}(n)$ with multiplicity at least $\frac{n+1}{2}$ or $\frac{n-2}{2}$ when n is odd or even, respectively. Furthermore, when n is odd, the integers $-3, -2 \ldots, \frac{n-11}{2}$ and $\frac{n-5}{2}, \ldots, n-5$ are additional integer eigenvalues of $\mathcal{Q}(n)$ and a family of eigenvectors associated with them will be presented. Finally, conjectures about the multiplicity of the aforementioned eigenvalues and about the non-existence of any other integer eigenvalue will be stated.

Acknowledgements: This work is supported by the Center for Research and Development in Mathematics and Applications (CIDMA) through the Portuguese Foundation for Science and Technology (FCT - Fundação para a Ciência e a Tecnologia), reference UIDB/04106/2020. I.S.C. also thanks the support of FCT - Fundação para a Ciência e a Tecnologia via the Ph.D. Scholarship PD/BD/150538/2019.

References

[1] Paul J. Campbell, *Gauss and the eight queens problem: A study in miniature of the propagation of historical error*, Historia Mathematica, 1977, 397–404. URL: https://doi.org/10.1016/0315-0860(77)90076-3

[2] D. M. Cardoso, I. S. Costa, R. Duarte, *Sharp bounds on the least eigenvalue of a graph determined from edge clique partitions*, arXiv preprint, 2022. URL: https://arxiv.org/abs/2201.01224

[3] J. Bell, B. Stevens, *A survey of known results and research areas for n-queens*, Discrete Math, Volume 309, 2009, 1–31.

Alternating Sign Matrices and Generalizations

Richard A Brualdi[1], *Geir Dahl*[2]

[1] *Department of Mathematics, University of Wisconsin, USA*
E-mail: brualdi@math.wisc.edu
[2] *Department of Mathematics, University of Oslo, Norway*
E-mail: geird@math.uio.no

Abstract

Initially we recall the notion of alternating sign matrices (ASMs). Then we introduce a generalization of ASMs called multiASMs and develop some of their properties. Classes of multiASMs with specified row and column sum vectors R and S extend the classes of $(0,1)$-matrices with specified R and S. The special case when $R = S$ is a constant vector, in particular all 2's, is treated in more detail. We briefly discuss the polytope spanned by a class of multiASMs and a Bruhat order on a class of multiASMs.

References

[1] R.A. Brualdi, G. Dahl. Multi-Alternating Sign Matrices. Submitted.

Rigid spectra - a surprising consequence of invertible subtrees

Shaun M. Fallat[1], *H. Tracy Hall*[2], *Rupert H. Levene*[3], *Seth A. Meyer*[4], *Shahla Nasserasr*[5], *Polona Oblak*[6], *Helena Šmigoc*[3]

[1] *Department of Mathematics and Statistics, University of Regina, Canada*
E-mail: shaun.fallat@uregina.ca
[2] *Hall Labs, LLC, USA*
E-mail: h.tracy@gmail.com
[3] *School of Mathematics and Statistics, University College Dublin, Ireland*
E-mail: rupert.levene@ucd.ie
E-mail: helena.smigoc@ucd.ie
[4] *Mathematics Discipline, St. Norbert College, USA*
E-mail: seth.meyer@snc.edu
[5] *School of Mathematical Sciences, Rochester Institute of Technology, USA*
E-mail: shahla@mail.rit.edu
[6] *Faculty of Computer and Information Science, Faculty of Mathematics and Physics, University of Ljubljana and Institute of Mathematics, Physics, and Mechanics, Slovenia*
E-mail: polona.oblak@fri.uni-lj.si

Abstract

Consider a simple graph G and the class of symmetric matrices whose off diagonal zero-nonzero pattern agrees with the adjacency matrix of G, but whose diagonal is unrestricted. The inverse eigenvalue problem for graphs asks what multisets of eigenvalues can occur for matrices in this class. This talk will discuss a recent development, *the invertible subtrees lemma* which takes combinatorial information about the graph and gives structural results on matrices achieving certain kinds of high eigenvalue multiplicities. In particular, it is one of the necessary ingredients to prove that there exist trees that have realizable unordered multiplicity lists that can only be achieved by a unique list of eigenvalues (up to shifting and scaling).

Acknowledgements: The authors are grateful to the organizers of the American Institute of Mathematics Research Community on the *Inverse eigenvalue problem for graphs*. Shaun Fallat's research is in part supported by an NSERC Discovery Grant, RGPIN–2019–03934. Polona Oblak received funding from Slovenian Research Agency (research core funding no. P1-0222 and project no. J1-3004).

References

[1] Shaun M. Fallat, H. Tracy Hall, Rupert H. Levene, Seth A. Meyer, Shahla Nasserasr, Polona Oblak, and Helena Šmigoc. Spectral arbitrariness for trees fails spectacularly. https://doi.org/10.48550/arxiv.2301.11073, 2023.

Multiplicative structures generated by alternating sign matrices

Cian O'Brien[1], *Rachel Quinlan*[2]

[1] *Department of Mathematics and Computer Studies, Mary Immaculate College, Limerick, Ireland*
E-mail: Cian.OBrien@mic.ul.ie
[2] *School of Mathematical and Statistical Sciences, University of Galway, Ireland*
E-mail: rachel.quinlan@universityofgalway.ie

Abstract

An alternating sign matrix (ASM) is a square $(0, 1, -1)$-matrix in which the row and column sums are all 1, and the non-zero entries alternate in sign through each row and column. ASMs include permutation matrices, and generalize permutation matrices in several precise and apparently distinct ways.

Unlike permutation matrices, ASMs may be singular. Also unlike permutation matrices, nonsingular ASMs typically have infinite order in the general linear group. Nevertheless, non-permutation ASMs of finite multiplicative order do exist. This talk will present some constructions of such matrices, and consider the question of how many distinct powers of a (non-permutation) ASM may themselves be ASMs.

References

[1] C. O'Brien, R. Quinlan. *Alternating sign matrices of finite multiplicative order. Linear Algebra Appl.*, 651: 332–358 (2022).

[2] R. Brualdi, J.Cooper. *Note on the spectral radius of alternating sign matrices. Linear Algebra Appl.*, 442: 99–105 (2014).

MSI02. Low-rank matrices and tensors: algorithms and applications

Dmitry Savostyanov, Sergei Dolgov

A Nyström-like randomized algorithm for low-rank approximation of tensors
Alberto Bucci, Leonardo Robol

Low-rank nonnegative matrix and tensor approximations: alternating projections and how to make them faster
Stanislav Budzinskiy, Sergey Matveev

Deep Importance Sampling Using Tensor Approximations
Sergey Dolgov, Tiangang Cui, Robert Scheichl

Low-rank tensor frames for the high-accuracy solution of elliptic and parabolic PDEs
Vladimir Kazeev

A weighted subspace exponential kernel for support tensor machines
Kirandeep Kour, Sergey Dolgov, Peter Benner, Martin Stoll, Max Pfeffer

Submatrices with the best-bounded inverses: revisiting the hypothesis
Yuri Nesterenko

Learning Feynman diagrams with tensor trains
Yuriel Nunez Fernandez, Matthieu Jeannin, Philipp Dumitrescu, Thomas Kloss, Jason Kaye, Olivier Parcollet, Xavier Waintal

Empirical Tensor Train Approximation in Optimal Control
Mahias Oster, Reinhold Schneider

A simple yet effective tensor-based ODE model for Deep Learning
Davide Palitta, Jemima Tabeart, Nick Vannieuwenhoven

A statistical POD approach for feedback boundary optimal control in fluid dynamics
Luca Saluzzi, Sergey Dolgov, Dante Kalise

Tensor product algorithms for Bayesian inference of networks from epidemiological data
Dmitry Savostyanov, Sergey Dolgov

Adaptive Undersampling in Spectromicroscopy
Oliver Townsend

A Nyström-like randomized algorithm for low-rank approximation of tensors.

Alberto Bucci[1]

[1] *University of Pisa, Italy.*
E-mail: alberto.bucci@phd.unipi.it

Abstract

Several applications lead to problems involving data or solutions that can be represented by tensors. Dealing with these multidimensional arrays is often prohibitively expensive and some type of compression in data-sparse formats has to be performed [1].

For this aim several low rank tensor factorizations have been proposed. In this work, we focus on the Tucker decomposition.

We generalize to tensors the method in [2] providing a randomized algorithm for the computation of the Tucker Decomposition: Multilinear Nyström method.

The accuracy of the approximation is close to the optimal truncated Higher Order SVD; moreover from a complexity point of view is cheaper than existing alternatives [3].

We discuss the advantages and disadvantages of imposing a Kronecker structure on the vectors chosen for the random sampling, and suggest good choices for such vectors depending on the available representation for the initial tensor data.

References

[1] T. Shi, A. Townsend *On the compressibility of tensors*. SIAM Journal on Matrix Analysis and Applications, 42(1):275–298, 2021.

[2] Y. Nakatsukasa *Fast and stable randomized low-rank matrix approximation*, arXiv preprint arXiv:2009.11392, 2020.

[3] S. Ahmadi-Asl, S. Abukhovich, M. G. Asante-Mensah, A.Cichocki, A. H. Phan, T. Tanaka, I. Oseledets. *Randomized Algorithms for Computation of Tucker Decomposition and Higher Order SVD (HOSVD)*.IEEE Access, 9:28684–28706, 2021.

Low-rank nonnegative matrix and tensor approximations: alternating projections and how to make them faster

Stanislav Budzinskiy[1], *Sergey Matveev*[2,3]

[1] *Faculty of Mathematics, University of Vienna, Austria*
E-mail: stanislav.budzinskiy@protonmail.ch
[2] *Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, Russia*
[3] *Marchuk Institute of Numerical Mathematics RAS, Russia*
E-mail: matseralex@cs.msu.ru

Abstract

Matrices and tensors with nonnegative entries are common in scientific computing and data analysis. Their low-rank approximations, however, often contain negative elements, which can cause numerical instabilities (e.g., when solving kinetic equations) or make the object lose its physical meaning as a whole (e.g., when it represents a probability distribution or a multispectral image). In the framework of *nonnegative matrix/tensor factorizations* [1], one specifically chooses nonnegative low-rank factors. While this eliminates the problem of negative entries, the nonnegative rank can be significantly larger than the usual numerical rank.

A more flexible approach was proposed in [2], where the factors are allowed to be arbitrary, but are optimized to make the low-rank matrix nonnegative. This can be achieved with *alternating projections* [3]: starting from some low-rank approximation of a nonnegative matrix, it is iteratively projected onto the nonnegative orthant and the smooth manifold of low-rank matrices.

For matrices, the truncated singular value decomposition (SVD) gives the *exact* projection in the Frobenius norm, but its cubic complexity makes the iterations computationally demanding. To reduce the cost, one can turn to faster *inexact* low-rank projections. In the tensor case, even the basic SVD-based low-rank approximation algorithms are inherently inexact.

We will discuss the numerical and theoretical aspects of how inexact low-rank projections affect the convergence of the alternating projections and reduce the computational complexity of finding low-rank nonnegative matrix and tensor approximations.

The talk is based on papers [4, 5], where we numerically study the alternating projections based on randomized sketching for matrices and tensors in Tucker and tensor train formats. We will also discuss some of the preliminary results of the ongoing research related to the theoretical convergence guarantees of inexact alternating projections and matrix/tensor cross approximations.

Acknowledgements: SM is supported by Russian Science Foundation (project 21-71-10072).

References

[1] N. Gillis *Nonnegative Matrix Factorization*. SIAM, Philadelphia, PA, 2020.

[2] B. Vanluyten, J. C. Willems, B. De Moor. Nonnegative matrix factorization without nonnegativity constraints on the factors, (2008).

[3] G.-J. Song, M. K. Ng. Nonnegative low rank matrix approximation for nonnegative matrices *Appl. Math. Lett.*, 105: 106300 (2020).

[4] S. Matveev, S. Budzinskiy. Sketching for a low-rank nonnegative matrix approximation: Numerical study. *Russ. J. Numer. Anal. Math. Modelling*, 38(2): 1–16 (2023).

[5] A. Sultonov, S. Matveev, S. Budzinskiy. Low-rank nonnegative tensor approximation via alternating projections and sketching. *Comp. Appl. Math.*, 42(68) (2023).

Deep Importance Sampling Using Tensor Approximations

Tiangang Cui[1], *Sergey Dolgov*[2], *Robert Scheichl*[3]

[1] *Monash University, Australia*
E-mail: tiangang.cui@monash.edu
[2] *University of Bath, United Kingdom*
E-mail: s.dolgov@bath.ac.uk
[3] *University of Heidelberg, Germany*
E-mail: r.scheichl@uni-heidelberg.de

Abstract

We propose a deep importance sampling method that is suitable in particular for estimating rare event probabilities in high-dimensional problems. We approximate the optimal importance distribution in a general importance sampling problem as the pushforward of a reference distribution under a composition of order-preserving transformations, in which each transformation is formed by a squared tensor-train decomposition of a ratio of unnormalized bridging density functions, such as tempered or smoothed versions of the target density. The use of composition of maps moving along a sequence of bridging densities alleviates the difficulty of directly approximating a concentrated target density function. To compute expectations over unnormalized probability distributions, we design a ratio estimator that estimates the normalizing constant using a separate importance distribution, again constructed via a composition of transformations in tensor-train format. This offers better theoretical variance reduction compared with self-normalized importance sampling, and thus opens the door to efficient computation of rare event probabilities in Bayesian inference problems. Numerical experiments on problems constrained by differential equations show little to no increase in the computational complexity with the event probability going to zero.

References

[1] T. Cui, S. Dolgov, R. Scheichl. Deep importance sampling using tensor-trains with application to a priori and a posteriori rare event estimation. *arXiv preprint*, 2209.01941 (2022).

Low-rank tensor frames for the high-accuracy solution of elliptic and parabolic PDEs

Vladimir Kazeev[1]

[1] *Faculty of Mathematics, University of Vienna, Austria*
E-mail: vladimir.kazeev@univie.ac.at

Abstract

Low-rank tensor decompositions, based on techniques of numerical linear algebra and optimization, realize adaptive low-parametric approximation for PDE problems that are capable of dramatically reducing the complexity of numerical solvers. One such a decomposition was proposed under the names of *matrix product states* (MPS) in computational quantum physics [13, 12, 11] and *tensor train* (TT) in computational mathematics [8, 7]. In particular, the *multilevel* MPS-TT representation [6, 5], building on the classical idea of Kronecker-product multilevel approximation [10, 9], allows to handle generic but extravagantly large discretizations and leads to data-driven computations based on effective discretizations adapted to the data instead of problem-dependent discretizations (approximation spaces) designed analytically. This approach has been shown, both theoretically and experimentally, to efficiently approximate functions with algebraic singularities [4] and highly-oscillatory solutions to multiscale diffusion problems [2, 3], achieving exponential convergence with respect to the total number of representation parameters.

The adaptive multilevel MPS-TT representation is advantageous compared to classical PDE discretizations when the number of levels employed (and hence the number of factors in the tensor decomposition involved) is large. This regime is associated with two notions of stability: not only the standard matrix conditioning of the discretized differential operator but also the stability of long ("deep") tensor factorizations [1]. In this talk, we present recent results on the use of the multilevel MPS-TT representation for the numerical solution of elliptic and parabolic PDE problems. We present a *low-rank tensor frame representation*, a generalization of the MSP-TT decomposition tailored for the stable discretization and solution of PDEs, with a clutch of algorithms based on matrix factorizations. The frame representation enables PDE solvers which are stable for large numbers of levels in the sense of both matrix and representation conditioning and, unlike the BPX-type preconditioner developed in [1], can be used in practice for 2D [4] and 3D multiscale problems and also for parabolic problems.

References

[1] Markus Bachmayr and Vladimir Kazeev. Stability and preconditioning of elliptic PDEs with low-rank multilevel structure. *Foundations of Computational Mathematics*, 20:1175–1236, 2020.

[2] Vladimir Kazeev, Ivan Oseledets, Maxim Rakhuba, and Christoph Schwab. QTT-finite-element approximation for multiscale problems I: model problems in one dimension. *Advances in Computational Mathematics*, 43(2):411–442, 2017.

[3] Vladimir Kazeev, Ivan Oseledets, Maxim V. Rakhuba, and Christoph Schwab. Quantized tensor FEM for multiscale problems: Diffusion problems in two and three dimensions. *Multiscale Modeling & Simulation*, 20(3):893–935, 2022.

[4] Vladimir Kazeev and Christoph Schwab. Quantized tensor-structured finite elements for second-order elliptic PDEs in two dimensions. *Numerische Mathematik*, 138:133–190, 2018.

[5] B. N. Khoromskij. $\mathcal{O}(d \log N)$-quantics approximation of N-d tensors in high-dimensional numerical modeling. *Constructive Approximation*, 34(2):257–280, 2011.

[6] I. V. Oseledets. Approximation of $2^d \times 2^d$ matrices using tensor decomposition. *SIAM Journal on Matrix Analysis and Applications*, 31(4):2130–2145, 2010.

[7] I. V. Oseledets. Tensor Train decomposition. *SIAM Journal on Scientific Computing*, 33(5):2295–2317, 2011.

[8] I. V. Oseledets and E. E. Tyrtyshnikov. Breaking the curse of dimensionality, or how to use SVD in many dimensions. *SIAM Journal on Scientific Computing*, 31(5):3744–3759, 2009.

[9] E. E. Tyrtyshnikov. Tensor approximations of matrices generated by asymptotically smooth functions. *Sbornik: Mathematics*, 194(5):941–954, 2003.

[10] C. F. Van Loan and N. Pitsianis. *Approximation with Kronecker Products*, pages 293–314. Springer Netherlands, Dordrecht, 1993.

[11] F. Verstraete and J. I. Cirac. Matrix product states represent ground states faithfully. *Phys. Rev. B*, 73:094423, 2006.

[12] Guifré Vidal. Efficient classical simulation of slightly entangled quantum computations. *Phys. Rev. Lett.*, 91(14):147902–147905, 2003.

[13] Steven R. White. Density matrix formulation for quantum renormalization groups. *Phys. Rev. Lett.*, 69(19):2863–2866, 1992.

A weighted subspace exponential kernel for support tensor machines

Kirandeep Kour[1], *Sergey Dolgov*[2], *Peter Benner*[1,3], *Martin Stoll*[3], *Max Pfeffer*[3]

[1] *Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany*
E-mail: kour@mpi-magdeburg.mpg.de benner@mpi-magdeburg.mpg.de
[2] *Department of Mathematical Sciences, University of Bath, United Kingdom*
E-mail: S.Dolgov@bath.ac.uk
[3] *Department of Mathematics, Technische Universität Chemnitz, Germany*
E-mail: martin.stoll@mathematik.tu-chemnitz.de
max.pfeffer@mathematik.tu-chemnitz.de

Abstract

High-dimensional data in the form of tensors are challenging for kernel classification methods. To both reduce the computational complexity and extract informative features, kernels based on low-rank tensor decompositions have been proposed. However, what decisive features of the tensors are exploited by these kernels is often unclear. In this talk, I would be presenting a proposed novel kernel that is based on the Tucker decomposition. For this kernel the Tucker factors are computed based on re-weighting of the Tucker matrices with tuneable powers of singular values from the HOSVD decomposition. This provides a mechanism to balance the contribution of the Tucker core and factors of the data. I benchmark support tensor machines with this kernel on several datasets. Firstly, I consider synthetic data where two classes differ in either Tucker factors or core. Then I compare our novel and certain existing kernels on real-world datasets. I show robustness of the new kernel with respect to both classification scenarios. The proposed kernel has demonstrated a higher test accuracy than the state-of-the-art tensor train multi-way multi-level kernel, and a significantly lower computational time.

Acknowledgements: Kirandeep Kour is supported by the International Max Planck Research School for Advanced Methods in Process and System Engineering-**IMPRS ProEng**, Magdeburg. Max Pfeffer is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – Projektnummer 448293816.

Submatrices with the best-bounded inverses: revisiting the hypothesis

Yuri Nesterenko[1]

[1] *Faculty of Computational Mathematics and Cybernetics, Moscow State University, Russia*
E-mail: y_nesterenko@mail.ru

Abstract

The following hypothesis was put forward by Goreinov, Tyrtyshnikov and Zamarashkin in [1].

For arbitrary semi-orthogonal $n \times k$ matrix a sufficiently "good" $k \times k$ submatrix exists. "Good" in the sense of having a bounded spectral norm of its inverse. The hypothesis says that for arbitrary $k = 1, \ldots, n-1$ the sharp upper bound is $\sqrt{n}$.

Supported by numerical experiments, the problem remains open for all non-trivial cases ($1 < k < n - 1$). During the talk, we will give the proof for the simplest of them ($n = 4$, $k = 2$) and discuss some observations about the general case.

Acknowledgements: I want to thank the authors of the original paper for fruitful discussions.

References

[1] S.A. Goreinov, E.E. Tyrtyshnikov, N.L. Zamarashkin, A theory of pseudo-skeleton approximation. *Linear Algebra Appl.*, 261: 1–21 (1997).

Learning Feynman diagrams with tensor trains

Y. N. Fernández[1,2], M. Jeannin[2], P. T. Dumitrescu[3], T. Kloss[1,2], J. Kaye[3], O. Parcollet[3,4], X. Waintal[2]

[1] Univ. Grenoble Alpes, CNRS, Institut Néel, 38000 Grenoble, France
E-mail: yurielnf@gmail.com
[2] Univ. Grenoble Alpes, CEA, Grenoble INP, IRIG, Pheliqs, F-38000 Grenoble, France
[3] Flatiron Institute, 162 5th Avenue, New York, NY 10010, USA
[4] Université Paris-Saclay, CNRS, CEA, 91191, Gif-sur-Yvette, France

Abstract

We use tensor cross interpolation algorithm to obtain high order perturbative diagrammatic expansions for the quantum many-body problem at very high precision. The approach is based on a tensor train representation of the sum of all Feynman diagrams. This representation is an effective separation of variables and therefore enables a direct calculation of the high dimensional integrals. It also yields to the full real time evolution of physical quantities in the presence of any time-dependent interaction. Our benchmarks on Anderson-like quantum impurity problems demonstrate that this technique supersedes diagrammatic Quantum Monte Carlo by orders of magnitude in precision and speed, with convergence rates $1/N^2$ or faster, where N is the number of function evaluations. The method also works in parameter regimes characterized by strongly oscillatory integrals in high dimension, which suffer from a catastrophic sign problem in Quantum Monte-Carlo.

Acknowledgements: The Flatiron Institute is a division of the Simons Foundation. Work partially supported by Plan France 2030 ANR-22-PETQ-0007 and the French-Japanese ANR QCONTROL, the European Union's Horizon 2020 research and innovation programme under Grant agreement No. 862683 (UltraFastNano).

References

[1] Y. N. Fernández, M. Jeannin, P. T. Dumitrescu, T. Kloss, J. Kaye, O. Parcollet, and X. Waintal. Learning Feynman Diagrams with Tensor Trains. *Phys. Rev. X 12*, 041018 (2022).

[2] S. Dolgov and D. Savostyanov. Parallel Cross Interpolation for High-Precision Calculation of High-Dimensional Integrals. *Comput. Phys. Commun. 246*, 106869 (2020).

[3] M. Maček, P. T. Dumitrescu, C. Bertrand, B. Triggs, O. Parcollet, and X. Waintal. Quantum Quasi-Monte Carlo Technique for Many-Body Perturbative Expansions. *Phys. Rev. Lett. 125*, 047702 (2020).

Empirical Tensor Train Approximation in Optimal Control

Mathias Oster[1], *Reinhold Schneider*[2]

[1] *Technical University Berlin, Germany*
E-mail: oster@math.tu-berlin.de
[2] *Technical University Berlin, Germany*
E-mail: schneidr@math.tu-berlin.de

Abstract

We will display two approaches in order to approximatively solve Bolza type finite horizon optimal control problems. The first approach is to solve the associated Bellman equation numerically by employing the Policy Iteration algorithm. In a second approach, we will introduce a semiglobal optimal control problem and use open loop methods arising from a Pontryagin maximum principle on a feedback level. To overcome computational infeasability we use low rank hierarchical tensor product approximation/tree-based tensor formats, in particular tensor trains (TT tensors) and multi-polynomials, together with high-dimensional quadrature, e.g. Monte-Carlo. By controlling a destabilized version of viscous Burgers and a diffusion equation with unstable reaction term numerical evidence is given.

References

[1] M. Oster, L. Sallandt, R. Schneider. Approximating the stationary Hamilton- Jacobi-Bellman equation by hierarchical tensor products. 2019

[2] M. Oster, L. Sallandt, R. Schneider. Approximating optimal feedback controllers of finite horizon control problems using hierarchical tensor formats. SIAM Journal on Scientific Computing, 2022

[3] K. Fackeldey, M. Oster, L. Sallandt, and R. Schneider. "Approximative Policy Iteration for Exit Time Feedback Control Problems Driven by Stochastic Differential Equations using Tensor Train Format". Multiscale Modeling & Simulation 20.1 (2022), 379–403.

A simple yet effective tensor-based ODE model for Deep Learning

Davide Palitta[1], *Jemima M. Tabeart*[2], *Nick Vannieuwenhoven*[3]

[1] *Dipartimento di Matematica, Università di Bologna, Bologna, Italy*
E-mail: davide.palitta@unibo.it
[2] *Mathematical Institute, University of Oxford, Oxford, UK*
E-mail: tabeart@maths.ox.ac.uk
[3] *KU Leuven, Department of Computer Science, Leuven, Belgium*
E-mail: nick.vannieuwenhoven@kuleuven.be

Abstract

In the past few years, applied mathematicians started looking at the forward propagation step of deep learning techniques in terms of discretization methods, e.g., forward Euler, applied to an unknown, underlying differential operator. Each layer of the network is seen as a time step of the discretization method [1, 2]. This point of view paved the way for so-called neural ordinary differential equations (ODE) [3, 4]. In the latter framework, the deep learning process is modeled by an ODE: Inputs are translated into initial values whereas outputs are viewed as the ODE solution evaluated at the final time step. Information propagates along the ODE flow in place of the net so that the extremely problem-dependent design of the latter is no longer needed. The training phase is now employed to learn the parameters definining the neural ODE. In this talk we present a novel tensor-based neural ODE, namely an ODE defined by tensors, to model a deep learning process. Preliminary results on classification problems show the potential of such new tool.

Acknowledgements: Work supported by the Polish National Agency for Academic Exchange under the funding scheme Una Europa Seed Funding - DIGITALIZED! grant SF21D6 "Tensor-based Optimal Control Approaches for Deep Learning (TOC4Deep).

References

[1] L. Ruthotto, E. Haber, *Deep Neural Networks Motivated by Partial Differential Equations*, J Math Imaging Vis, 62 (2020), pp. 352–364.

[2] E. Haber, L. Ruthotto, *Stable architectures for deep neural networks*, Inverse Problems, 34 (2017).

[3] R. T. Q. Chen, Y. Rubanova, J. Bettencourt, and D. K. Duvenaud, *Neural Ordinary Differential Equations*, Advances in Neural Information Processing Systems, 31 (2018).

[4] P. Kidger, *On Neural Differential Equations*, PhD thesis, University of Oxford (2021)

A statistical POD approach for feedback boundary optimal control in fluid dynamics

Sergey Dolgov[1], *Dante Kalise*[2], *Luca Saluzzi*[3]

[1] *Department of Mathematical Sciences, University of Bath, North Rd, BA2 7AY Bath, United Kingdom* (sd901@bath.ac.uk)
[2] *Department of Mathematics, Imperial College London, South Kensington Campus, SW7 2AZ London, United Kingdom* (dkaliseb@imperial.ac.uk)
[3] *Department of Mathematics, Imperial College London, South Kensington Campus, SW7 2AZ London, United Kingdom* (lsaluzzi@ic.ac.uk)

Abstract

Hamilton-Jacobi-Bellman (HJB) equation plays a central role in optimal control and differential games, enabling the computation of robust controls in feedback form. The main disadvantage for this approach depends on the so-called curse of dimensionality, since the HJB equation and the dynamical system live in the same, possibly high dimensional, space. In this talk I will consider feedback boundary optimal control problems arising from fluid dynamics and their reduction by the means of a Statistical Proper Orthogonal Decomposition (SPOD) method. The Proper Orthogonal Decomposition (POD) is a well-known technique in the Model Order Reduction community used to reduce the complexity of intensive simulations. The SPOD approach is characterized by the introduction of stochastic terms in the model (*e.g.* in the initial condition or in the boundary conditions) to enrich the knowledge of the Full Order Model, useful for the definition of a more reliable controlled reduced dynamics. In the offline stage of the method we consider different realizations of the artificial random variables and we compute the corresponding optimal trajectory via the Pontryagin Maximum Principle (PMP), which will form our snapshots set. Afterwards, we construct the reduced basis and we consider the corresponding reduced dynamical system. At this point the HJB can be solved in a reduced domain through the application of a data-driven Tensor Train Gradient Cross [2] based on samples derived by either PMP or the State-Dependent Riccati Equation. Finally, I will show its effectiveness on the optimal control of the incompressible Navier-Stokes equation in a backward step domain.

References

[1] S. Dolgov, D. Kalise, L. Saluzzi, A statistical POD approach for boundary optimal control in fluid dynamics, preprint, 2023.

[2] S. Dolgov, D. Kalise, L. Saluzzi, Data-driven Tensor Train Gradient Cross Approximation for HJB Equations. To appear on *SIAM J. Sci. Comp.*, 2023.

Tensor product algorithms for Bayesian inference of networks from epidemiological data

Dmitry Savostyanov[1]

[1] *Department of Mathematical Sciences, University of Essex, UK*
E-mail: d.savostyanov@essex.ac.uk

Epidemiological modelling is crucial to inform healthcare policies and to support decision making for disease prevention and control. The recent outbreak of COVID-19 pandemic raised a significant scientific and public debate regarding the quality of the mathematical models used to predict the effect of the pandemics and to choose an appropriate response strategy. To accurately capture how the disease spreads, we have to move beyond a usual assumption that the population is connected homogeneously (well–mixed), and towards network models of epidemics. Unfortunately, their complexity grows exponentially with the size of the network — these models suffer from the curse of dimensionality and usually rely on further approximations to make them practically solvable. In this talk we discuss how epidemiological models on networks can be solved accurately using the recently proposed algorithms based on low–rank tensor product factorisations. We also discuss the inverse problem of inferring a contact network from epidemiological data, for which we employ Bayesian optimisation techniques.
This is joint work with Sergey Dolgov (University of Bath, UK).

Acknowledgements: This work is supported by the Leverhulme Trust Research Fellowship RF-2021-258.

Adaptive Undersampling in Spectromicroscopy

Oliver Townsend[1], *Silvia Gazzola*[2], *Sergey Dolgov*[3], *Paul Quinn*[4]

[1] *Dpt. of Mathematics, University of Bath, UK*
E-mail: ot280@bath.ac.uk
[2] *Dpt. of Mathematics, University of Bath, UK*
E-mail: S.Gazzola@bath.ac.uk
[3] *Dpt. of Mathematics, University of Bath, UK*
E-mail: S.Dolgov@bath.ac.uk
[4] *Diamond Light Source, UK*
E-mail: author3@

Abstract

X-ray spectromicroscopy is a powerful tool for studying material distributions, which is extracted from the data using a combination of PCA and cluster analysis. However, the traditional data collection setting has some significant weaknesses (e.g. long scanning times and material degradation due to x-ray radiation).

In this talk, we present a novel approach for undersampling, reconstructing, and analysing X-ray spectromicroscopic measurements based on low-rank matrix completion. The new method allows the selection of robust sampling pattern, matrix rank and undersampling ratio, while minimising the impact of undersampling on the cluster analysis. Results obtained on real data will be illustrated.

References

[1] O. Townsend, S. Gazzola, S. Dolgov, & P. Quinn. *Undersampling raster scans in spectromicroscopy for a reduced dose and faster measurements.* Optics Express, 30(24), 43237-43254. https://doi.org/10.1364/OE.471663, 2022.

MSI03. Semidefinite matrices: geometry and optimization

João Gouveia, Bruno F. Lourenço

Deterministic Approximation Algorithms for Volumes of Spectrahedra
Mahmut Levent Doğan, Jonathan Leake, Mohan Ravichandran

The Difference-of-Convex Algorithm and Quantum Conditional Entropy
Oisín Faust, Hamza Fawzi, James Saunderson

Classifying Linear Matrix Inequalities via Abstract Operator Systems
Tim Netzer, Martin Berger, Tom Drescher

A semidefinite program for least distortion embeddings of flat tori into Hilbert spaces
Marc Christian Zimmermann, Arne Heimendahl, Moritz Lücke, Frank Vallentin

Deterministic Approximation Algorithms for Volumes of Spectrahedra

Mahmut Levent Doğan[1], *Jonathan Leake*[2], *Mohan Ravichandran*[3]

[1] *Technische Universität Berlin, Institut für Mathematik, Strasse des 17. Juni 136, 10623, Berlin, Germany*
E-mail: dogan@math.tu-berlin.de
[2] *University of Waterloo, Department of Combinatorics and Optimization, 200 University Ave W, Waterloo, ON, Canada*
E-mail: jonathan.leake@uwaterloo.ca
[3] *Department of Mathematics, Bogazici University, Bebek, Istanbul*
E-mail: mohan.ravichandran@gmail.com

Abstract

In this talk, we will present a method for computing asymptotic formulas and approximations for the volumes of spectrahedra, based on the maximum-entropy principle from statistical physics. The method gives an approximate volume formula based on a single convex optimization problem of minimizing $-\log\det P$ over the spectrahedron. Spectrahedra can be described as affine slices of the convex cone of positive semi-definite (PSD) matrices, and the method yields efficient deterministic approximation algorithms and asymptotic formulas whenever the number of affine constraints is sufficiently dominated by the dimension of the PSD cone.

Our approach is inspired by the work of Barvinok and Hartigan [1] who used an analogous framework for approximately computing volumes of polytopes. Spectrahedra, however, possess a remarkable feature not shared by polytopes, a new fact that we also prove: central sections of the set of density matrices (the quantum version of the simplex) all have asymptotically the same volume. This allows for very general approximation algorithms, which apply to large classes of naturally occurring spectrahedra.

Acknowledgements: The first author is supported by the ERC under the European's Horizon 2020 research and innovation programme (grant 787840). The second author was partially supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy - The Berlin Mathematics Research Center MATH+ (EXC-2046/1, project ID: 390685689). The third author would like to thank the Institute of Mathematical Sciences, Chennai, for hosting him in Spring 2022, when part of this work was done. He also gratefully acknowledges financial support from the Bogazici University Solidarity fund.

References

[1] A. Barvinok, J.A. Hartigan. Maximum entropy Gaussian approximations for the number of integer points and volumes of polytopes. *Advances in Applied Mathematics*, 45: 252–289 (2010).

The Difference-of-Convex Algorithm and Quantum Conditional Entropy

Oisín Faust[1], *Hamza Fawzi*[2], *James Saunderson*[2]

[1] *DAMTP, University of Cambridge, United Kingdom*
E-mail: opbf2@maths.cam.ac.uk
[2] *DAMTP, University of Cambridge, United Kingdom*
E-mail: h.fawzi@damtp.cam.ac.uk
[3] *ECSE, Monash University, Australia*
E-mail: james.saunderson@monash.edu

Abstract

The difference-of-convex (DC) algorithm is a conceptually simple method for the minimization of (non)convex functions that are expressed as the difference of two convex functions. The DC algorithm can be reinterpreted as the Bregman proximal point algorithm. This viewpoint allows for the straightforward derivation of convergence guarantees for the DC algorithm. We present several conditions that ensure a linear convergence rate, namely a new DC Polyak-Łojasiewicz condition, as well as a relative strong convexity assumption which does not require smoothness. In the second part of the talk, we will consider an application in quantum statistical mechanics. We will discuss a simple first-order algorithm for computing the convex conjugate and proximal operator of the conditional entropy of a bipartite quantum system.

References

[1] Oisín Faust, Hamza Fawzi, James Saunderson. A Bregman Divergence View on the Difference-of-Convex Algorithm. *AISTATS 2023*

Classifying Linear Matrix Inequalities via Abstract Operator Systems

Tim Netzer[1]

[1] *Universität Innsbruck, Department of Mathematics, Austria*
E-mail: tim.netzer@uibk.ac.at

Abstract

A spectrahedron is, by definition, the solution set of a linear matrix inequality. Spectrahedra are precisely the feasible sets of semidefinite programming, and thus deciding whether a set is a spectrahedron is a relevant problem for optimization. It is also interesting and important to classify the different possible linear matrix inequalities defining a given spectrahedron. In this talk we explain how properties of abstract operator systems help classifying linear matrix inequality definitions of sets. Our main focus is on polyhedral cones, the 3-dimensional Lorentz cone, where we can completely describe all defining linear matrix inequalities, and on the cone of positive semidefinite matrices.

A semidefinite program for least distortion embeddings of flat tori into Hilbert spaces

Arne Heimendahl[1], *Moritz Lücke*[2], *Frank Vallentin*[3], *Marc Christian Zimmermann*[4]

[1] *Department Mathematik/Informatik, Universität zu Köln, Germany*
E-mail: arne.heimendahl@uni-koeln.de
[2] *Department Mathematik/Informatik, Universität zu Köln, Germany*
E-mail: moritz.luecke1997@gmail.com
[3] *Department Mathematik/Informatik, Universität zu Köln, Germany*
E-mail: frank.vallentin@uni-koeln.de
[4] *Department Mathematik/Informatik, Universität zu Köln, Germany*
E-mail: marc.christian.zimmermann@gmail.com

Abstract

We derive and analyze an infinite-dimensional semidefinite program which computes least distortion embeddings of flat tori $\mathbb{R}^n/L$, where L is an n-dimensional lattice, into Hilbert spaces.

This enables us to provide a constant factor improvement over the previously best lower bound on the minimal distortion of an embedding of an n-dimensional flat torus.

As further applications we prove that every n-dimensional flat torus has a finite dimensional least distortion embedding, that the standard embedding of the standard tours is optimal, and we determine least distortion embeddings of all 2-dimensional flat tori.

Acknowledgements: This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie agreement No 764759. F.V. is partially supported by the SFB/TRR 191 "Symplectic Structures in Geometry, Algebra and Dynamics", F.V. and M.C.Z. are partially supported "Spectral bounds in extremal discrete geometry" (project number 414898050), both funded by the DFG. A.H. is partially funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy - Cluster of Excellence Matter and Light for Quantum Computing (ML4Q) EXC 2004/1 - 390534769.

References

[1] A. Heimendahl, M. Lücke, F. Vallentin, M.C.Zimmermann *A semidefinite program for least distortion embeddings of flat tori into Hilbert spaces.* arXiv:2210.11952v1, 2022.

MSI04. Matrix equations

Andrii Dmytryshyn

A new low-rank solver for algebraic Riccati equations based on the matrix sign function and principal pivot transforms
Peter Benner, Federico Poloni

Chebyshev HOPGD for parameterized linear systems
Siobhan Correnty, Melina A. Freitag, Kirk M. Soodhalter

A mixed-precision algorithm for the Sylvester equation
Andrii Dmytryshyn, Massimiliano Fasi, Nicholas J. Higham, Xiaobo Liu

On a new family of low-rank algorithms for large-scale algebraic Riccati equations
Heike Fassbender, Christian Bertram

Deflating subspaces of palindromic pencils and the T-Riccati matrix equation
Bruno Iannazzo, Beatrice Meini, Federico Poloni

Inexact low-rank ADI for large-scale Sylvester equations
Patrick Kürschner

Compress-and-restart block Krylov subspace methods for Sylvester matrix equations
Kathryn Lund, Daniel Kressner, Stefano Massei, Davide Palitta

On computing modified moments for half--range Hermite and Pollaczek--Hermite weights in floating point arithmetic0
Nicola Mastronardi, Teresa Laudadio, Paul Van Dooren

A delayed shift technique for M-matrix algebraic Riccati equations
Federico Poloni, Elena Addis, Bruno Iannazzo

Numerical solution of a class of quasi-linear matrix equations
Valeria Simoncini, Margherita Porcelli

Efficient iterative methods for the solution of Generalized Lyapunov Equations: Block vs. point Krylov projections, and other controversial decisions
Daniel Szyld, Stephen Shank, Valeria Simoncini

Balanced Truncation Model Reduction of Parametric Differential-Algebraic Systems
Matthias Voigt, Jennifer Przybilla

A new low-rank solver for algebraic Riccati equations based on the matrix sign function and principal pivot transforms

Peter Benner[1], *Federico Poloni*[2]

[1] *Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany*
E-mail: benner@mpi-magdeburg.mpg.de
[2] *University of Pisa, Italy*

Abstract

We discuss an algorithm to compute the inverse of quasi-definite matrices with low-rank diagonal blocks. This employs principal pivot transforms (PPTs) to derive a new, backward stable variant of symmetric Gauß-Jordan elimination. Employing this in the Newton iteration for computing the matrix sign function of a $2n \times 2n$ Hamiltonian matrix (which is quasi-definite after multiplication by $J = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}$) allows to keep the off-diagonal blocks in low-rank form without re-factoring them as needed in previous attempts to preserve the low-rank structure explicitly. This inversion method then can be used to derive a novel low-rank solver for algebraic Riccati equations based on the sign function method. The new method is superior to previous sign function-based methods with respect to the number of floating point operations as well as numerical accuracy, as demonstrated by several numerical examples.

Chebyshev HOPGD for parameterized linear systems

Siobhán Correnty, Melina A. Freitag, and Kirk M. Soodhalter

[1] *Department of Mathematics, Royal Institute of Technology (KTH), Sweden*
E-mail: correnty@kth.se
[2] *Institute of Mathematics, University of Potsdam, Germany*
E-mail: melina.freitag@uni-potsdam.de
[3] *School of Mathematics, Trinity College Dublin, Ireland*
Email: ksoodha@maths.tcd.ie

Abstract

We are interested in solutions to parameterized linear systems of the form

$$A(\mu_1, \mu_2)x(\mu_1, \mu_2) = b, \tag{1}$$

for $(\mu_1, \mu_2) \in \mathbb{R}^2$, where $A(\mu_1, \mu_2) \in \mathbb{R}^{n\times n}$ nonsingular with a nonlinear dependence on the parameters, $x(\mu_1, \mu_2) \in \mathbb{R}^n$, and $b \in \mathbb{R}^n$. This work combines companion linearization with the Krylov subspace method preconditioned BiCG and a decomposition of a tensor matrix of precomputed snapshots. A reduced order model of $x(\mu_1, \mu_2)$ is constructed, which can be evaluated in a cheap way for many values of the parameters.

Previously proposed methods require sampling, e.g., the computation of a finite element solution for each snapshot in the tensor matrix, which this method avoids. Specifically, the snapshots are generated as in [1], where one parameter is frozen at a time, and the method returns solutions to (1) as a one variable function of the other parameter. The decomposition is performed using the procedure presented in [2], resulting a rank m approximation of the form

$$x^m(\mu_1, \mu_2) = \sum_{k=1}^{m} \Phi^k F_1^k(\mu_1) F_2^k(\mu_2) \approx x(\mu_1, \mu_2), \tag{2}$$

where $\Phi^k \in \mathbb{R}^n$, $F_1^k : \mathbb{R} \to \mathbb{R}$, and $F_2^k : \mathbb{R} \to \mathbb{R}$. An interpolation of the functions F_1^k, F_2^k in (2) is used to produce approximations to (1) outside of the snapshots.

References

[1] S. Correnty, E. Jarlebring, D. B. Szyld. Preconditioned Chebyshev BiCG for parameterized linear systems. *Preprint on arXiv* (2022). https://arxiv.org/abs/2212.04295

[2] Y. Lu, N. Blal, A. Gravouil. Adaptive sparse grid based HOPGD: Toward a nonintrusive strategy for constructing a space-time welding computational vademecum. *Int. J. Numer. Methods Eng.*, 114:1438–1461 (2018).

A mixed-precision algorithm for the Sylvester equation

Andrii Dmytryshyn[1], *Massimiliano Fasi*[2], *Nicholas J. Higham*[3], *Xiaobo Liu*[3]

[1] *School of Science and Technology, Örebro University, Sweden*
E-mail: andrii.dmytryshyn@oru.se
[2] *Department of Computer Science, Durham University, UK*
E-mail: massimiliano.fasi@durham.ac.uk
[3] *Department of Mathematics, The University of Manchester, UK*
E-mail: nick.higham@manchester.ac.uk, xiaobo.liu@manchester.ac.uk

Abstract

We consider the use of mixed precision to solve the Sylvester matrix equation $AX + XB = C$. When only one level of precision is available, the method of choice is the one developed by Bartels and Stewart [1]. Their algorithm consists of three steps: first, we compute the Schur decompositions $A =: U_A T_A U_A^*$ and $B =: U_B T_B U_B^*$, where U_A and U_B are unitary and T_A and T_B are upper quasi-triangular; then, we use a substitution algorithm to solve for Y the quasi-triangular equation $T_A Y + Y T_B = U_A^* C U_B$; and finally, we compute the solution as $X := U_A Y U_B^*$.

How can we improve the performance of this method when, in addition to the current working precision, a lower-precision arithmetic is also available? An effective way to achieve this is to reduce the precision used to carry out the most expensive step of the algorithm, which is the initial computation of the two Schur decompositions. The use of low precision introduces a loss of accuracy, and we need to adapt the subsequent stages of the algorithm to make up for it. First, we replace the substitution algorithm that solves the quasi-triangular equation with an iterative refinement scheme that runs in working precision but only uses the low-precision Schur factors of A and B. But the Bartels–Stewart algorithm hinges on U_A and U_B being unitary, and if the Schur decompositions are computed using low precision, then recovering X as $U_A Y U_B^*$ will cause all accuracy to be lost. We propose two inexpensive solutions that overcome this issue: one is based on the inversion of U_A and U_B, the other on their re-orthonormalization, and they both rely on working-precision arithmetic.

In our numerical experiments, the new methods are as accurate as the Bartels–Stewart algorithm run in working precision, but they can be faster if the low-precision arithmetic is sufficiently cheaper than the working-precision one.

References

[1] R. H. Bartels, G. W. Stewart. Algorithm 432: Solution of the matrix equation $AX + XB = C$, *Comm. ACM*, 15: 820–826 (1972).

On a new family of low-rank algorithms for large-scale algebraic Riccati equations

Christian Bertram[1], *Heike Faßbender*[1]

[1] *Institute for Numerical Analysis, TU Braunschweig, Germany*
E-mail: h.fassbender@tu-braunschweig.de

Abstract

Finding the unique stabilizing solution $X = X^H$ of large-scale continuous-time algebraic Riccati equations (CAREs) $0 = R(X) := A^H X + XA + C^H C - XBB^H X$ with a large, sparse matrix $A \in \mathbb{C}^{n\times n}$, and matrices $B \in \mathbb{C}^{n\times m}$ and $C \in \mathbb{C}^{p\times n}$ is of interest in a number of applications. Here, B and C^H are assumed to have full column and row rank, resp., with $m, p \ll n$. The unique stabilizing solution $X = X^H$ is positive semidefinite and makes the closed-loop matrix $A - BB^H X$ stable. Even so A is large and sparse, the solution X will still be a dense matrix in general. But our assumptions on B and C often imply that the sought-after solution X will have a low numerical rank (that is, its rank is $\ll n$). This allows for the construction of iterative methods that approximate X with a series of low rank matrices stored in low-rank factored form.

To be precise, we focus on Hermitian low-rank approximations X_j to X of the form $X_j = Z_j Y_j Z_j^H$, where $Z_j \in \mathbb{C}^{n\times k_j}$ is a matrix with only few columns and $Y_j \in \mathbb{C}^{k_j\times k_j}$ is a small square Hermitian matrix. There are several methods which produce such a low-rank approximation (e.g., rational Krylov subspace methdos, low-rank Newton-Kleinman methods and Newton-ADI-type methods), see, e.g., [4] for an overview. In particular, [3, Theorem 2] states that the approximation X_j^{cay} of the Riccati solution obtained by the Cayley transformed Hamiltonian subspace iteration [5] and the approximation X_j^{qadi} obtained by the qADI iteration [6, 7] are equal to the approximation X_j^{radi} obtained by the RADI method [3]

$$X_j^{\text{radi}} = X_j^{\text{cay}} = X_j^{\text{qadi}}$$

(if the initial approximation in all algorithms is zero and the same shifts are used). Beyond that, if rank $C = 1$ and the shifts are chosen in a specific way,

$$X_j^{\text{radi}} = X_j^{\text{cay}} = X_j^{\text{qadi}} = X_j^{\text{inv}}$$

where X_j^{inv} is the approximation obtained by the invariant subspace approach [1]. Parts of these connections have already been described in [2, 5].

Our approach is based on a block rational Arnoldi decomposition and an associated block rational Krylov subspace spanned by A^H and C^H. The approximations X_j as well as $\|R(X_j)\|_F$ can be computed fast and efficiently. In particular, our approach gives a whole new family of algorithmic descriptions of the same approximation sequence X_j to the Riccati solution as four above mentioned algorithms for CARE, that is,

$$X_j = X_j^{\text{radi}} = X_j^{\text{cay}} = X_j^{\text{qadi}}.$$

We will suggest a new algorithm which allows for a computationally slightly more efficient way to compute X_j than the other four algorithms. The focus of the talk will be on the theoretic background of the family of algorithms rather than on a comparison with other known algorithms.

References

[1] L. Amodei, J.-M. Buchot. *An invariant subspace method for large-scale algebraic Riccati equation.* Applied Numerical Mathematics, 60(11):1067–1082, 2010. Special Issue: 9th IMACS International Symposium on Iterative Methods in Scientific Computing (IISIMSC 2008).

[2] P. Benner, Z. Bujanović. *On the solution of large-scale algebraic Riccati equations by using low-dimensional invariant subspaces.* Linear Algebra and its Applications, 488:430–459, 2016.

[3] P. Benner, Z. Bujanović, P. Kürschner, J. Saak. *RADI: a low-rank ADI-type algorithm for large scale algebraic Riccati equation*s. Numerische Mathematik 138(2):301–330, 2018.

[4] P. Benner, Z. Bujanović, P. Kürschner, J. Saak. *A Numerical Comparison of Different Solvers for Large-Scale, Continuous-Time Algebraic Riccati Equations and LQR Problems.* SIAM Journal on Scientific Computing, 42(2):A957–A996, 2020.

[5] Y. Lin, V. Simoncini. *A new subspace iteration method for the algebraic Riccati equation.* Numerical Linear Algebra with Applications, 22(1):26–47, 2015.

[6] N. Won, V. Balakrishnan. *Quadratic alternating direction implicit iteration for the fast solution of algebraic Riccati equations.* In 2005 International Symposium on Intelligent Signal Processing and Communication Systems, pages 373–376, 2005.

[7] N. Wong, V. Balakrishnan. *Fast positive-real balanced truncation via quadratic alternating direction implicit iteration.* IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 26(9):1725–1731, 2007

Deflating subspaces of palindromic pencils and the T-Riccati matrix equation

Bruno Iannazzo[1], Beatrice Meini[2], Federico Poloni[2]

[1] Università degli Studi di Perugia
[2] Università di Pisa

Abstract

The solution of the Nonsymmetric algebraic $\top$-Riccati equation ($\top$-NARE)

$$X^\top BX - X^\top A - DX - C = 0, \tag{1}$$

where X is the unknown matrix and $A, B, C, D \in \mathbb{R}^{n\times n}$ are coefficients can be related to the computation of certain deflating subspaces of the $\top$-palindromic matrix pencil

$$\varphi(z) = M + zM^\top, \qquad M = \begin{bmatrix} C & D \\ A & -B \end{bmatrix}, \tag{2}$$

where the superscript $\top$ denotes transposition.

In particular, if $\varphi(z)$ is regular and X is a solution of (1) then the columns of $\left[\begin{smallmatrix} I \\ X \end{smallmatrix}\right]$ span a deflating subspace of $\varphi(z)$ and a kind of converse result holds. This relation has been exploited to develop $\top$-NARE solvers based on efficient algorithms for computing deflating subspaces of a pencil, such as the (palindromic) QZ algorithm and Doubling Algorithms [1].

By further exploiting this relation, we develop some theoretical and computational results.

On the theoretical side, we describe both necessary and sufficient conditions for the existence of solutions expressed in terms of the matrix coefficients and we provide conditions under which the pencil $\varphi(z)$ has no eigenvalues on the unit circle. These results are very similar to the well-known conditions for the existence of solution and the absence of critical eigenvalues in the celebrated continuous-time algebraic Riccati equations [2].

On the computational side, since the palindromic QZ algorithm of [1] requires a swapping of eigenvalues in the antitriangular form of the pencil $\varphi(z)$ to get the required deflating subspace, we update the swapping strategy reducing the cost from $O(n^4)$ to $O(n^3)$ arithmetic operations in the worst case. Moreover, we provide new algorithms based in quadraticizations of the pencil $\varphi(z)$, with the subsequent application of the Cyclic Reduction algorithm in two different ways and a new algorithm based on a contour integral representation of the orthogonal projection on the required deflating subspace of $\varphi(z)$.

References

[1] P. Benner, B. Iannazzo, B. Meini, D. Palitta. Palindromic linearization and numerical solution of nonsymmetric algebraic T-Riccati equations. *BIT*, 62 (2022) pp. 1649–1672.

[2] D. A. Bini, B. Iannazzo, B. Meini. *Numerical Solution of Algebraic Riccati Equations*, SIAM, Philadelphia, 2012.

[3] B. Iannazzo, B. Meini, F. Poloni. Invariant subspaces of T-palindromic pencils and algebraic T-Riccati equations. *arXiv preprint*, submitted for publication 2023.

Inexact low-rank ADI for large-scale Sylvester equations

Patrick Kürschner[1]

[1] *Leipzig University of Applied Sciences (HTWK Leipzig), Germany*
E-mail: patrick.kuerschner@htwk-leipzig.de

Abstract

The low-rank ADI iteration is an often used algorithm to compute low-rank solution approximations for large-scale matrix equations such as algebraic Riccati, Lyapunov, and Sylvester equations. In every step of this iteration, a large-scale linear system of equations has to be solved which is the most expensive part of the whole process. For Lyapunov equations,

$$AX + XA^T + ff^T = 0,$$

we recently presented in [1] accuracy criteria for the arising linear systems in the low-rank Lyapunov ADI iterations.
Here, we look at Sylvester equations

$$AX + XB + fg^T = 0$$

and the associated low-rank ADI for Sylvester equations [2], [3]. There, a pair of two different large-scale linear systems

$$(A + \beta_k I_n)v_k = s_{k-1} \quad (B + \alpha_k I_m)^T w_k = t_{k-1}, \quad k \geq 1$$

has to be solved in every step for v_k, w_k. We investigate the situation when those inner linear systems are solved inexactly by an iterative methods such as, e.g., preconditioned Krylov subspace methods. We present estimates for the required accuracies regarding the inner linear systems which dictate when the employed inner Krylov subspace methods can be safely terminated. The goal is to save some computational effort without endangering the functionality of the low-rank Sylvester-ADI method. Ideally, the inexact ADI method mimics the convergence behavior of the more expensive exact ADI method.

References

[1] P. Kürschner and M. Freitag. *Inexact methods for the low rank solution to large scale Lyapunov equations.* BIT, 60:1221–1259, 2020.

[2] P. Benner, R.-C. Li, and N. Truhar. *On the ADI method for Sylvester equations.* J. Comput. Appl. Math., 233(4):1035–1045, 2009.

[3] P. Benner and P. Kürschner. *Computing Real Low-rank Solutions of Sylvester equations by the Factored ADI Method.* Comput. Math. Appl., 67(9):1656–1672, 2014.

Compress-and-restart block Krylov subspace methods for Sylvester matrix equations

Daniel Kressner[1], *Kathryn Lund*[2], *Stefano Massei*[3], *Davide Palitta*[4]

[1] *Institute for Mathematics, EPF Lausanne, Lausanne, Switzerland*
E-mail: daniel.kressner@epfl.ch;
[2] *Computational Methods in Systems and Control Theory, MPI Magdeburg, Magdeburg, Germany*
E-mail: lund@mpi-magdeburg.mpg.de
[3] *Dipartimento di Matematica, Università di Bologna, Bologna, Italy*
E-mail: davide.palitta@unibo.it
[3] *Dipartimento di Matematica, Università di Pisa, Pisa, Italy*
E-mail: stefano.massei@unipi.it

Abstract

Block Krylov subspace methods (KSMs) comprise building blocks in many state-of-the-art solvers for large-scale matrix equations as they arise, for example, from the discretization of partial differential equations. While extended and rational block Krylov subspace methods provide a major reduction in iteration counts over polynomial block KSMs, they also require reliable solvers for the coefficient matrices, and these solvers are often iterative methods themselves. It is not hard to devise scenarios in which the available memory, and consequently the dimension of the Krylov subspace, is limited. In such scenarios for linear systems and eigenvalue problems, restarting is a well-explored technique for mitigating memory constraints. In this work, such restarting techniques are applied to polynomial KSMs for matrix equations with a compression step to control the growing rank of the residual. An error analysis is also performed, leading to heuristics for dynamically adjusting the basis size in each restart cycle. A panel of numerical experiments demonstrates the effectiveness of the new method with respect to extended block KSMs.

Acknowledgements: W ork supported in part by SNSF, "Fast algorithms from low-rank updates", grant number: 200020 178806, and in part by Charles University PRIMUS/19/SCI/11 grant.

References

[1] D. Kressner, K. Lund, S. Massei, and D. Palitta. Compress-and-restart block Krylov subspace methods for Sylvester matrix equations. Numerical Linear Algebra with Applications. 28(1), e2339, 2021.

On computing modified moments for half–range Hermite and Pollaczek–Hermite weights in floating point arithmetic

Teresa Laudadio[1], *Nicola Mastronardi*[1], *Paul Van Dooren*[3]

[1] *Istituto per le Applicazioni del Calcolo "M. Picone", Consiglio Nazionale delle Ricerche, Bari, Italy*
E-mail: teresa.laudadio@cnr.it, nicola.mastronardi@cnr.it
[2] *Department of Mathematical Engineering, Catholic University of Louvain, Louvain-la-Neuve, Belgium*
E-mail: paul.vandooren@uclouvain.be

Abstract

In this talk we consider the problem of computing two different kinds of integrals :

$$\int_{a_i}^{b_i} f(x)w_i(x)dx, \quad i = 1, 2, \qquad \begin{array}{|c|c|c|c|}\hline i & w_i(x) & a_i & b_i \\ \hline 1 & e^{-x^2} & 0 & \infty \\ \hline 2 & e^{-x^2-\frac{1}{x^2}} & -\infty & \infty \\ \hline \end{array} \tag{1}$$

where $w_1(x)$ and $w_2(x)$ are the Hermite and Pollaczek–type weights, respectively [4, 5].

Although $w_1(x)$ and $w_2(x)$ are both positive weights, the associated system of orthogonal polynomials is not known [3, 4, 5]. Therefore, it is not possible to compute the nodes and weights of the corresponding Gaussian rules in the standard way [3].

Usually, two possible approaches are considered for computing an approximation of (1) :

- computing the Gaussian quadrature rule associated to the weight $w_i(x), i = 1, 2$, via the modified Chebyshev algorithm [3];
- approximating (1) by a product quadrature rule [1, 3].

In both cases, it is needed to compute the modified moments :

$$\mathcal{M}_\ell^{(i)} = \int_{a_i}^{b_i} \tilde{p}_\ell(x)w_i(x)dx, \qquad \ell = 0, 1, 2, \ldots, \ i = 1, 2,$$

where $\tilde{p}_\ell(x)$, $\ell = 0, 1, 2, \ldots$, is a system of orthonormal polynomials with respect to another positive weight $\tilde{w}(x)$ in a interval $[\tilde{a}, \tilde{b}]$, satisfying a known three–term recurrence relation.

In case $i = 1$, i.e., the half–range Hermite weight, we consider the system of orthonormal Laguerre polynomials $\{\mathcal{L}_\ell(x)\}_{j=0}^{\infty}$ as $\{\tilde{p}_\ell(x)\}_{\ell=0}^{\infty}$. A system of two recurrence relations can be derived

$$\begin{aligned}
\mathcal{M}_0^{(1)} &= \frac{\sqrt{\pi}}{2}, \quad \mathcal{N}_0^{(1)} = \frac{1}{2}, \\
\mathcal{M}_1^{(1)} &= -\frac{1}{2} + \frac{\sqrt{\pi}}{2}, \quad \mathcal{N}_1^{(1)} = \frac{1}{2} - \frac{\sqrt{\pi}}{4}, \\
\ell\mathcal{M}_\ell^{(1)} &= (2\ell-1)\mathcal{M}_{\ell-1}^{(1)} - \mathcal{N}_{\ell-1}^{(1)} - (\ell-1)\mathcal{M}_{\ell-2}^{(1)}, \quad \ell \geq 2, \\
\ell\mathcal{N}_\ell^{(1)} &= -\frac{\ell}{2}\mathcal{M}_{\ell-1}^{(1)} + \frac{\ell-1}{2}\mathcal{M}_{\ell-2}^{(1)} + (2\ell-1)\mathcal{N}_{\ell-1}^{(1)} - (\ell-1)\mathcal{N}_{\ell-2}^{(1)},
\end{aligned}$$

with

$$\mathcal{M}_\ell^{(1)} = \int_0^\infty \mathcal{L}_\ell(x)e^{-x^2}dx, \quad \mathcal{N}_\ell^{(1)} = \int_0^\infty x\mathcal{L}_\ell(x)e^{-x^2}dx, \quad \ell, 0, 1, \ldots$$

In exact arithmetic, the sequences $\left\{\mathcal{M}_\ell^{(1)}\right\}_{\ell=0}^{\infty}$ and $\left\{\mathcal{N}_\ell^{(1)}\right\}_{\ell=0}^{\infty}$ go to zero for $\ell \to \infty$ [2, 6]. Unfortunately, their straightforward implementation in `Matlab` with double precision turns out to be unstable, and the two sequences diverge.

In this talk we will show that, given $n > 1$, the vector of modified moments $\left[\mathcal{M}_0^{(1)}, \mathcal{M}_1^{(1)}, \ldots, \mathcal{M}_n^{(1)}\right]^T$ is the solution of a matrix equation involving totally nonnegative matrices. Therefore, these moments can be computed to high relative accuracy.

The case $i = 2$, i.e., the Pollaczek–Hermite weight, can be handled in a similar way, choosing the orthonormal Hermite polynomials $\{\mathcal{H}_\ell(x)\}_{j=0}^{\infty}$ as $\{\tilde{p}_\ell(x)\}_{\ell=0}^{\infty}$ for computing the corresponding modified moments.

Acknowledgements: Work partially supported by Istituto Nazionale di Alta Matematica "F. Severi"

References

[1] P.J. Davis, P. Rabinowitz *Methods of Numerical Integration.* 2nd Edition, Academic Press, New York, 1984.

[2] W. Gautschi. Algorithm 957: Evaluation of the Repeated Integral of the Coerror Function by Half–Range Gauss-Hermite Quadrature, *ACM Trans. Math. Softw.*, 42: 1–10 (2016).

[3] W. Gautschi *Orthogonal Polynomials: Computation and Approximation.* Oxford University Press, Oxford, UK, 2004.

[4] G. Mastroianni, I. Notarangelo. Lagrange interpolation at Pollaczek-Laguerre zeros on the real semiaxis, *J. Approx. Theory*, 245: 83–100 (2019).

[5] T. Laudadio, N. Mastronardi, D. Occorsio. Computing integrals on $\mathbb{R}$ with a Pollaczek–Hermite weight without using extended precision. submitted to Appl. Numer. Math.

[6] T. Laudadio, N. Mastronardi, P. Van Dooren. On computing modified moments for half-range Hermite weights. submitted to Numer. Algorithms.

A delayed shift technique for M-matrix algebraic Riccati equations

Elena Addis[1], *Bruno Iannazzo*[2], *Federico Poloni*[3]

[1] *Department of Mathematics and Computer Science, University of Florence, Italy*
E-mail: elena.addis@unifi.it
[2] *Department of Mathematics, University of Perugia, Italy*
E-mail: bruno.iannazzo@unipg.it
[3] *Department of Computer Science, University of Pisa, Italy*
E-mail: federico.poloni@unipi.it

Abstract

The shift technique is a method to accelerate iterations to solve a vast family of matrix equations. In this talk we focus on nonsymmetric algebraic Riccati equations

$$XBX - XA - DX - C = 0, \tag{1}$$

with the additional property that

$$M = \begin{bmatrix} A & -B \\ C & D \end{bmatrix} \in \mathbb{R}^{(m+n)\times(m+n)}$$

is an M-matrix. Such equations appear most notably in probability applications related to so-called *fluid queue models*. For these equations, the structured doubling algorithm generates sequences E_k, F_k, G_k, H_k of nonnegative matrices, such that H_k converges to the minimal solution of (1) in an entrywise-monotonic fashion. It can be proved that the algorithm is forward-stable in an entrywise fashion, i.e., for sufficiently large k one has $|X - H_k|_{ij} \leq (H_k)_{ij} O(\mathsf{u})$, where $O(\mathsf{u})$ stands for a moderate multiple of the machine precision.

Applying the shift technique corresponds to replacing M with a rank-1 modification $\hat{M} = M = \eta v p^T$, with $v, p, \in \mathbb{R}^{m+n}$ and $\eta > 0$, and following the same procedure to generate a different sequence $\hat{E}_k, \hat{F}_k, \hat{G}_k, \hat{H}_k$. Carrying out the shift technique in a way that preserves the entrywise stability is not obvious, as $\hat{M}$ does not have the same sign structure as M.

We show that one can perform the operations in a different order, first computing the matrix of (unshifted) initial values

$$P_0 = \begin{bmatrix} E_0 & G_0 \\ H_0 & F_0 \end{bmatrix}$$

and then recovering its analogue $\hat{P}_0$ via a rank-1 modification. This route preserves better the sign structure, and allows one to carry out a detailed forward error analysis.

Acknowledgements: Work partially supported by INDAM/GNCS and project PRA_2020_61 of the university of Pisa.

References

[1] Addis, E, Elementwise accurate algorithms for nonsymmetric algebraic Riccati equations associated with M-matrices. PhD thesis, University of Florence.

[2] Bean, N G; O'Reilly, M M; Taylor, P G, Algorithms for return probabilities for stochastic fluid flows, Stoch. Models 21, No. 1, 149-184 (2005).

[3] Guo, X-X; Lin, W-W; Xu, S-F, A structure-preserving doubling algorithm for nonsymmetric algebraic Riccati equation, Numer. Math., 103, 393-412, (2006)

[4] Nguyen, G T; Poloni, F, Componentwise accurate fluid queue computations using doubling algorithms. Numer. Math. 130, No. 4, 763-792 (2015).

[5] Xue, J; Xu, S; Li, R-C, Accurate solutions of M-matrix algebraic Riccati equations, Numer. Math., 120, 671-700, (2012).

Numerical solution of a class of quasi-linear matrix equations

Margherita Porcelli[1], *Valeria Simoncini*[2]

[1] *Dipartimento di Matematica, AM*^{2}*, Alma Mater Studiorum - Università di Bologna, 40126 Bologna, and ISTI–CNR, Pisa, talia.*
E-mail: margherita.porcelli@unibo.it
[2] *Dipartimento di Matematica, AM*^{2}*, Alma Mater Studiorum - Università di Bologna, 40126 Bologna, and IMATI-CNR, Pavia, Italia*
E-mail: valeria.simoncini@unibo.it

Abstract

Given the matrix equation $\boldsymbol{AX} + \boldsymbol{XB} + f(\boldsymbol{X})\boldsymbol{C} = \boldsymbol{D}$ in the unknown $n \times m$ matrix $\boldsymbol{X}$, we analyze existence and uniqueness conditions, together with computational solution strategies for $f : \mathbb{R}^{n\times m} \to \mathbb{R}$ being a linear or nonlinear function. We characterize different properties of the matrix equation and of its solution, depending on the considered classes of functions f. Our analysis mainly concerns small dimensional problems, though several considerations also apply to large scale matrix equations.

This work is based on the article [1].

References

[1] M. Porcelli, and V. Simoncini. Numerical solution of a class of quasi-linear matrix equations *Linear Algebra and Its Appl.*, 664C (2023) pp. 349-368.

Efficient iterative methods for the solution of Generalized Lyapunov Equations: Block vs. point Krylov projections, and other controversial decisions

Stephen D. Shank[1], *Valeria Simoncini*[2], *Daniel B. Szyld*[1]

[1] *Temple University USA*
E-mail: shank@temple,edu, szyld@temple.edu
[2] *Universita di Bologna, Italy*
E-mail: valeria.simoncini@unibo.it

Abstract

There has been a flurry of activity in recent years in the area of solution of matrix equations. In particular, a good understanding has been reached on how to approach the solution of large scale Lyapunov equations. An effective way to solve Lyapunov equations of the form $A^TX + XA + C^TC = 0$, where A and X are $n \times n$, is to use Galerkin projection with appropriate extended or rational Krylov subspaces. These methods work in part because the solution is known to be symmetric positive definite with rapidly decreasing singular values, and therefore it can be approximated by a low rank matrix $X_k = Z_k Z_k^T$. Thus the computations are performed usually with storage which is lower rank, i.e., much lower than order of n^2.

Generalized Lyapunov equations have additional terms. In this talk, we concentrate on equations of the following form

$$A^TX + XA + \sum_{j=1}^{m} N_j X N_j^T + C^TC = 0,$$

Such equations arise for example in stochastic control.

In the present work, we propose a return to classical iterative methods, and consider instead stationary iterations. The classical theory of splittings applies here, and we present a new theorem on the convervegence when the linear system at each step is solved inexactly.

Several theoretical and computational issues are discussed so as to make the iteration efficient. Numerical experiments indicate that this method is competitive vis-à-vis the current state-of-the-art methods, both in terms of computational times and storage needs.

Balanced Truncation Model Reduction of Parametric Differential-Algebraic Systems

Jennifer Przybilla[1], *Matthias Voigt*[2]

[1] *Computational Methods in Systems and Control Theory, Max Planck Institute for Dynamics of Complex Technical Systems, Magdeburg, Germany*
E-mail: przybilla@mpi-magdeburg.mpg.de
[2] *Faculty of Mathematics and Computer Science, UniDistance Suisse, Brig, Switzerland*
E-mail: matthias.voigt@fernuni.ch

Abstract

In this talk we discuss a procedure to apply balanced truncation to parameter-dependent differential-algebraic systems. For this the solutions of multiple projected Lyapunov equations for different parameter values are required for the truncation procedure. As this process would lead to high computational costs if we perform it for a large number of parameters, we combine this approach with the reduced basis method that determines a reduced representation of the Lyapunov equation solutions within the desired parameter domain. Residual-based error estimators are then used to evaluate the quality of the approximations. After introducing the procedure for a general class of differential-algebraic systems we turn our focus to systems with a certain structure, for which the method can be applied particularly efficiently. We illustrate the effectiveness of our approach on several models from fluid dynamics and mechanics. The results are available in [1].

References

[1] J. Przybilla and M. Voigt. *Model reduction of parametric differential-algebraic systems by balanced truncation*, arXiv preprint arXiv:2108.08646, 2021.

MSI05. Realization formulas, rational inner functions, and real algebraic geometry

Ryan Tully-Doyle, James Pascoe, Kelly Bickel

Clark measures associated with RIFs
Linus Bergqvist, John Anderson, Kelly Bickel, Joe Cima, Alan Sola

A moment theoretic approach to estimate the cardinality of certain algebraic varieties
Raul Curto, Seonguk Yoo

Free Extreme points of free spectrahedrops and generalized free spectrahedra
Eric Evert

Packages of curves associated with the numerical range
Pamela Gorkin, Gregory Adams

Hankel forms over a free monoid
Michael Jury

Ranks of linear matrix pencils separate simultaneous similarity orbits
Igor klep, Harm Derksen, Visu Makam, Jurij Volčič

Spectrahedral Shadows and Completely Positive Maps on Real Closed Fields
Mario Kummer, Manuel Bodirsky, Andreas Thom

Realizations of rational inner functions in the full Fock space
Robert Martin

Projection Theorems in Free Semialgebraic Geometry
Tim Netzer, Tom Drescher, Andreas Thom

The wonders of the Cesaro matrix
William Ross

Nonnegative polynomials, sums of squares and sums of nonnegative circuit polynomials- a story of three convex cones
Moritz Schick, Mareike Dressler, Salma Kuhlmann

On the minimum number of Toeplitz factors of a matrix
Daniel Seco

Facial structure of matrix convex sets
Tea Štrekelj, Igor Klep

Contractive realizations of rational functions on polynomially defined domains and contractive determinantal representations of stable polynomials
Victor Vinnikov

Assistant Professor
Jurij Volcic, Igor Klep, Victor Magron, Jie Wang

Clark measures associated with RIFs

Linus Bergqvist[1]

[1] *Department of Mathematics, Stockholm University, Sweden*
E-mail: linus@math.su.se

Abstract

In this talk we introduce and discuss Clark measures associated with rational inner functions (RIFs) on the polydisk. In particular we discuss general two-variable RIFs, including those with singularities, and general d-variable rational inner functions with no singularities. We give precise descriptions of support sets and weights for such Clark measures in terms of level sets and partial derivatives of the associated RIF.

The talk is based on joint work with John T. Anderson, Kelly Bickel, Joseph A. Cima, and Alan A. Sola.

References

[1] John T. Anderson, Linus Bergqvist, Kelly Bickel, Joseph A. Cima, and Alan A. Sola *Clark Measures for Rational inner functions II: General bidegrees and high dimensions.* (preprint)

A moment theoretic approach to estimate the cardinality of certain algebraic varieties

Raúl E. Curto[1] *and Seonguk Yoo*[2]

[1] *Department of Mathematics, The University of Iowa, United States*
E-mail: raul-curto@uiowa.edu
[2] *Department of Mathematics Education and RINS, Gyeongsang National University, Republic of Korea*
E-mail: seyoo@gnu.ac.kr

Abstract

For $n \in \mathbb{N}$, we consider the algebraic variety $\mathcal{V}$ obtained by intersecting $n+1$ algebraic curves of degree n in $\mathbb{R}^2$, when the leading terms of the associated bivariate polynomials are all different. We provide a new proof, based on the Flat Extension Theorem from the theory of truncated moment problems, that the cardinality of $\mathcal{V}$ cannot exceed $\binom{n+1}{2}$. In some instances, this provides a slightly better estimate than the one given by Bézout's Theorem. Our main result contributes to the growing literature on the interplay between linear algebra, operator theory, and real algebraic geometry.

References

[1] R.E. Curto and S. Yoo, A moment theoretic estimate for the cardinality of certain algebraic varieties, *New York J. Math.* 28(2022) 357 – 366.

Free Extreme points of free spectrahedrops and generalized free spectrahedra

Eric Evert[1]

[1] *Northwestern University, USA*
E-mail: eric.evert@northwestern.edu

Abstract

Matrix convexity generalizes convexity to the dimension free setting and has connections to many mathematical and applied pursuits including operator theory, quantum information, noncommutative optimization, and linear control systems. In the setting of classical convex sets, extreme points are central objects which exhibit many important properties. For example, the well-known Minkowski theorem shows that any element of a closed bounded convex set can be expressed as a convex combination of extreme points. Extreme points are also of great interest in the dimension free setting of matrix convex sets; however, here the situation requires more nuance.

In the dimension free setting, there are many different types of extreme points. Of particular importance are free extreme points, a highly restricted type of extreme point that is closely connected to the dilation theoretic Arveson boundary. If free extreme points span a matrix convex set through matrix convex combinations, then they satisfy a strong notion of minimality in doing so. However, not all closed bounded matrix convex sets even have free extreme points. Thus, a major goal is to determine which matrix convex sets are spanned by their free extreme points.

Building on a recent work of J. W. Helton and the speaker [1] which shows that free spectrahedra, i.e., dimension free solution sets to linear matrix inequalities, are spanned by their free extreme points, this talk establishes two additional classes of matrix convex sets which are the matrix convex hull of their free extreme points. Namely, we show that closed bounded free spectrahedrops, i.e, closed bounded projections of free spectrahedra, are the span of their free extreme points. Furthermore, we show that if one considers linear operator inequalities that have compact operator defining tuples, then the resulting "generalized" free spectrahedra are spanned by their free extreme points.

References

[1] E. Evert, J.W. Helton. Arveson extreme points span free spectrahedra. *Math. Ann.*, 375, 629–653, 2019.

Packages of curves associated with the numerical range

Pamela Gorkin[1], *Gregory Adams*[2]

[1] *Department of Mathematics, Bucknell University, USA*
E-mail: pgorkin@bucknell.edu
[2] *Department of Mathematics, Bucknell University, USA*
E-mail: adams@bucknell.edu

Abstract

Gau and Wu studied compressions of the shift operator and showed that they have a Poncelet-like property; that is the numerical range of such a curve has the property that it is inscribed in a closed convex polygon that is itself inscribed in the unit circle. At the same time, Boris Mirman considered what he called UB-matrices, or matrices for which the addition of a (certain) column and row dilates it to a unitary matrix. Mirman also showed that the numerical range of such a matrix has the Poncelet property. These two results are the same, but they were developed quite differently. Later, complex function theory began to play an important role when it was shown that the vertices of the polygon can be obtained using Blaschke products. Thinking of the circumscribing polygon as joining lines between consecutive points, Mirman considered what he called "Poncelet packages of curves", or curves that are obtained from (possibly non-convex) polygons that result by drawing lines that skip over points. Our talk will begin by considering this history and major consequences of each viewpoint. We then discuss how a package of curves also makes sense for a general matrix.

References

[1] Daepp, Ulrich; Gorkin, Pamela; Voss, Karl Poncelet's theorem, Sendov's conjecture, and Blaschke products. J. Math. Anal. Appl. 365 (2010), no. 1, 93–102.

[2] Daepp, Ulrich; Gorkin, Pamela; Shaffer, Andrew; Voss, Karl Finding ellipses. What Blaschke products, Poncelet's theorem, and the numerical range know about each other. Carus Mathematical Monographs, 34. MAA Press, Providence, RI, 2018.

[3] Gau, Hwa-Long; Wu, Pei Yuan Numerical range and Poncelet property. Taiwanese J. Math. 7 (2003), no. 2, 173–193.

[4] Gau, Hwa-Long; Wu, Pei Yuan, Numerical range of a normal compression. Linear Multilinear Algebra 52 (2004), no. 3-4, 195–201.

[5] Gau, Hwa-Long; Wu, Pei Yuan, Numerical range of a normal compression. II. Linear Algebra Appl. 390 (2004), 121–136.

[6] Gau, Hwa-Long; Wu, Pei Yuan Numerical ranges and compressions of S_n-matrices. Oper. Matrices 7 (2013), no. 2, 465–476.

[7] Gau, Hwa-Long; Wu, Pei Yuan Numerical range and Poncelet property. Taiwanese J. Math. 7 (2003), no. 2, 173–193.

[8] B. Mirman, Numerical ranges and Poncelet curves. Linear Algebra Appl., 281, (1998). 59–85.

[9] B. Mirmanm UB-matrices and conditions for Poncelet polygon to be closed. Linear Algebra Appl., 360, (2003). 123–150.

[10] B. Mirman, V. Borovikov, L. Ladyzhensky and R. Vinograd, Numerical ranges, Poncelet curves, invariant measures. Linear Algebra Appl., 329, (2001). 61–75.

[11] Mirman, Boris Sufficient conditions for Poncelet polygons not to close. Amer. Math. Monthly 112 (2005), no. 4, 351–356.

Hankel forms over a free monoid

Michael T. Jury[1]

[1] *University of Florida, USA*

E-mail: mjury@ufl.edu

Abstract

We discuss analytic aspects (boundedness, etc.) of Hankel forms defined over a free monoid, from the point of view of nc function theory. In particular we investigate the success and failure of various ways of generalizing the Nehari theorem in this setting. The problem is motivated by applications to weighted finite automota (WFA).

Acknowledgements: Work (partially) supported by NSF grant DMS-2154494

Ranks of linear matrix pencils separate simultaneous similarity orbits

Harm Derksen[1], *Igor Klep*[2], *Visu Makam*[3], *Jurij Volčič*[4]

[1] *Department of Mathematics, Northeastern University, Boston, MA, USA*
E-mail: ha.derksen@northeastern.edu
[2] *Faculty of Mathematics and Physics, University of Ljubljana, Slovenia*
E-mail: igor.klep@fmf.uni-lj.si
[3] *Radix Trading Europe B. V., Amsterdam, Netherlands*
E-mail: visu@umich.edu
[3] *Department of Mathematics, Drexel University, Pennsylvania, USA*
E-mail: jurij.volcic@drexel.edu

Abstract

This talk will present a solution to the two-sided version and provide a counterexample to the general version of the 2003 conjecture by Hadwin and Larson [HL2003]. Consider evaluations of linear matrix pencils

$$L = T_0 + x_1 T_1 + \cdots + x_m T_m$$

on matrix tuples as

$$L(X_1, \ldots, X_m) = I \otimes T_0 + X_1 \otimes T_1 + \cdots + X_m \otimes T_m.$$

It will be shown that ranks of linear matrix pencils constitute a collection of separating invariants for simultaneous similarity of matrix tuples. That is, m-tuples A and B of $n \times n$ matrices are simultaneously similar if and only if

$$\operatorname{rk} L(A) = \operatorname{rk} L(B)$$

for all linear matrix pencils L of size mn. Time permitting, variants of this property will also be established for symplectic, orthogonal, unitary similarity, and for the left-right action of general linear groups. Furthermore, a polynomial time algorithm for orbit equivalence of matrix tuples under the left-right action of special linear groups will be deduced.

Acknowledgements: HD was supported by the National Science Foundation grants IIS-1837985 and DMS-2001460. IK was supported by the Slovenian Research Agency grants J1-2453, N1-0217, J1-3004 and P1-0222. VM was supported by the University of Melbourne and the National Science Foundation grant CCF-1900460. JV was supported by the National Science Foundation grant DMS-1954709 and the Slovenian Research Agency grant J1-3004.

References

[DKMV23] H. Derksen, I. Klep, V. Makam, J. Volčič: *Ranks of linear matrix pencils separate simultaneous similarity orbits*, Adv. Math. 415 (2023) 108888, 20pp.

[HL2003] D. Hadwin, D. R. Larson: *Completely rank-nonincreasing linear maps*, J. Funct. Anal. 199 (2003) 210–227.

Spectrahedral Shadows and Completely Positive Maps on Real Closed Fields

Manuel Bodirsky[1], *Mario Kummer*[1], *Andreas Thom*[1]

[1] *Fakultät Mathematik, Technische Universität Dresden, Germany*
E-mail:
manuel.bodirsky@tu-dresden.de
mario.kummer@tu-dresden.de
andreas.thom@tu-dresden.de

Abstract

Semidefinite programming is a generalization of linear programming whose feasible sets are called *spectrahedral shadows*. These are convex semialgebraic sets that (are the image under an affine linear map of a set that) can be described by symmetric linear matrix inequalities. [4] asked whether every convex semialgebraic set is a spectrahedral shadow. Later [3] conjectured that the answer to this question is in fact *yes*. This conjecture was recently disproved by [5]. Further counter-examples were subsequently given by [2] and [1]. However, the techniques used in these articles were essentially the same as the ones developed by [5]. In a joint work with Manuel Bodirsky and Andreas Thom we provide new techniques for proving that a certain semialgebraic set is not a spectrahedral shadow. We use these to prove that the set of all *copositive matrices* of size m, i.e. the cone of all symmetric $m \times m$ matrices A such that $x^t Ax \geq 0$ for all $x \in \mathbb{R}^m_{\geq 0}$, is not a spectrahedral shadow whenever $m \geq 5$.

Acknowledgements: Work (partially) supported by DFG grant 421473641.

References

[1] Bettiol, R.G., Kummer, M., and Mendes, R.A.E. (2021). Convex algebraic geometry of curvature operators. *SIAM J. Appl. Algebra Geom.*, 5(2), 200–228.

[2] Fawzi, H. (2021). The set of separable states has no finite semidefinite representation except in dimension 3×2. *Comm. Math. Phys.*, 386(3), 1319–1335.

[3] Helton, J.W. and Nie, J. (2009). Sufficient and necessary conditions for semidefinite representability of convex hulls and sets. *SIAM J. Optim.*, 20(2), 759–791.

[4] Nemirovski, A. (2007). Advances in convex optimization: conic programming. In *International Congress of Mathematicians. Vol. I*, 413–444. Eur. Math. Soc., Zürich.

[5] Scheiderer, C. (2018). Spectrahedral shadows. *SIAM J. Appl. Algebra Geom.*, 2(1), 26–44.

Realizations of rational inner functions in the full Fock space

Robert T.W. Martin[1]

[1] *Department of Mathematics, University of Manitoba, Canada*
E-mail: Robert.Martin@umanitoba.ca

Abstract

A bounded analytic function in the complex unit disk is *inner* if it has unimodular non-tangential boundary values (almost everywhere) on the unit circle (with respect to Lebesgue measure). Equivalently, an analytic function, h, in the disk, is inner if multiplication by h defines an isometry on the Hardy space of square–summable Taylor series in the complex unit disk.

A natural multivariate and non-commutative generalization of the Hardy space is then the full Fock space, or *free Hardy space* of square–summable power series in several non-commuting variables. We will completely characterize the minimal realization formulas of non-commutative rational inner multipliers of the free Hardy space and describe some applications.

References

[1] M. T. Jury, R. T. W. Martin and E. Shamovich, Non-commutative rational Clark measures. *Canad. J. Math.*, In Press. (2022).

[2] M. T. Jury and R. T. W. Martin, Sub-Hardy Hilbert spaces in the non-commutative unit row–ball. In *Function spaces, theory and applications*, *Fields Institute Proceddings*, In Press. (2023).

Projection Theorems in Free Semialgebraic Geometry

Tim Netzer[1]

[1] *Universität Innsbruck, Department of Mathematics, Austria*
E-mail: tim.netzer@uibk.ac.at

Abstract

An important result in real algebraic geometry is the projection theorem: every projection of a semialgebraic set is again semialgebraic. This theorem and some of its conclusions lie at the basis of many other results, for example the decidability of the theory of real closed fields, and almost all Positivstellensätze. We explain to which extend a projection theorem is possible for non-commutative (=free) semialgebraic sets. First we review and extend some results that count against a full free projection theorem. For example, it is undecidable whether a free statement holds for all matrices of at least one size. We then explain a weaker version of the projection theorem: projections along linear and separated variables yields a semi-algebraically parametrized free semi-algebraic set. This is joint work with Tom Drescher and Andreas Thom.

The wonders of the Cesàro matrix

William T. Ross[1]

[1] *Department of Mathematics and Statistics, University of Richmond, Virginia, USA*
E-mail: wross@richmond.edu

Abstract This talk will present be a survey, along with some new results, about the classical Cesàro matrix. Topics to be discussed are the norm, spectrum, commutant (known) along with the square roots and invariant subspaces (new).

Nonnegative polynomials, sums of squares and sums of nonnegative circuit polynomials - a story of three convex cones

Mareike Dressler [1], *Salma Kuhlmann*[2], *Moritz Schick*[3]

[1] *School of Mathematics and Statistics, University of New South Wales, Australia*
E-mail: m.dressler@unsw.edu.au
[2] *Department of Mathematics and Statistics, University of Konstanz, Germany*
E-mail: Salma.Kuhlmann@uni-konstanz.de
[3] *Department of Mathematics and Statistics, University of Konstanz, Germany*
E-mail: Moritz.Schick@uni-konstanz.de

Abstract

An n-variate homogeneous polynomial (or form) over $\mathbb{R}$ is positive semidefinite, if it takes nonnegative values on $\mathbb{R}^n$. The set of such forms of degree $2d$ is a convex cone denoted by $P_{n,2d}$. In general, checking membership in $P_{n,2d}$ is difficult, therefore one often considers membership in appropriate convex subcones. On the one hand, the sums of squares cone $\Sigma_{n,2d}$ has a long history in Mathematics, with results going back to Hilbert's seminal work in [3]. On the other hand, the sums of nonnegative circuits cone $C_{n,2d}$ is rather newly established, first formally defined in [4].

In this talk, we introduce all three cones and show how they can be set-theoretically separated from each other. Further, motivated by [2], we study the convex hull of $\Sigma_{n,2d} \cup C_{n,2d}$, i.e. the Minkowski sum $(\Sigma + C)_{n,2d} := \Sigma_{n,2d} + C_{n,2d}$. Due to a result in [1], $(\Sigma + C)_{n,2d}$ is a proper subcone of $P_{n,2d}$ if and only if $n \geq 3, 2d \geq 4$ and $(n, 2d) \neq (3, 4)$. For those $n, 2d$, we add explicit forms that separate $P_{n,2d}$ from $(\Sigma + C)_{n,2d}$ and $(\Sigma + C)_{n,2d}$ from $\Sigma_{n,2d} \cup C_{n,2d}$.

References

[1] G. Averkov. Optimal size of linear matrix inequalities in semidefinite approaches to polynomial optimization. *SIAM J. Appl. Algebra Geometry*, 3:128-151, 2019.

[2] M. Dressler. *Sums of nonnegative circuit polynomials: geometry and optimization.* PhD thesis, Goethe-Universität Frankfurt am Main, 2018.

[3] D. Hilbert. Über die Darstellung definiter Formen als Summe von Formenquadraten. *Mathematische Annalen,* 32:342-250, 1888.

[4] S. Iliman and T. de Wolff, Amoebas, nonnegative polynomials and sums of squares supported on circuits. *Res. Math. Sci.,* 3, 2016. 3:9.

On the minimum number of Toeplitz factors of a matrix

Daniel Seco[1]

[1] *Dpt. of Mathematical Analysis, University of La Laguna, Spain*
E-mail: dsecofor@ull.edu.es

Abstract

In an article by Ye and Lim [1], it is shown that every $n \times n$ matrix can be factorized as the product of at most $2n + 5$ Toeplitz matrices. It is also shown there that generic matrices have a better behavior ($\lfloor \frac{n}{2} \rfloor + 1$) and it is conjectured that the generic bound is actually always true. We show a counterexample to this conjecture.

Acknowledgements: Work supported by the Agencia Estatal de Investigación, Spain, through Ramón y Cajal grant RYC2021-034744-I; and through grant PID2019-106433GB-I00/AEI/10.13039/501100011033.

References

[1] K. Ye and L.-H. Lim. Every matrix is a product of Toeplitz matrices. *Found. Comput. Math.*, 16 (3) 577–598 (2016).

Facial structure of matrix convex sets

Igor Klep [1], *Tea Štrekelj*[2]

[1] *Department of Mathematics, Faculty of Mathematics and Physics, University of Ljubljana, Slovenia*
E-mail: igor.klep@fmf.uni-lj.si
[2] *Institute of Mathematics and Physics, Ljubljana, Slovenia*
E-mail: tea.strekelj@fmf.uni-lj.si

Abstract

In this talk we discuss the notions of exposed points and (exposed) faces in the matrix convex setting. Matrix exposed points in finite dimensions were first defined by Kriel in 2019. We show how this notion can be extended to matrix convex sets in infinite-dimensional vector spaces and present a connection between matrix exposed points and matrix extreme points: a matrix extreme point is ordinary exposed if and only if it is matrix exposed. This leads to a Krein-Milman type result for matrix exposed points that is due to Straszewicz-Klee in classical convexity: a compact matrix convex set is the closed matrix convex hull of its matrix exposed points. At the end we discuss matrix exposed points of spectrahedra as well as of matrix state spaces of C^*-algebras.

We introduce several notions of a fixed-level as well as a multicomponent matrix face and matrix exposed face to extend the concepts of a matrix extreme point and a matrix exposed point, respectively. Their properties resemble those of (exposed) faces in the classical sense and they give rise to the noncommutative counterpart of the classical theory connecting (archimedean) faces of compact convex sets and (archimedean) order ideals of the corresponding function systems.

Acknowledgements: Work partially supported by the ARRS young research program.

References

[1] I. Klep, T. Štrekelj, *Facial structure of matrix convex sets*, J. Funct. Anal. 283 (2022) 109601, 55pp.

Contractive realizations of rational functions on polynomially defined domains and contractive determinantal representations of stable polynomials

Victor Vinnikov[1]

[1] *Department of Mathematics, Ben Gurion University of the Negev, Beer-Sheva, Israel*
E-mail: vinnikov@math.bgu.ac.il

Abstract

I will discuss polynomials that are stable (i.e., have no zeroes) on a tubular domain in $\mathbb{C}^n$ and determinantal representations that certify their positivity, with a special attention to the poly-upper halfplane $\{(z_1, \ldots, z_n) \colon \Im z_j > 0,\ j = 1, \ldots, n\}$ (and more generally tubular domains over a homogeneous cone in $\mathbb{R}^n$). One way to construct such determinantal representations is by using contractive realizations of rational functions on the domain (or on a bounded realization thereof) that can be constructed using appropriate hermitian sum of squares decompositions (similar to the Positivstellensätze in real algebraic geometry) and the "lurking contraction" argument of multivariable operator theory.

This talk is based on joint work with Hugo Woerdeman.

Positivity of state polynomials with applications

Igor Klep[1], *Victor Magron*[2], *Jurij Volčič*[3], *Jie Wang*[4]

[1] *Faculty of Mathematics and Physics, University of Ljubljana, Slovenia*
E-mail: igor.klep@fmf.uni-lj.si
[2] *LAAS-CNRS & Institute of Mathematics from Toulouse, France*
E-mail: vmagron@laas.fr
[3] *Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China*
E-mail: jurij.volcic@drexel.edu
[4] *Department of Mathematics, Drexel University, Pennsylvania*
E-mail: wangjie212@amss.ac.cn

Abstract

This talk addresses state polynomials, i.e., polynomials in noncommuting variables and formal states of their products. The arising theory mirrors and employs the techniques from both the commutative and the noncommutative aspects of these objects. A state analog of Artin's solution to Hilbert's 17th problem is proved showing that state polynomials, positive over all matrices and matricial states, are sums of squares with denominators. Further, archimedean Positivstellensätze in the spirit of Putinar and Helton-McCullough are presented leading to a hierarchy of semidefinite relaxations converging monotonically to the optimum of a state polynomial subject to state constraints. This hierarchy can be seen as a state analog of the Lasserre hierarchy for optimization of polynomials, and the Navascués-Pironio-Acín scheme for optimization of noncommutative polynomials. The motivation behind this theory arises from the study of polynomial Bell inequalities and their quantum violations in quantum networks.

References

[1] I. Klep, V. Macron, J. Volčič, J. Wang: *State polynomials: positivity, optimization and nonlinear Bell inequalities*, arXiv 2301.12513

Madrid June 12-16

ILAS 2023

25th Conference of the International Linear Algebra Society

June 12th - 16th, 2023

Organizers and sponsors

International Linear Algebra Society

Montes

Politécnica — Universidad Politécnica de Madrid

Universidad de Alcalá — Departamento de Física y Matemáticas

Universidad Complutense Madrid

Saint Louis University Madrid

UNED

uc3m

Universidad de Alcalá

Instituto de Matemática Interdisciplinar

SEMA

siam Society for Industrial and Applied Mathematics

Taylor & Francis Group an informa business

Elsevier

Madrid Convention Bureau

Comunidad de Madrid

www.ilas2023.es